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Some empirics on economic growth under heterogeneous technology

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Abstract

A new econometric approach to testing for economic growth convergence is overviewed. The method is applicable to panel data, involves a simple regression based one-sided t-test, and can be used to form a clustering algorithm to assess the existence of growth convergence clubs. The approach allows for heterogeneous technology, utilizes some new asymptotic theory for nonlinear dynamic factor models, and is easy to implement. Some background growth theory is given which shows the form of augmented Solow regression (ASR) equations in the presence of heterogeneous technology and explains sources of potential misspecification that can arise in conventional formulations of ASR equations that are used to analyze growth convergence and growth determinants. A short empirical application is given illustrating some aspects of the methodology involving technological heterogeneity and learning in growth patterns for selected groups of countries.

JEL classification: 040; C32; C33

Keywords: Convergence parameter; Conditional convergence; Determinants of growth; Economic growth; Growth convergence; Heterogeneity; Learning; Neoclassical economics; Speed of learning; Transition curve; Transitional divergence

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1. Introduction

An important task in the economic growth literature is to explain the substantial heterogeneity in income performance across countries. A number of studies have attempted to model this heterogeneity empirically, running cross section, time series and panel regressions with various data sets over the past two decades. Most of these studies assume homogeneity in technological progress, which is convenient in economic growth theory models but seems unrealistic in the context of many empirical applications. For examples, see Barro and Sala-i-Martin (1992), Mankiw et al. (1992) among many others. Much of this theoretical and empirical literature has been expertly overviewed in Durlauf et al. (2005) and will not be reviewed here. The purpose of the present paper is to examine the impact of heterogeneity in technical progress on growth and econometric tests of convergence, transition and the determinants of growth.

Under a Cobb–Douglas production function, the per capita real income y_{it} for country i at time t can be decomposed so that

$$\log y_{it} = \log \tilde{y}_{it} + \log A_{it},$$

involving technology, A_{it}, and effective per capita real income, $\tilde{y}_{it}$, which is a function of real effective capital, $\tilde{k}_{it}$. When technology is homogenous across countries, so that $\log A_{it} = \log A_{jt}$ for $i \neq j$, the cross sectional income differential is explained only by differences in relative real effective income. Homogenous technology across countries is, of course, a rather strong and unrealistic assumption and it is of interest to relax the condition in models that are explicitly designed for empirical use.

Once the condition is relaxed, the possibility of heterogeneous technology in growth raises some immediate issues. One issue concerns the validity of conventional growth convergence tests and another relates to the appropriate formulation of the cross sectional regression equations that are conventionally used to investigate growth determinants – the so-called 'augmented Solow regressions' (ASRs). Phillips and Sul (2006a) address the first issue, showing that, under technological heterogeneity, neither standard panel unit root tests (using relative log income differentials) nor conventional ASRs are appropriate vehicles for testing for growth convergence. Another paper (Phillips and Sul, 2006b) deals with the second issue, showing that commonly used formulations of ASR equations are misspecified and inappropriate for exploring growth determinants.

The present paper overviews some of the results in the aforementioned papers and provides some new empirical illustrations involving technological heterogeneity and learning. The plan of the paper is as follows. The next section explains the sources of misspecification in conventional augmented Solow regression and the consequent difficulties in using these regressions to test growth convergence and find growth determinants. Section 3 outlines a new empirical approach to studying growth convergence with heterogeneous technology and discusses a new clustering algorithm which can be applied to find growth convergence clubs. Section 4 illustrates a new empirical analysis of growth determinants, exploring the relationship between learning and the adoption of advanced technology. Section 5 concludes.

2. Homogeneous vs. heterogenous technology

The production function in the neoclassical theory of growth with labor augmented technological progress can be written as

$$\begin{aligned}&\tilde{y}=f(\tilde{k})\\&\tilde{y}=Y/LHA,\quad \tilde{k}=K/LHA,\quad y=\tilde{y}HA=\tilde{y}A,\end{aligned}\tag{1}$$

where Y is total output, L is the quantity of labor input, H is the stock of human capital, A is the state of technology, K is physical capital, and $\tilde{y}$ is output per effective labor unit. In the last part of (1), H is normalized to unity by broadly defining technology A to encompass the effects of human capital.

Taking logarithms of (1) and adding subscripts i and t for individual country and year, respectively, we re-write the neoclassical production function as

$$\log y_{it}=\log A_{it}+\log\tilde{y}_{it},\quad i=1,\ldots,N;\ t=1,\ldots,T\tag{2}$$

with $\log\tilde{y}_{it}=\log f(\tilde{k}_{it})$. Phillips and Sul (2006a,b) allowed for heterogenous technological progress across countries using a nonlinear factor structure of the form

$$\log A_{it}=\log A_{i0}+\gamma_{it}\log\psi_t\quad\text{and}\quad\log\psi_t=\xi t\tag{3}$$

so that

$$\log A_{it}=\log A_{i0}+x_{it}t.\tag{4}$$

where $x_{it}=\gamma_{it}\xi$. The common factor $\log\psi_t=\xi t$ represents publically available advanced technology and is assumed to follow a linear trend. Hence, when $x_{it}=x$ for all i, technical progress is captured by the usual common exponential path; when $x_{it}\to x_i$ as $t\to\infty$, technical progress follows individual exponential paths asymptotically for large t; and, when $x_{it}\to x_i=x$ as $t\to\infty$ for all i, a common path applies asymptotically. In the general formulation (3) and (4), the time varying idiosyncratic component γ_{it} may be interpreted as representing the economic distance between the individual technology of country i ($\log A_{it}$) and the advanced common technology.

With exogenous but heterogenous saving and population growth rates and under Cobb–Douglas production (with share parameter α), the dynamic evolution of the capital stock is determined according to the differential equation

$$\dot{k}_{it}=s_ik_{it}^{\alpha}-(n_i+x_{it}+\delta)k_{it}.\tag{5}$$

where s_i is the saving rate, α is the capital share, n_i is the population growth rate, and δ is the discount rate. Correspondingly, we find that log per capita real income, $\log y_{it}$, evolves under cross section heterogenous technology progress according to the dynamic path (or DGP)

$$\log y_{it}=\log\tilde{y}_i^*+\left[\log\tilde{y}_{i0}-\log\tilde{y}_i^*\right]\mathrm{e}^{-\beta_{it}t}+x_{it}t,\tag{6}$$

where

$$\beta_{it}=\beta_i-\frac{1}{t}\log\left\{1-d_{i1}\int_0^t\mathrm{e}^{\beta_im}(x_{im}-x)\,\mathrm{d}m\right\},\tag{7}$$

$d_{i1}=1/(\log k_{i0}-\log k_i^*)$, $\beta_i=(1-\alpha)\ (n_i+x_i+\delta)$, x_i is the technological growth rate in the steady state, and y_i^* and k_i^* (respectively, y_{i0} and k_{i0}) are the steady state (initial) levels of per capita real effective income and capital.[1]

[1] As shown in Phillips and Sul (2006b), under Cobb–Douglas production, $k_i^*=(\frac{s_i}{n_i+x_i+\delta})^{1/(1-\alpha)}$. Derivations of (6) and (7) are given in the same reference.

By direct calculation, the average growth rate between time q and $t+q$ under heterogenous technology progress is found to satisfy the equation

$$\frac{\log y_{it+q} - \log y_{iq}}{t} = -b_{it} \log \tilde{y}_i^* + b_{it} \log y_{iq} - \left(\frac{1}{t} + b_{it}\right) \log A_{iq} + \frac{1}{t} \log A_{it+q}, \tag{8}$$

where

$$b_{it} = -\left(\frac{1 - \exp\left(-\beta_{it}^+ t\right)}{t}\right) < 0 \quad \text{if } \beta_{it}^+ > 0, \tag{9}$$

with $\beta_{it}^+ = \beta_{it+q}(t+q)/t - \beta_{iq} q/t$.

From (8), a generalized form for an augmented Solow regression (ASR) that applies under heterogenous technology can be deduced, leading to the empirical regression equation

$$t^{-1}(\log y_{it+q} - \log y_{iq}) = c_0 + b_1 \log y_{iq} + b_2 z_{iq} + b_3 z_{it+q} + e_i, \tag{10}$$

with covariates z_{iq} and z_{it+q} proxing the corresponding technology variables in (8). Conventional Solow regression equations may be derived as particular cases of (10). The model may be estimated using cross section observations $i = 1, \ldots, N$ and time series data over $t = 1, \ldots, T$.

Table 1 outlines the major differences among three models that have been used in practical work: the homogeneous technology, saving and population growth model studied by Barro (1991), which we refer to as Hom 1; the homogeneous technology with heterogenous saving and population growth model suggested in Mankiw et al. (1992, MRW), referred to as Hom 2; and the heterogenous technology, saving and population growth model developed in Phillips and Sul (2006a,b), referred to as Het. These models differ in terms of their assumptions concerning technology, and these differences become manifest in the empirical regression (10) in terms of the implied coefficients. The following is a brief discussion of the implications of these differences for the three models.

Table 1
Pitfalls in augmented Solow regression (ASR) under homogeneous and heterogenous technology

	Hom 1 (Barro)	Hom 2 (MRW)	Het (PS)
DGP: $\log y_{it} = \log \tilde{y}_i^* + [\log \tilde{y}_{i0} - \log \tilde{y}_i^*]\mathrm{e}^{-\beta_{it}t} + x_{it}t$			
(a) Assumptions	$A_i = A$, $s_i = s$, $n_i = n$	$A_i = A$	NA
(b) Restriction on the implied DGP	$\tilde{y}_i^* = \tilde{y}^*$, $\beta_{it} = \beta$, $x_{it} = x$	$\beta_{it} = \beta_i$, $x_{it} = x$	NA
ASR: $t^{-1}(\log y_{it+q} - \log y_{iq}) = c_0 + b_1 \log y_{iq} + b_2 z_{iq} + b_3 z_{it+q} + e_i$			
(c) Restriction on the ASR	$b_2 = b_3 = 0$	$b_3 = 0$,	
		$z_{iq} = \frac{1}{T}\sum z_{it}$	NA
		$z_{iq} = f(\log \tilde{y}_i^*)$	$z_{it} = f(\log \tilde{y}_i^*, \log A_{it})$
(d) Condition for convergence	$\beta > 0$; $b_1 < 0$	$\beta_i > 0$; $b_1 < 0$	$\log A_{it} \to \log A_t$
(e) Consistency properties	$E_N \hat{b}_1 = b_1 + \mathrm{O}(\frac{1}{T})$		$E_N \hat{b}_2 = \mathrm{O}(\frac{1}{T})$
		$E_N \hat{b}_2 = \mathrm{O}(\frac{1}{T})$	$E_N \hat{b}_3 = b_3 + \mathrm{O}(\frac{1}{T})$
		$E_{N,T} \hat{b}_2 = 0$	$E_{N,T} \hat{b}_2 = 0$
			$E_{N,T} \hat{b}_3 = 0$

2.1. Model assumptions and restrictions

Hom 1 assumes homogeneous technology, saving, and population growth. Hence the initial income difference can be explained only by the initial real effective income difference, $\log \tilde{y}_{i0}$, and the initial measure of technological progress, $\log A_{i0}$. Since all economies have the same steady state, real effective income converges. Moreover, the convergence speed, β_{it}, is also homogenous. Hom 2 relaxes the homogeneity restriction on the saving and population growth rate parameters in Hom 1. When savings and population growth rates may differ across economies, so may real effective income, including the steady state path. More importantly, the speed of convergence parameter β_{it} is also heterogenous, and is a function of the population growth rate and the technical progress rate, as seen in (7).

2.2. The ASR specification and interpretation of the covariates

Hom 1 does not use control variables, so the true values of b_2 and b_3 are zero in the ASR specification (10) for this model. Hom 2 does not include the last observation z_{it+q}, which is a control or proxy variable for $\log \tilde{y}_i^*$ and $\log A_{it+q}$. Moreover, MRW suggests the use of the time series average of z_{it} to proxy a steady state value, so that in the place of z_{iq}, MRW use $z_{i\cdot} = T^{-1}\sum_{t=1}^{T} z_{it}$. Note also that Barro and Sala-i-Martin (1992) use the initial observation for total years of schooling.

The Het specification does not require any of these restrictions. However, the economic interpretation of the z_{it} variables is somewhat different in this model. In Hom 2, z_{iq} is a proxy variable for $\log \tilde{y}_i^*$ only, whereas in the Het model, the variable z_{it} serves both as a proxy for $\log \tilde{y}_i^*$ and $\log A_{iq}$ (through z_{iq}) and as a proxy for $\log A_{it+q}$ (through z_{it+q}). Hence, under heterogeneous technology progress, conventional specifications of the ASR empirical regression are misspecified due to the error endogeneity arising from the omission of the variable z_{it+q} in the regression.

2.3. The convergence condition

In Hom 1, convergence requires $\beta > 0$ or $b_1 = -t^{-1}(1 - \mathrm{e}^{-\beta t}) < 0$. This condition is known as the 'beta'-convergence. If the growth path is concave, then an initially poor country will grow faster under this condition than an initially rich county. The non-augmented Solow regression, under the restrictions $b_2 = b_3 = 0$, is used to test $b_1 < 0$ and thereby tests the concavity of $\log y_{it}$ under these side restrictions.

The convergence condition for Hom 2 is conditional on $\ln \tilde{y}_i^*$, and the level of log per capita real income may not converge even though the growth rate converges. So the negative sign in the regression coefficient $\hat{b}_1$ does not necessarily imply level convergence. Also, when $x_{it} = x$, the relative income differential between economies, $(\log y_{it} - \log y_{jt})$, is explained only by the initial real effective per capita income levels.

In the Het model, a negative sign for $\hat{b}_1$ does not imply convergence because of the possibility of heterogenous technology progress. When $x_{it} \neq x$ during transition periods, the relative technological differential between x_{it} and x_{jt} (and the historical trajectory of this differential) contributes to the income difference. Note that when $\beta_{it} > 0$, $\mathrm{e}^{-\beta_{it} t} \to 0$ as $t \to \infty$, and if the convergence rate of this exponential term in (6) is fast relative to the convergence rate of x_{it}, then the main long run determinant of the relative income difference is the difference in the rates of technological accumulation. In this case, the relative income difference between two economies may be well explained by the relative difference in technology accumulation. For large t, $\log y_{it}$ eventually follows a long run path determined by the term $x_{it}t$ in (6). Hence, analyzing the dynamics of $\log A_{it}$ and the past history of x_{it} are key elements in understanding transitional income dynamics. In particular, the convergence condition for the Het model is $\lim_{t\to\infty} \log A_{it} = \lim_{t\to\infty} \log A_{jt}$ for $i \neq j$. It follows that the ASR regression model is not a good vehicle for testing growth convergence when technology is heterogenous across countries and over time.

2.4. Consistency

Under the given conditions for each model, some large cross section ($N \to \infty$) properties are derived in Phillips and Sul (2006b). First, for all models, none of the coefficients are consistently estimated for fixed T. Denote $\text{plim}_{N\to\infty}$ as E_N, and $\text{plim}_{N,T\to\infty}$ as $E_{N,T}$. The bias in the coefficient estimate $\hat{b}_1$ arises from correlation between the initial income level, $\log y_{iq}$, and the regression error. The misspecification occurs because initial technology is heterogenous, so that $\log A_{iq} = \log A_q + \epsilon_i$, and ϵ_i is absorbed into the regression error. Since initial technology is positively correlated with initial income, the estimate $\hat{b}_1$ is biased. For model Hom 2 and model Het, the true DGP based on (8) and (10) consists of product forms involving observable proxy variables and coefficients b_{it}, which are not necessarily homogeneous across i or t. Hence, in the regression in (10), the fitted regression coefficients can be thought of as cross sectional averages, so that the regression error absorbs deviations from the mean. Phillips and Sul (2006b) show that in the regression model Hom 2, $\text{plim}_{N\to\infty}\hat{b}_2 \neq b_2$ even if the conditions for the Hom 2 model hold. Instead, $\text{plim}_{N\to\infty}\hat{b}_2$ depends on the ratio of the third central moment to the second central moment of the z_{iq} covariates. The source of the inconsistency is the fact that $\text{E}(b_i z_i) \neq 0$ and the sign of the bias in the fitted coefficient $\hat{b}_2$ is dependent on the distribution of z_{iq}. Hence, the ASR regression under the Hom 2 conditions is misspecified and involves inconsistencies. Relaxing the homogeneity condition for technological progress does not resolve this problem. For the Het model, the true sign becomes $b_2 < 0$, but $b_3 > 0$. Conventional ASR regressions do not include the z_{it+q} covariate term, and these regressions therefore suffer omitted variable bias under heterogeneous technology. The inclusion of the last observation z_{it+q} may reduce this bias in the regression, but the regression coefficient $\hat{b}_2$ is still affected by the endogeneity bias induced by z_{iq} and the fact that $b_2 = \frac{1}{t} + b_{it}$ from (9).

These findings indicate that there are difficulties in the empirical implementation of ASR regressions both for testing growth convergence and in searching for growth determinants. The findings apply whether technological progress is heterogenous or homogeneous.

3. Some new empirics on growth convergence

Phillips and Sul (2003, 2006a,c) show how log per capita real income can be reformulated in terms of the time varying common factor representation

$$\log y_{it} = b_{it}\mu_t, \tag{11}$$

where b_{it} measures the share of the common trend μ_t experienced by economy i. The coefficient b_{it} therefore captures the individual transition path of economy i as it moves in relation to global technology or a common growth path that is determined by μ_t. During transition, b_{it} depends on the speed of convergence parameter β_{it}, the rate of technical progress parameter, x_{it}, and the initial technical endowment and steady state levels.

By adapting (6) to a factor representation and assuming a common advanced global technology factor $\mu_t = \xi t$, we can find an explicit expression for b_{it} as follows:

$$\log y_{it} = a_{it} + x_{it}t, \quad a_{it} = \log \tilde{y}_i^* + \left[\log \tilde{y}_{i0} - \log \tilde{y}_i^*\right] \mathrm{e}^{-\beta_{it}t} = \left(\frac{a_{it}}{\xi t} + \gamma_{it}\right)\xi t = b_{it}\mu_t,$$

with $\gamma_{it} = x_{it}/\xi$. Hence, for large t, we have

$$b_{it} = \frac{a_{it}}{\xi t} + \gamma_{it} = \gamma_{it} + \mathrm{O}(t^{-1}) \simeq \gamma_{it}. \tag{12}$$

The time varying parameter γ_{it} effectively measures the ratio of individual technology, $\log A_{it} = \log A_{i0} + x_{it}t$, to advanced common technology $\mu_t = \xi t$. More complex forms of advanced technological progress may be considered in a similar way. The loading coefficient, b_{it}, retains a similar interpretation and relationship with γ_{it} under such complications provided $a_{it} = o(\mu_t)$.

Relative income differentials can then be written as

$$\log y_{it} - \log y_{jt} = \left(b_{it} - b_{jt}\right)\mu_t,$$

so that in the long run these income differences are explained only by technology differences between the two countries. Growth convergence therefore requires the following condition:

$$\lim_{t\to\infty} b_{it} = b, \tag{13}$$

or equivalently,

$$\lim_{t\to\infty} \frac{\log y_{it}}{\log y_{jt}} = 1. \tag{14}$$

Note that condition (14) does not imply overall level convergence of the $\log y_{it}$ over i. For example, suppose

$$b_{it} = b + c_i t^{-\alpha}. \tag{15}$$

Then b_{it} converges to b whenever $\alpha > 0$ and condition (14) is satisfied. However, the relative income difference has the form

$$\log y_{it} - \log y_{jt} = (c_i - c_j)\xi t^{1-\alpha}. \tag{16}$$

Now if $\alpha = 1$, $\lim_{k\to\infty}(\log y_{it+k} - \log y_{jt+k}) = (c_i - c_j)\xi$ and $\log y_{it+k} - \log y_{jt+k}$ diverges when $\alpha < 1$. Phillips and Sul (2006c) therefore call condition (14) ‘relative’ convergence. For large t, relative convergence with b_{it} of the form (15) with $\alpha > 0$ implies that the relative growth rate differential tends to zero, as is apparent from (16) upon differentiation.

3.1. The relative transition curve

The estimation of b_{it} is impossible without imposing some restrictions on (11) since the number of unknowns in the model exceeds the number of observations. Accordingly, Phillips and Sul (2006a,c) suggest a modeling approach based on the following relative measure

$$h_{it} = \frac{\log y_{it}}{N^{-1}\sum_{i=1}^{N}\log y_{it}} = \frac{b_{it}}{N^{-1}\sum_{i=1}^{N}\log b_{it}}, \tag{17}$$

which eliminates the common growth component by scaling and measures the transition element b_{it} for economy i relative to the cross section average. Over time, the variable h_{it} traces out an individual trajectory for each i relative to the average, so we call h_{it} the 'transition path'. At the same time, h_{it} measures economy i's relative departure from the common steady state growth path μ_t. Thus, any divergences from μ_t are reflected in the transition paths h_{it}. While many paths are possible, a case of particular interest and empirical importance occurs when an economy slips behind in the growth tables and diverges from others in particular group. We may then use the transition path to measure the extent of the divergent behavior and to assess whether or not the divergence is transient.

When there is a common (limiting) transition behavior across economies, we have $h_{it} = h_t$ across i; and when there is ultimate growth convergence we have

$$h_{it} \to 1, \quad \text{for all } i, \text{ as } t \to \infty. \tag{18}$$

This framework for studying growth convergence admits the existence of an empirical family of relative transitions, where the curves traced out by the h_{it} may differ across i in the short run, while allowing for ultimate convergence when (18) holds in the long run. This apparatus turns out to be convenient for studying a number of issues, including growth convergence under heterogeneous technologies, transition behavior of various types (including transitional divergence), and the determinants of growth.

Sometimes, it is useful to study transition curves in selected subgroups of the cross section population of individuals. There are several ways to modify the transition curve so that appropriate benchmarks are used. Here we illustrate one way of comparing transitions in two different sets of economies. The empirical example is given in Fig. 8 of Phillips and Sul (2006a), showing that Latin American countries in general have not started to catch up with OECD countries. To formalize this comparison, we can consider a panel of countries which consist of two subgroups, involving the OECD (G_1) nations and the Latin American countries (G_2). We may be interested in the overall economic performance of the Latin American countries against that of the OECD. In this case, we can calculate the relative transition curve for each Latin American country j as

$$\eta_{jt} = \left[\frac{\log y_{jt}}{(G_1 + G_2)^{-1}\sum_{i\in G_1\cup G_2}\log y_{it}}\right] \quad \text{for } j \in G_2.$$

Next, if can compute an average measure of relative economic performance for the Latin American countries by taking the cross sectional average of the η_{jt} as

$$\eta_{G_2,t} = \frac{1}{G_2}\sum_{j\in G_2}^{G_2}\left[\frac{\log y_{jt}}{(G_1 + G_2)^{-1}\sum_{i\in G_1\cup G_2}\log y_{it}}\right],$$

where the OECD nations serve as the numeraire economy. As another example, for large distinctive economies such as China and India, we may be interested in the economic performance of these individual countries compared with the OECD. In this case, the second group G_2 is a singleton and we may compute $\eta_{\text{China},t}$, and $\eta_{\text{India},t}$ as in $\eta_{G_2,t}$ above. The relative economic performance of China and India was compared with that of the OECD in Fig. 8 in Phillips and Sul (2006a) by using this type of modification.

A further example is as follows. Suppose that we are interested in comparisons between each of the Latin American countries and the specific economy of Mexico. In this case, the countries of interest are the other Latin American countries and the numeraire is the singleton Mexico. So, the size of G_1 is one and the other Latin American countries are allocated to G_2. Then we calculate $\eta_{G_2,t}$. Similarly, we may set Spain to be the numeraire country and re-calculate $\eta_{G_2,t}$. These two relative measures are plotted in Fig. 1. The scale of each graph differs and is shown on the right and left axes. When the other Latin American countries are compared with Spain, we notice from the figure that the relative transition curve started to diverge (i.e. move away from unity) around 1975. Correspondingly, relative to Spain, the economic performance of the other Latin American has been deteriorating since 1975. Meanwhile, as the figure also shows, the relative economic performance of the other Latin American countries to Mexico is U-shaped. So, after 1987, the economic performance of the other Latin American countries began to catch up with Mexico.

3.2. Convergence testing

Phillips and Sul (2006c) developed a new convergence test and a panel data clustering algorithm based on the time varying factor presentation (11). In that framework, the null hypothesis is formulated as

$$H_0 : \text{Convergence for all } i, \text{ vs } H_A : \text{No convergence for some } i. \tag{19}$$

An explicit form of the null hypothesis used in Phillips and Sul (2006c) is given as follows

$$H_0 : b_{it} = b + \psi_{it}, \quad \psi_{it} = N(0, \sigma_{it}^2 \log t), \quad \sigma_{it}^2 = \sigma_i^2 t^{-2\alpha}. \tag{20}$$

According to this formulation, the coefficient b_{it} may not be equal to b at time t, but its variance is shrinking at the rate $\mathrm{O}(t^{-2\alpha} \log t)$ as $t \to \infty$. This parametric structure leads

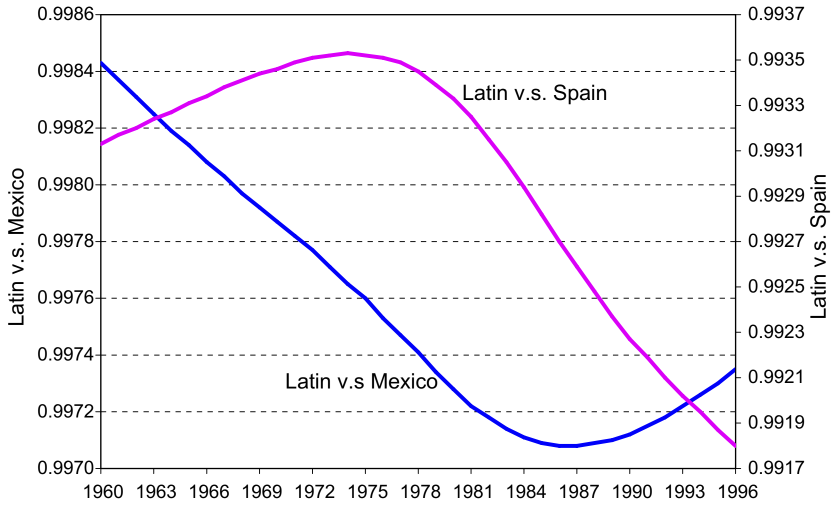

Fig. 1. Two relative measures of transition for Latin American countries.

to a direct mechanism for testing the null hypothesis in (11) by running the following simple least squares regression

$$\log\left(\frac{H_1}{H_t}\right) - 2\log\log t = a + b\log t + u_t, \tag{21}$$

where

$$H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{it} - 1)^2, \tag{22}$$

and h_{it} is defined in (17). The regression equation (21) is called a $\log t$ regression because of the $\log t$ regressor. The presence of the log–$\log t$ term on the left side of the equation arises because of the $\log t$ component in the variance (20) and is helpful in assuring good power properties of the test. The regression model (21) and its asymptotic properties are fully explored in Phillips and Sul (2006c), where it is shown how it can be used in the empirical analysis of convergence and as a clustering algorithm. The technique has the advantage of great simplicity in its practical implementation.

In the context of the regression model (21), the null and alternative hypotheses in (19) can be written as

$$H_0 : b \geqslant 0, \quad H_A : b < 0.$$

The null weak inequality null hypothesis H_0 can be tested by using a conventional one-sided t-test constructed with a heteroskedasticity and autocorrelation consistent (HAC) estimate from the residuals $\hat{u}_t$ in this regression. The null hypothesis H_0 implies relative convergence rather than the absolute convergence. A test for absolute convergence is also possible, in which case the null hypothesis must be changed to $H_0 : b \geqslant 2$, which applies when the unknown common factor follows either a random walk with a drift or contains a linear trend.

Rejection of the null hypothesis H_0 does not rule out the possibility of club convergence. In fact, the regression t-test in (21) may be used as the basis of an algorithm for assessing club convergence and clustering. In particular, Phillips and Sul (2006c) propose a step by step procedure for evaluating evidence in support of panel data clustering. The following is a brief outline of the steps involved and their motivation. Full details, simulations and several empirical illustrations are given in the source article.

1. (Last income ordering): Order the individuals in the panel according to the last observation. This ordering forms the first stage and prepares the panel for a cluster analysis.
2. (Core group formation): The second step is to form a primary cluster of individuals that comprise a core convergence subgroup against which the other individuals may be compared. The approach is to run a sequence of $\log t$ regressions and calculate the convergence test statistic $t_k = t(G_k)$ for subgroups based on the k highest individuals (for some $N > k \geqslant 2$) in the panel from the ordering obtained in step 1. The core group size k^* is selected by maximizing t_k over k according to the criterion

$$k^* = \arg\max_k\{t_k\} \quad \text{subject to } \min\{t_k\} > -1.65. \tag{23}$$

The maximum criterion is designed to locate a primary cluster with a high degree of confidence. This core convergence subgroup is denoted G_{k^*}. If the condition is too severe and no group is found, the step can be re-run with a different collection of primary individuals. Details are given in Phillips and Sul (2006c).

3. (Sieve individuals for club membership): The third step, as the name suggests, sifts through individuals one at time to check for possible membership of the primary cluster. This check is conducted using a $\log t$ regression. Suppose $G_{k^*}^c$ is the complementary set to the core group G_{k^*}. Adding one individual in $G_{k^*}^c$ at a time to the k^* core members of G_{k^*}, the $\log t$ test is run and the individual included in the convergence club if $\hat{t} > 0$, using the positive sign test to assure a high degree of confidence in the inequality null.
4. (Recursion and stopping): The final step searches the remaining individuals in the panel for subgroup clusters by repeating steps 2 and 3 in the above algorithm. When no other clusters are found in this process, the remaining individuals are assumed to display divergent behavior.

   Phillips and Sul (2006a) gave several empirical illustrations of this clustering methodology. For instance, in an application to cross country growth, the algorithm located three convergent clubs and one divergent group among 88 countries in the Penn World Tables in terms of real per capita GDP over the period 1960–1996.

4. Some new empirics on growth determinants

Empirical applications involving regression equations such as (8) require some proxy variables to represent technology. Phillips and Sul (2006b) used a decomposition of log per capita real income into components involving technology and real effective per capita income in order to obtain a suitable proxy. Positing a capital share in GDP of 1/3, an approximate version of log technology may be constructed as

$$\log \hat{A}_{it} = \log y_{it} - \frac{1}{3} \log \hat{k}_{it}, \tag{24}$$

where the approximate capital ratio is calculated as

$$\log \hat{k}_{it} = \log \left(\frac{\hat{K}_{it}}{L_{it}} \right). \tag{25}$$

Following the conventional perpetual inventory method, we have

$$K_{it} = I_{it} + (1 - \delta) K_{it-1} \quad \text{or} \quad K_{it} = (1 - B + \delta B)^{-1} I_{it},$$

where B is the backshift operator and I_{it} is investment. By letting $\hat{K}_{i0} = I_{i0}$, we obtain $\hat{K}_{it} = (1 - B + \delta B)^{-1} I_{it}$, which may be used as the generating mechanism.

Here we suggest an alternative way of approximating technology. From (12), it appears that the transition parameter may itself be a good proxy for relative technology as $t \to \infty$. In particular,

$$h_{it} = \frac{b_{it}}{N^{-1} \sum_{i=1}^N b_{it}} \simeq \frac{\gamma_{it}}{N^{-1} \sum_{i=1}^N \gamma_{it}} = \frac{\log A_{it}}{N^{-1} \sum_{i=1}^N \log A_{it}}. \tag{26}$$

So the transition curve may be regarded as an alternative proxy for the time path of relative technology. This measure is sufficient for many empirical applications and will be used in the illustration that follows.

Phillips and Sul (2006a) plotted relative transition curves for China, India, the Asian Dragons, the NIEs, the Latin American countries and the Sub-Saharan African countries against the benchmark of the OECD countries. These curves can be thought as proxes for relative technological progress. In related work, Phillips and Sul (2006b) showed the importance of human capital in economic performance by the use of approximate technology as measured in (24).

Here we show some empirical relationships between the relative transition curves defined in (26) and the speed of learning. Examining such relationships is of interest because poor countries may grow faster as those countries learn faster and are more able to adopt advanced technology. The relationships therefore provide an empirical mechanism for studying growth determinants.

A country's speed of learning at time t, S_{it}, may be approximated in terms of human capital by the measure

$$S_{it} = H_{it} - H_{i1},$$

where H_{it} is the per capita total years of schooling for the ith country at time t. Then, total years of schooling between 1 and t provide background information on relative degrees of learning. Correspondingly, the average speed of learning or creation in the OECD nations may be measured by

$$S_t^{\mathrm{OECD}} = \frac{1}{\#(\mathrm{OECD})}\left(\sum_{i\in\mathrm{OECD}} (H_{st} - H_{s1})\right).$$

With these two measures, we can proxy the differential between the speed of learning of individual countries and that of the OECD by

$$\Phi_{it} = (H_{it} - H_{i1}) - S_t^{\mathrm{OECD}}. \tag{27}$$

This differential measure is constructed in absolute terms and is useful because, for some countries, H_{i1} is zero and we cannot use logarithms or ratios. However, when $H_{i1} > 0$, we can use relative measures, analogous to those in the construction of the relative transition curve as follows

$$s_{it} = \frac{H_{it}}{H_{i1}}, \quad s_t^{\mathrm{OECD}} = \frac{1}{\#(\mathrm{OECD})}\sum_{j\in\mathrm{OECD}} \frac{H_{jt}}{H_{j0}},$$

and

$$\phi_{it} = \frac{H_{it}/H_{i1}}{\frac{1}{\#(\mathrm{OECD})+1}\left\{\sum_{j\in\mathrm{OECD}} \frac{H_{jt}}{H_{j0}} + \frac{H_{it}}{H_{i0}}\right\}} \simeq \frac{H_{it}/H_{i1}}{s_t^{\mathrm{OECD}}}.$$

In applications, we have found little difference between the measures ϕ_{it} and Φ_{it}, but Φ_{it} avoids difficulties with zero initial conditions.

Figs. 2–4 display some empirical relationships between Φ_{it} and h_{it} for India, Sub-Saharan Africa, Latin American and Carribean (LAC) countries, the Asian Dragons and the newly industrialized economies (NIEs).[2] In general, when $\Phi_{it} > 0$ (i.e, when the relative speed of learning for the corresponding country or subgroup is faster than the OECD average) the relative transition curves have positive slopes. Otherwise, the slopes of the relative transition curves are negative. The relationships are evident in all the figures, but are

[2] See Phillips and Sul (2006a) for details of the individual countries.

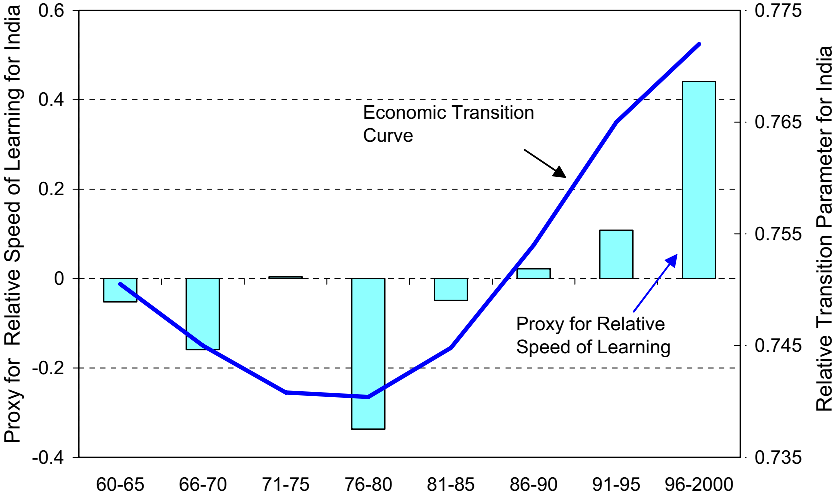

Fig. 2. India's economic transition and speed of learning.

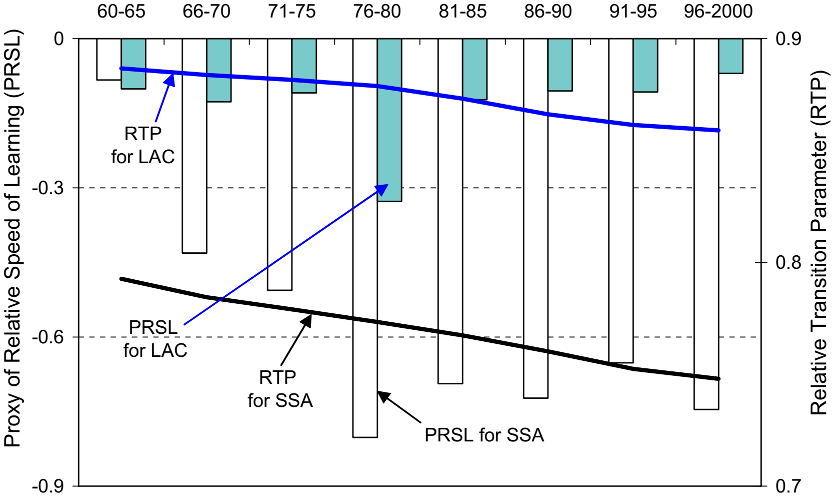

Fig. 3. Economic transition and speed of learning: the LACs and SSA.

particularly strong in the case of India, the NIEs, and the Asian Dragons. The strong negative effects in the case of Sub-Saharan Africa are also evident.

Further regression analysis can be done to quantify these relationships and estimate the impact probability of human learning on technology adoption. One approach is to use a discrete choice framework. Start by defining an index function for technological advance as follows

$$I(\Delta h_{it}) = \begin{cases} 1 & \text{if } \Delta h_{it} \geqslant 0, \\ 0 & \text{if } \Delta h_{it} < 0. \end{cases}$$

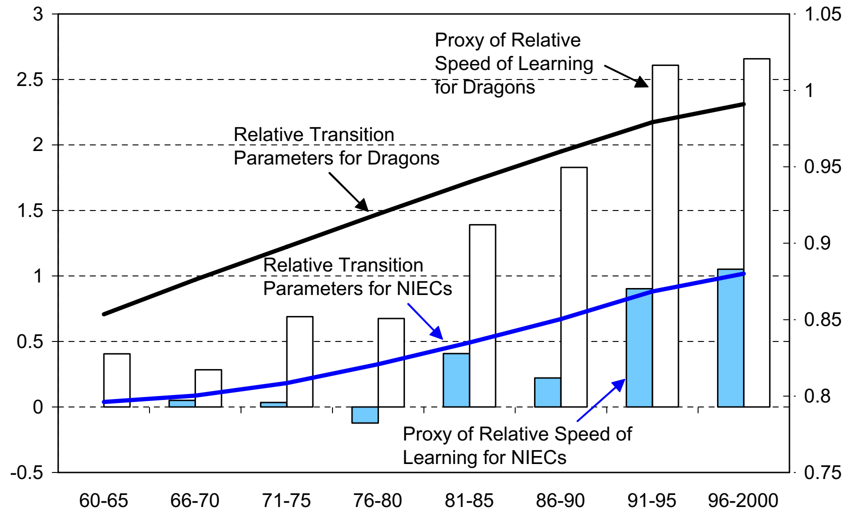

Fig. 4. Economic transition and speed of learning: the Asian Dragons and NIECs.

Next, assume that the impact of learning on technology can be captured in a panel relationship of the form

$$\Delta h_{it} = a_i + \beta \Phi_{it} - \varepsilon_{it},$$

where a_i is a fixed effect, β is a parameter, and ε_{it} are stationary random quantities with distribution function F. Then, $E\{I(\Delta h_{it})\} = P\{I(\Delta h_{it}) = 1\} = F(a_i + \beta \Phi_{it})$, and we can run the following limited dependent variable regression with individual fixed effects.

$$I(\Delta h_{it}) = F(a_i + \beta \Phi_{it}) + u_{it}$$

to quantify the impact of learning on technology and growth. The simplest approach is to use a logit or probit specification. However, to gain realism, such a model needs to allow for cross section dependence and possible serial correlation in the present context. A limit theory for panel discrete choice modeling with these complications and especially cross section dependence is still to be developed. So, modeling along these lines is left for future research.

Economic transitions of the type shown in Figs. 2–4 raise questions about the many potentially relevant factors that can influence such transitions. The short illustration just given focuses on explaining economic transitions using human capital in terms of educational attainment. Clearly, this is only one of a very large group of potential factors that range over economic, social, cultural and political facets of individual countries. There is plenty of scope for using the above methodology to assess explanatory evidence from alternative sources.

5. Conclusion

The paper overviews a new approach to modeling and analyzing economic transition behavior in the presence of common growth characteristics. The model is a nonlinear factor model with a growth component and a time varying idiosyncratic component that allows for quite general heterogeneity across individuals and over time. The formulation is particularly useful in measuring transition towards a long run growth path or individual transitions over time relative to some common trend, representative or aggregate variable. The formulation also gives rise to a simple and convenient time series regression test for convergence. The methods discussed have many potential applications outside of the growth context. Some natural candidates occur in financial economics, where long, wide panels of asset returns are commonly available and interest centres on finding empirical clusters of related financial assets. Other potential applications occur in labor, micro-econometrics, and spatial econometrics. Some further empirical illustrations are given in Phillips and Sul (2006a,b).

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