Econometrics I, 2011 Spring
The First Exam

Part I: Definition

1. (1 point) Symmetric idempotent Matrix: $\mathbf{A}'\mathbf{A} =$

2. (1 point) Unitary Matrix: $\mathbf{A}'\mathbf{A} =$

3. (1 point) $tr\,(a\mathbf{A}) =$

4. (1 point) $||\mathbf{e}|| =$

5. (1 point) $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) =$

6. (1 point) Spectral (Eigen) Decomposition: $\mathbf{A} =$

7. (1 point) Normal distribution:

$$f\left(x|\mu,\sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\qquad}{\qquad}\right)$$

8. (1 point) If z is a $N(0,1)$ and x is χ_n^2 and is independent of z, then the ratio

$$\frac{z}{\sqrt{x/n}} \sim$$

9. (2 point) x is a lognormal variable of which pdf is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left(-\frac{1}{2}\left\{\frac{\ln x - \mu}{\sigma}\right\}^2\right).$$

   Then

$$V(x) =$$

$$Median\,(x) =$$

10. (2 point) Convergence in probability:

11. (2 point) Kolmogorov's Strong LLN:

12. (2 point) Lindegerg-Feller CLT:

Part II: (Simple Calculation)

Q1. (4 point) The regression model is given by

$$y = \mathbf{X}'\boldsymbol{\beta} + u$$

Show that

$$\mathbf{X}'\hat{u} = 0$$

where

$$\hat{u} = y - \mathbf{X}'\hat{\boldsymbol{\beta}}, \quad \hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}$$

Q2. (5 point) Show that $\rho^t = o(1)$, if $0 < \rho < 1$, but $\rho^t = O(1)$ if $\rho = 1$.

Q3. (5 point) Show the order of the sequence of $\sum_{t=1}^{T} \rho^t$ when $\rho < 1$ and $\rho = 1$.

Q4. (5 point) Show the order in probability of $x_n \sim N\left(0, \frac{\sigma^2}{n}\right)$.

Part III: Derivation Let

$$y_{it} = \mu_i + \varepsilon_{it}, \quad \varepsilon_{it} \sim iid\left(0, T^{\alpha}\right).$$

Q1: (5 point) You want to estimate the overall mean of y_{it}, that is, $\bar{y}_{NT} = \frac{1}{NT}\sum y_{it}$. Let $\alpha = 0$ so that $\varepsilon_{it} \sim iid(0,1)$. Derive the limiting distribution of $\bar{y}_{NT}$.

Q2: (5 point) Let $\alpha = -1$. Derive the limiting distibution of the cross sectional average, $\bar{y}_{tN} = \frac{1}{N}\sum_{i=1}^{N} y_{it}$.

Q3. (5 point) Let $\alpha = -1$. Derive the limiting distibution of the overall $\bar{y}_{TN}$.

Part IV (Extra Credit) Consider the following simple regression

$$y_{it} = \beta\frac{1}{t} + u_{it}, \quad u_{it} \sim iid\left(0, \sigma^2\right)$$

Now we take the cross sectional average of y_{it} and run

$$\bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{it},$$

and

$$\bar{y}_t = \beta\frac{1}{t} + \bar{u}_t$$

Q1: (5 point) Let $N = 1$. That is, you have only one time series y_t. Consider you run

$$y_t = \beta \frac{1}{t} + u_t.$$

Derive the limiting distribution of $\hat{\beta}$ as $T \to \infty$.

Q2: (5 point) Now N is increasing. As $N \to \infty$ but fixed T, what is the limiting distribution of $\hat{\beta}$ in $\bar{y}_t = \beta \frac{1}{t} + \bar{u}_t$.