The First Minor Exam

Part I: Definition (one point each. Use one word or sentence)

1. Idempotent Matrix: $\mathbf{AA} =$
2. Orthogonal Matrix: $\mathbf{A}^{-1} =$
3. Unitary Matrix: $\mathbf{A}'\mathbf{A} =$
4. Skewness:
5. Moment Generating Function:
6. Characteristic Function:

Part II: Simple Calculation (2 points each) Don't prove it. Just write answers only

1. Find eigen values of $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$. $\lambda_1 =$ $\lambda_2 =$
2. Let $x \sim N(0,3)$. Then $x^2 \sim$
3. Let $x_i \sim iidN(0,3)$ for $i = 1,2$. Then $\frac{x_1^2}{x_2^2} \sim$
4. Let $x \sim LN(\mu, \sigma^2)$, then $ax \sim$

Part III: Derivation (6 points. Write down all derivations) Let $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ and consider the following problem $\arg\min_\beta L = \mathbf{u}'\mathbf{u}$. Show the solution satisfies $\hat{\boldsymbol{\beta}} = (X'X)^{-1}\mathbf{X}'\mathbf{y}$

$$S(\beta) = \mathbf{u}'\mathbf{u} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) = \mathbf{y}'\mathbf{y} -$$

$$\frac{\partial S(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \qquad\qquad = 0$$

so that $\hat{\boldsymbol{\beta}} = (X'X)^{-1}\mathbf{X}'\mathbf{y}$.

Part IV: Extra Credit (2 points each) Let $x \sim N(\mu, \sigma^2)$. Write down moment and cumulant generating functions.