

# Theory of Computational Complexity

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## *Errata*

Page 21, line 4: “ $A \in DTIME(c \cdot t(n))$ ” should read “ $L(M) \in DTIME(c \cdot t(n))$ ”

Page 47, line 11 from the bottom: “ $EXP$ ” should read “ $EXPPOLY$ ”

Page 56, line 13: Both occurrences of “ $x[j]$ ” should be changed to “ $x[k]$ ”

Page 67, line 24: Delete “at most  $3K_T/2$  and, hence,”

Page 87, line 17: Add the following sentence at the end:

We also let 3-CNF-TAU<sub>k</sub> (3-DNF-TAU<sub>k</sub>) denote the problem of determining whether a given 3-CNF (3-DNF, respectively) formula  $F$  satisfies

$$(\forall \tau_1 : X_1 \rightarrow \{0, 1\})(\exists \tau_2 : X_2 \rightarrow \{0, 1\}) \cdots \\ (Q_{k+1} \tau_k : X_k \rightarrow \{0, 1\}) F|_{\tau_1, \tau_2, \dots, \tau_k} = 1,$$

where  $Q_k = \exists$  if  $k$  is odd, and  $Q_k = \forall$  if  $k$  is even.

Page 87, Corollary 3.14: The second half of part (a) should read

“and 3-DNF-TAU<sub>k</sub> is  $\leq_m^P$ -complete for  $\Pi_k^P$ .”

The second half of part (b) should read

“and 3-CNF-TAU<sub>k</sub> is  $\leq_m^P$ -complete for  $\Pi_k^P$ .”

Page 110, Exercise 3.13: This exercise should be changed to the following:

**3.13** Prove that the game GEOGRAPHY played on undirected graphs is solvable in polynomial time.

Page 114, line 8 from the bottom: “In particular,  $|S_{n-1}| = \varphi(n-1) > 0$ ” should be replaced by the following paragraph (in the following,  $a|b$  denotes that  $a$  divides  $b$ ):

Since, for all  $d$  that do not divide  $n-1$ , we have  $S_d = \emptyset$ , we get  $\sum_{d|(n-1)} |S_d| = |Z_n^*| = n-1$ . In the above, we have proved that if  $d$  divides  $n-1$ , then either  $S_d = \emptyset$  or  $|S_d| = \varphi(d)$ . Thus, by Euler’s Theorem, for each  $d$  that divides  $n-1$ , we must have  $|S_d| = \varphi(d)$ , for otherwise  $\sum_{d|(n-1)} |S_d|$  would be less than  $n-1$ . It follows that  $|S_{n-1}| = \varphi(n-1) > 0$ .

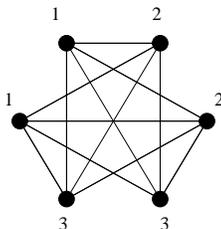
Page 115, line 18: “less than  $n-1$ ” should read “less than or equal to  $(n-1)/2$ ”

Page 121, line 8 from the bottom: “approximation to  $f$ ” should read “approximation to  $f^{-1}$ ”

Page 204, line 13: “ $\langle 0, w \rangle \neq B$ ” should read “ $\langle 0, w \rangle \notin B$ ”

Page 204, line 16: “then  $\langle \psi, w \rangle \in B$ ” should read “then  $\langle \phi, w \rangle \in B$ ”

Page 206, Figure 6.5: The figure at the right-hand side should be changed to



Page 218, line 1: “Each subcircuit  $C_i$ ” should read “Each subcircuit  $C_{i+1}$ ”

Page 239, line 9: “ $\mathcal{D} = P^{\mathcal{C}}$ ” should read “ $\mathcal{D} = P^{\mathcal{C}}$  for some class  $\mathcal{C}$  that contains  $P$  as a subclass”

Page 258, line 22: “there is a co-sparse set” should read “there is no co-sparse set”

Page 258, line 23: “is also em no” should read “is also NO”

Page 291, Lemma 8.5: Add two more items to Lemma 8.5:

(c) For all  $x, y > 1$  such that  $\gcd(x, n) = \gcd(y, n) = 1$ ,  $(\frac{x}{n}) \cdot (\frac{y}{n}) = (\frac{xy}{n})$ .

(d) For all odd  $n > 2$ ,  $(\frac{2}{n}) = (-1)^{(n^2-1)/8}$ .

Page 291, line 14 from the bottom: “Lemma 8.5(b) above shows” should read “Lemma 8.5 above shows”

Page 306, line 12: “ $F|_{x=1} \in \text{SAT}$ ” should read “ $F|_{x_1=1} \in \text{SAT}$ ”

Page 333, line 11: “a cover of  $H_k$ ” should read “the path  $(u, u_1, v_1, v)$  and a cycle cover of  $H_k - \{u, u_1, v_1, v\}$ ”

Page 421: The first half of the proof of Theorem 11.24 has an error and should be changed as follows:

Lines 26 to 27: Replace “We assume” by “We assume that  $\Phi$  has  $k(h+1)^m$  variables and  $(h+1)^m$  clauses. In addition, we assume”

Line 27: Replace “where  $\Pi$ ” by “where each  $f_i$ ,  $1 \leq i \leq k$ , defines Boolean assignments to  $(h+1)^m$  variables, and  $\Pi$ ”

Lines 31 to 33: Replace the two sentences “Then,  $V_k$  simulates  $V \dots$  uses  $f_i(\vec{a})$  for it.” by “Then,  $V_k$  simulates  $V$  with the following modification:

It uses  $\Pi_{k+1}$  as the additional proof, and replaces  $\bar{G}^f(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v})$  by

$$\bar{G}_1^{f_1, \dots, f_k}(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{v}) = \prod_{j=1}^3 \prod_{i=1}^k \bar{\theta}_{ij}(\vec{u}_j, \vec{v})(\bar{\varphi}_j(\vec{v}) - f_i(\vec{u}_j)),$$

where  $\theta_{ij}(s, t)$  is equal to 1 if the  $s$ th variable in the  $i$ th block is the  $j$ th variable in  $C_t$ , and equal to 0 otherwise. Note that the sum check proof system with respect to function  $\bar{G}_1^{f_1, \dots, f_k}$  now requires three values of  $f_i(s)$  for  $1 \leq i \leq k$  and  $s \in \mathcal{F}^m$ .”

Page 432, line 6 from the bottom: “and  $F$  has” should read “and  $F_x$  has”

Page 432, line 5 from the bottom: “total of  $d_1 n^c$  clauses).” should read “total of  $d_1 n^c$  clauses) if  $x \notin L$ .”

Page 436, line 13: Delete the period at the end of the formula for  $S'$ .

Page 465, lines 26 to 28: Change this reference item to

Kozen, D. [1977], Lower bounds for natural proof systems, *Proceedings of the 18th IEEE Symposium on Foundations of Computer Science*, IEEE Computer Society Press, Los Angeles, pp. 254–266.

Updated June 30, 2006