## Midterm Exam

Lasers and Modern Optics
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Thursday, October 26, 2006 - Tuesday, November 7, 2006

Instructions: Work through the following problem neatly and professionally, and without collaboration of any kind. You may consult your own notes, and any published textbook. Please be neat and professional in your presentation. Include plots and words to expand on your results. A perfect score on this exam is 40 points.

## 1. Beam Optics and Laser Resonators (10 points)

Consider the following laser resonator comprising a curved mirror (radius Rc ) and a flat mirror separated by a length (Lc). The flat mirror is the output coupler from which the laser beam emerges.

a. Analyze this cavity and derive the ranges for Rc and Lc that support a Gaussian beam.
b. Pick one of these stable resonators (choose numbers that satisfy the condition for part a) and solve for the mode of this cavity. Specifically, plot the spotsize, $w(z)$, and the radius of curvature, $R(z)$, of the mode everywhere in the cavity.
c. Put a focusing lens immediately after the output coupler of the resonator as shown below. Design the lens to minimize the focused spot size:
i. What is the minimum spot produced by this configuration?
ii. Where is the minimum spot size located?
iii. Does the radius of curvature go flat at this same point?
iv. The electric field is stronger at the focus of the lens than immediately out of the laser. What is this electric field enhancement factor?


## 2. Laser Diodes (10 points)

For lasers ( $100-\mathrm{um}$ wide apertures, $\mathrm{w}=100 \mathrm{um}$ ) with uncoated facets, $\mathrm{R}=0.3$, and operating at $\lambda=0.808 \mathrm{um}$, the following data was obtained from length studies.

| $\mathrm{L}(\mathrm{cm})$ | $\alpha(1 / \mathrm{cm})$ | $\mathrm{I}_{\mathrm{th}}(\mathrm{A})$ | $\mathrm{J}_{\mathrm{th}}\left(\mathrm{A} / \mathrm{cm}^{2}\right)$ | $\left.\eta_{d}\left[\frac{d P}{d I}\right) \frac{q}{h \nu}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.1 | 12 | 0.311 | 311 | 0.56 |
| 0.15 | 8 | 0.392 | 261 | 0.5 |
| 0.2 | 6 | 0.478 | 239 | 0.45 |

(a) Find the internal efficiency, $\eta \mathrm{i}$, and internal loss, $\alpha \mathrm{i}$.
(b) Find the modal gain coefficient, $\Gamma g 0$, and the transparency current, Jtr. Use the logarithmic approximation for the gain, $g=g_{0} \ln \left(\eta_{i, J} / J_{t r}\right)$. The gain is in terms the current density, J.
(c) For a Power level of $\mathrm{P}=1 \mathrm{~W}$, graphically determine the device length, L , that provides the minimum operating current, I.

## 3. Quantum States (10 points)

Solve the steady state Schrodinger wave equation for an electron in an infinite well of length a (particle in a box).
$\frac{-i \hbar^{2}}{2 m} \frac{\partial^{2} \Psi}{\partial x^{2}}+(V-E) \Psi=0$

Inside the well, $\mathrm{V}=0$, outside the well, $\mathrm{V}=\infty$

Show that the boundary conditions demand quantized energy states, $n=1,2,3$ with energies En given by:
$E_{n}=\frac{h^{2} n^{2}}{8 m a^{2}}$

## 4. FM Microchip Laser (10 points)

Consider the following laser comprised of a sliver of Er:YVO4 epoxied to Lithium Niobate (LiNb03), and pumped by a laser diode. The Er doped crystal is the gain medium. Lithium Niobate has an index of refraction that changes with voltage.
a. What is the approximate output wavelength of the laser?
b. What is the longitudinal mode spacing of the laser if there is no voltage applied to the lithium niobate?
c. Derive an equation to describe what happens to the lasing frequency as a voltage signal is put on the lithium niobate.
d. Comment on the utility of this laser.


Figure 3.1: The electro-optic tunable microchip laser


