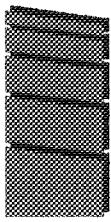
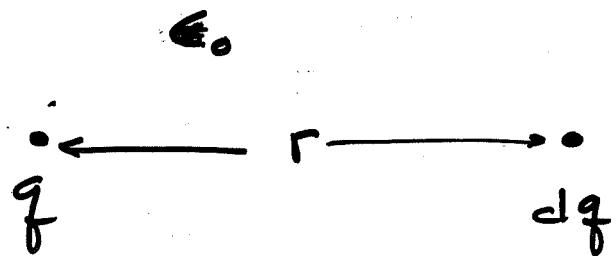


Polarization



Consider Two Charges:



The Coulomb Force Between These Two Charges is :

$$\tilde{F}_c = \frac{q dq}{4\pi\epsilon_0 r^2} \hat{a}_r$$

Note that the Force is, as always, A VECTOR.

If I take the limit as $dq \rightarrow 0$
I'm left with the Electric Field

$$\tilde{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

POLARIZATION

$$\nabla \times \underline{\underline{E}} = -\frac{\partial \underline{\underline{B}}}{\partial t}$$

$$\nabla \cdot \underline{\underline{D}} = \rho$$

$$\nabla \times \underline{\underline{H}} = \underline{\underline{J}} + \frac{\partial \underline{\underline{D}}}{\partial t}$$

$$\nabla \cdot \underline{\underline{B}} = 0$$

$$\rightarrow \nabla^2 \underline{\underline{E}} - \frac{1}{c^2} \frac{\partial^2 \underline{\underline{E}}}{\partial t^2} = 0$$

THE ELECTRIC FIELD IS A VECTOR

≠ SOMETIMES YOU HAVE TO ADD
VECTOR COMPONENTS TOGETHER

$$\rightarrow \underline{\underline{E}}(z, t) = E_x' \cos(\omega t - kz + \delta_x) \hat{\alpha}_x \\ + E_y' \cos(\omega t - kz + \delta_y) \hat{\alpha}_y$$

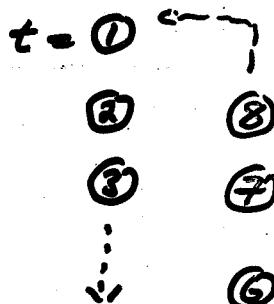
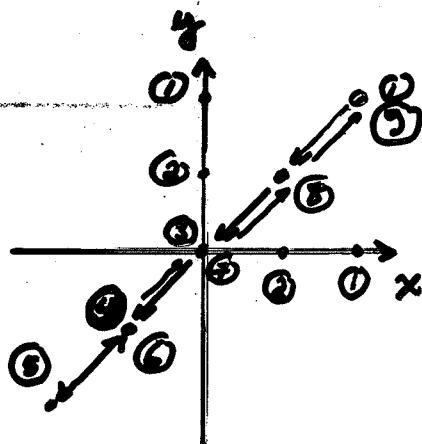
Some Examples:

GO TO $Z=0$ SET $S_x = 0$

$$|E'_x| = |E'_y|$$

$$\tilde{E}(t) = E' \left[\cos(\omega t) \hat{a}_x + \cos(\omega t + \delta_y) \hat{a}_y \right]$$

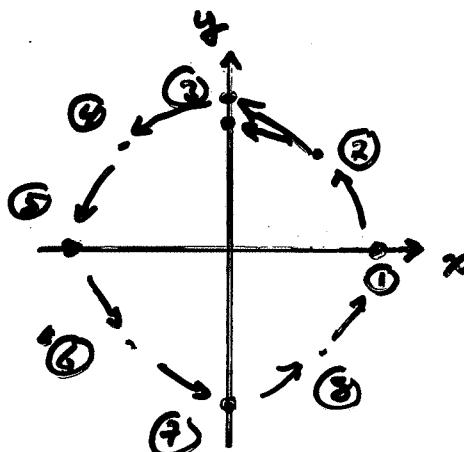
$$\underline{\delta_y = 0}$$



"LINEAR POLARIZATION"

$$\underline{\delta_y = -\frac{\pi}{2}}$$

$$\cos(\omega t - \frac{\pi}{2}) \\ = \sin(\omega t)$$



"CIRCULAR POLARIZATION"

IF $|E'_x| \neq |E'_y|$ THEN ELLIPSE INSTEAD OF CIRCLE ... "ELLiptical POLARIZATION"

Now, in slightly more general notation:

$$\underline{E}(z, t) = \operatorname{Re} \left[\underline{A} e^{i(\omega t - kz)} \right]$$

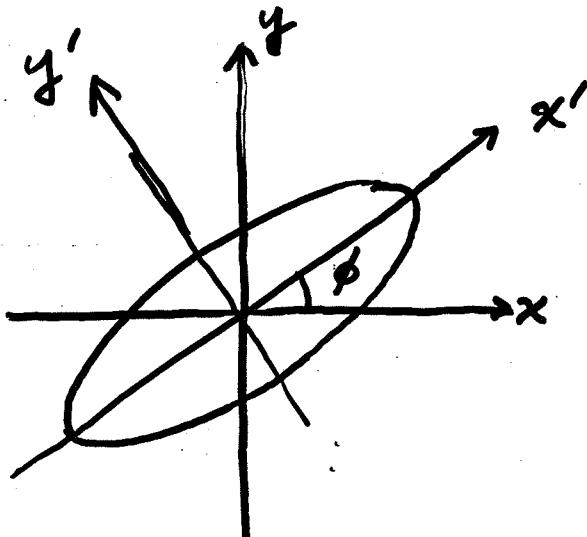
$$\Rightarrow \underline{A} = A_x e^{i\delta_x} \hat{a}_x + A_y e^{i\delta_y} \hat{a}_y$$

We can write the shape of the trajectory, in general, as

$$\left(\frac{Ex}{Ax} \right)^2 + \left(\frac{Ey}{Ay} \right)^2 - 2 \frac{\cos \delta}{Ax \cdot Ay} E_x E_y = \sin^2 \delta$$

$$\delta = \delta_y - \delta_x$$

The General
Trajectory
is an
Ellipse →



ROTATE THE AXES SO $x' \neq y'$

$$\left(\frac{E_x'}{a}\right)^2 + \left(\frac{E_y'}{b}\right)^2 = 1$$

WHERE:

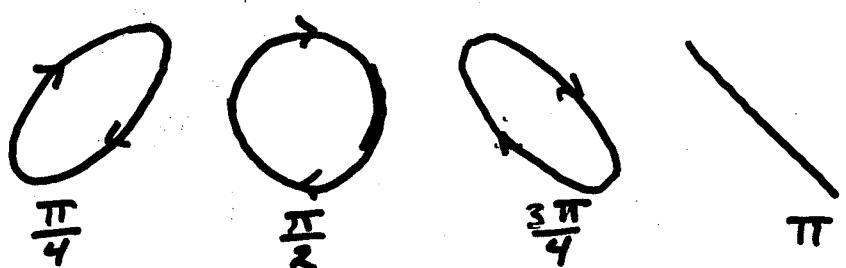
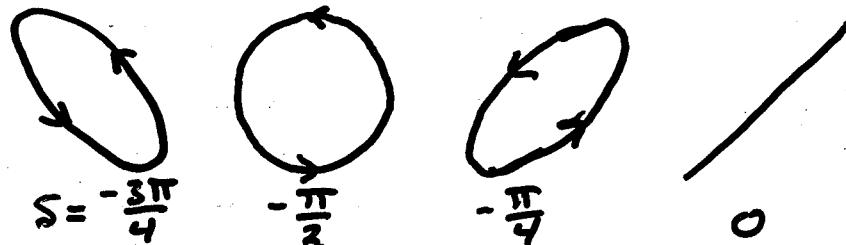
$$a^2 = A_x^2 \cos^2 \phi + A_y^2 \sin^2 \phi + 2A_x A_y \cos S \cos \phi \sin \phi$$

$$b^2 = A_x^2 \sin^2 \phi + A_y^2 \cos^2 \phi - 2A_x A_y \cos S \cos \phi \sin \phi$$

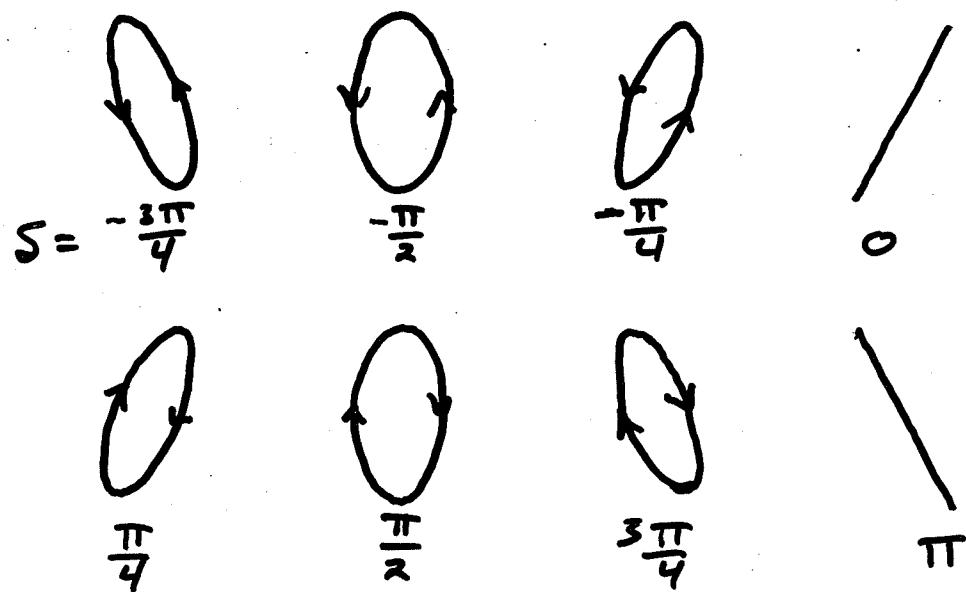
≠

$$\tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2} \cos S$$

$$|E_x| = |E_y|$$



$$|E_x| = \frac{1}{2}|E_y|$$



Linearly Polarized Light

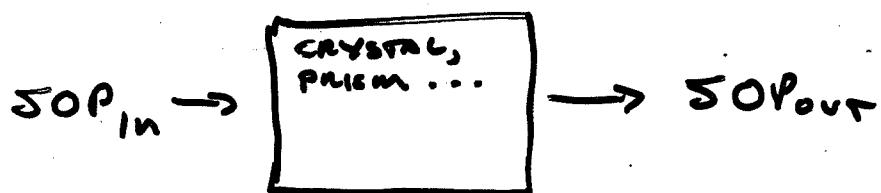
$$S = m\pi$$

Circularly Polarized Light

$$S = \pm \pi \neq |E_x| = |E_y|$$

$$\text{ellipticity } e = \pm \frac{b}{a}$$

To Solve Polarization Problems:



- Break SOP_{in} into its two components
- Calculate the
 - a) Amplitude change
 - b) Travel time (phase) $\Gamma = (n_x - n_y) \frac{s}{\lambda}$for each component
- Add the two output components together to get SOP_{out}

The Jones Vectors:

$$\underline{v} = \begin{bmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{bmatrix}$$

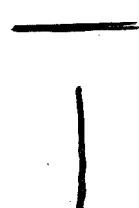
- DESCRIBE
THE CRYSTAL,
POLARIZATION, ETC

SOP

\underline{v}

AS 2×2

MATRICES



$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- MULTIPLY SEVERAL
MATRICES FOR
SYSTEMS



$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$



$$\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$



$$\frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix}$$



$$\frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2i \end{pmatrix}$$



$$\frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -i \end{pmatrix}$$