

## FIELDS AND WAVES - SOLUTIONS

## CHAPTER 1

$$\text{Ans 1.1: } \nabla \times H = J_{ic} + \frac{\partial D}{\partial t}$$

Taking the divergence on both sides

$$\begin{aligned} \nabla \cdot (\nabla \times H) &= \nabla \cdot J_{ic} + \nabla \cdot \frac{\partial D}{\partial t} \\ &= \nabla \cdot J_{ic} + \frac{\partial}{\partial t} \nabla \cdot D \end{aligned}$$

Using  $\nabla \cdot (\nabla \times A) = 0$  and  $\nabla \cdot D = q_{ev}$ , we can write

$$0 = \nabla \cdot J_{ic} + \frac{\partial}{\partial t} (q_{ev})$$

$$\Rightarrow \boxed{\nabla \cdot J_{ic} = -\frac{\partial q_{ev}}{\partial t}}$$

$$\text{Ans 1.2: } \nabla \times E = -M_i - \frac{\partial B}{\partial t}$$

Taking surface integral on both sides

$$\iint_S (\nabla \times E) \cdot dS = - \iint_S M_i \cdot dS - \frac{\partial}{\partial t} \iint_S B \cdot dS$$

Applying Stokes' Theorem from Eq (1-7) in book to left side of equation leads to

$$\oint_C E \cdot dl = - \iint_S M_i \cdot dS - \frac{\partial}{\partial t} \iint_S B \cdot dS$$

Using the same procedure, we can write

$$\oint_{\text{C}} \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{T}_i \cdot d\mathbf{s} + \iint_S \mathbf{T}_c \cdot d\mathbf{s} + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot d\mathbf{s}$$

For the remaining 3 equations of Table 1-1, we proceed as follows:

$$\nabla \cdot \mathbf{D} = q_{\text{var}}$$

Taking a Volume integral on both sides, we get

$$\iiint_V \nabla \cdot \mathbf{D} dv = \iiint_V q_{\text{var}} ds = Q_e$$

Applying Divergence theorem (eq 1-8) on left hand side,

$$\iint_S \mathbf{D} \cdot d\mathbf{s} = Q_e$$

Using the same procedure,

$$\iint_S \mathbf{B} \cdot d\mathbf{s} = Q_m$$

$$\text{and } \iint_S \mathbf{T}_{ic} \cdot d\mathbf{s} = \frac{\partial}{\partial t} \iiint_V q_{\text{var}} dv = \frac{-\partial Q_e}{\partial t}$$

Ans 1.3

(a)  $\mathbf{D} = \hat{a}_x (3+x)$

$$\begin{aligned} Q_e &= \iint_S \mathbf{D} \cdot d\mathbf{s} = \left. \iint_S \hat{a}_x (3+x) \right|_{z=0} \cdot (-\hat{a}_x dy dz) + \left. \iint_S \hat{a}_x (3+x) \right|_{z=1} dy dz \\ &= -3 + 4 \\ &= 1 \end{aligned}$$

(b)  $\mathbf{D} = \hat{a}_y (4+yz^2)$

$$\begin{aligned} \partial e = \oint \vec{D} \cdot d\vec{s} &= \int \int \left[ \hat{a}_x (4+yz^2) \right] \Big|_{y=0} \cdot (-\hat{a}_y dx dz) + \int \int \left[ \hat{a}_y (4+yz^2) \right] \Big|_{y=0} \\ &\quad \cdot \hat{a}_x dy dz \\ &= -4 + 5 \\ &= 1 \end{aligned}$$

Ans 1.5  $\nabla \cdot E = 0$  for a source free and homogeneous medium.

Thus:

$$\nabla \cdot E = \left[ \hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z} \right] \cdot \left[ \hat{a}_x A(x+yz) + \hat{a}_y (x-y) \right] \cos \omega t = 0$$

$$A \frac{\partial}{\partial x} (x+yz) + B \frac{\partial}{\partial y} (x-y) = A(1) + B(-1) = 0 \Rightarrow A = B$$

Ans 1.8  $\vec{J}_d = \hat{a}_x y z + \hat{a}_y y^2 + \hat{a}_z x y z$ ,  $I_d = \oint \vec{J}_d \cdot d\vec{s}$

$$I_d = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_x y z) \Big|_{y=-1/2} \cdot (-\hat{a}_x dy dz) + \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_x y z) \Big|_{y=1/2} \cdot (\hat{a}_x dy dz)$$

$$+ \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_y y^2) \Big|_{y=-1/2} \cdot (-\hat{a}_y dx dz) + \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_y y^2) \Big|_{y=1/2} \cdot (\hat{a}_y dx dz)$$

$$+ \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_z x y z) \Big|_{z=-1/2} \cdot (-\hat{a}_z dx dy) + \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} (\hat{a}_z x y z) \Big|_{z=1/2} \cdot (\hat{a}_z dx dy)$$

$$I_d = 0 + 0 + 0 = 0.$$

$$\text{Ans 1.13} \quad J = \hat{a}_z 10^4 e^{-10^6 y} \cos(2\pi \times 10^9 t)$$

$$\begin{aligned} \mathcal{T}(y = 0.25 \times 10^{-3}) &= \hat{a}_z 10^4 e^{-10^6 (2.5 \times 10^{-4})} \cos(2\pi \times 10^9 t) \\ &= \hat{a}_z 10^4 e^{-250} \cos(2\pi \times 10^9 t) \approx 0 \end{aligned}$$

$$I = \iint_S \mathcal{T} dS = 2 \int_0^{2.5 \times 10^{-4}} \int_0^{5 \times 10^{-3}} [\hat{a}_z 10^4 e^{-10^6 y} \cos(2\pi \times 10^9 t)] \cdot \hat{a}_z dy$$

$$I = 2(5 \times 10^{-3})(10^4) \cos(2\pi \times 10^9 t) \int_0^{2.5 \times 10^{-4}} e^{-10^6 y} dy = 10^{-4} \cos(2\pi \times 10^9 t)$$

$$\text{Ans 1.14} \quad E = \hat{a}_y E_0 \sin\left(\frac{\pi}{a} x\right) \cos(\omega t - \beta_z z) = \operatorname{Re}[\hat{a}_y E_0 \sin\left(\frac{\pi}{a} x\right) e^{j(\omega t - \beta_z z)}]$$

$$= \operatorname{Re}[\underline{E} e^{j\omega t}]$$

$$\text{where } \underline{E} = \hat{a}_y E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}$$

$$H = \frac{-1}{j\mu_0 \omega} \nabla \times \underline{E} = \hat{a}_x \frac{1}{j\omega \mu_0} \cdot \frac{\partial E_y}{\partial z} - \hat{a}_z \frac{1}{j\omega \mu_0} \cdot \frac{\partial E_y}{\partial x}$$

$$= -\hat{a}_x \frac{\beta_z}{\omega \mu_0} E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z} + \hat{a}_z \frac{E_0}{\omega \mu_0} \left(\frac{\pi}{a}\right) \cos\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}$$

$$H = \operatorname{Re}[H e^{j\omega t}] = -\hat{a}_x \frac{\beta_z}{\omega \mu_0} E_0 \sin\left(\frac{\pi}{a} x\right) \cos(\omega t - \beta_z z)$$

$$+ \hat{a}_z \frac{E_0}{\omega \mu_0} \left(\frac{\pi}{a}\right) \cos\left(\frac{\pi}{a} x\right) \cos(\omega t + \frac{\pi}{2} - \beta_z z)$$

$$(b.) \quad \text{Using } \nabla \times H = j\omega \epsilon_0 E \Rightarrow \hat{a}_y \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) = j\omega \epsilon_0 E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}$$

$$\hat{a}_y \frac{1}{\omega \mu_0} \left[ \beta_z^2 + \left(\frac{\pi}{a}\right)^2 \right] \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z} = j\omega \epsilon_0 E_0 \sin\left(\frac{\pi}{a} x\right) e^{-j\beta_z z}$$

$$\omega_{eo} = \frac{1}{\mu_0} \left[ \beta_z^2 + \left( \frac{\eta}{a} \right)^2 \right] \Rightarrow \beta_z = \sqrt{\omega^2 \mu_0 \epsilon_0 - \left( \frac{\eta}{a} \right)^2}$$

Ans 1.18  $E = R_e [E e^{i\omega t}] = R_e [(E_R + jE_x) (\cos \omega t + j \sin \omega t)]$

$$= R_e [(E_R \cos \omega t - E_x \sin \omega t) + j(E_R \sin \omega t + E_x \cos \omega t)]$$

$$E = E_R \cos \omega t - E_x \sin \omega t$$

Similarly,

$$E = \frac{1}{2} [E e^{i\omega t} + (E e^{i\omega t})^*] = \frac{1}{2} [E e^{i\omega t} + E^* e^{-i\omega t}]$$

$$= \frac{1}{2} [(E_R + jE_x) (\cos \omega t + j \sin \omega t) + (E_R - jE_x) (\cos \omega t - j \sin \omega t)]$$

$$= \frac{1}{2} [2(E_R \cos \omega t - E_x \sin \omega t)]$$

$$= E_R \cos \omega t - E_x \sin \omega t.$$

Ans 1.29 To determine the coefficients  $P_0$  and  $T_0$ , we apply the boundary conditions along the interface at  $z=0$ . To do this, we first find the corresponding magnetic field components. This is accomplished using Maxwell's equation of  $\nabla \times E = -j\omega \mu_0 H_0 \Rightarrow H_0 = \frac{-1}{j\omega \mu_0} \nabla \times E$ . Doing this for each component:

$$H^x = \hat{a}_y \sqrt{\frac{\epsilon_0}{\mu_0}} E_0 e^{-jP_0 z}, \quad H^y = -\hat{a}_y \sqrt{\frac{\epsilon_0}{\mu_0}} E_0 P_0 e^{+jP_0 z},$$

$$H^z = \hat{a}_y \sqrt{\frac{\epsilon_0}{\mu_0}} E_0 T_0 e^{-jP_0 z}$$

Applying the boundary conditions on the continuity of the tangential electric and magnetic fields along the interface at  $z=0$  leads to

$$1 + \Gamma_0 = T_0 \quad - \text{from continuity of electric fields}$$

$$1 - \Gamma_0 = \sqrt{\epsilon_r} T_0 = \sqrt{81} T_0 \quad - \text{from continuity of magnetic fields.}$$

Solving this we get,

$$T_0 = \frac{2}{1 + \sqrt{\epsilon_r}} = \frac{2}{1 + \sqrt{81}} = \frac{2}{1 + 9} = 0.2$$

$$\Gamma_0 = T_0 - 1 = 0.2 - 1 = -0.8.$$

Ans 1.37 :  $E = \hat{a}_y E_0 \sin\left(\frac{\pi}{a}x\right) \sin\left(\frac{\pi}{c}z\right), \quad \omega_3 = \omega = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{D}{a}\right)^2 + \left(\frac{D}{c}\right)^2}$

a.  $H = \frac{-1}{j\omega \mu_0} \nabla \times E = \frac{-1}{j\omega \mu_0} \left[ -\hat{a}_x \frac{\partial E_y}{\partial z} + \hat{a}_z \frac{\partial E_y}{\partial x} \right]$

$$= \hat{a}_x \frac{E_0}{j\omega \mu_0} \left( \frac{\pi}{c} \right) \sin\left(\frac{\pi}{a}x\right) \cos\left(\frac{\pi}{c}z\right)$$

$$- \hat{a}_z \frac{E_0}{j\omega \mu_0} \left( \frac{\pi}{a} \right) \cos\left(\frac{\pi}{a}x\right) \sin\left(\frac{\pi}{c}z\right)$$

b.  $P_s = \frac{1}{2} \iiint_V [H^* \cdot M_i + E \cdot J_i^*] \cdot dV = 0.$

c.  $P_e = \iint_{S_a} \left( \frac{1}{2} E \times H^* \right) \cdot dS = \iint_{S_a} S \cdot ds, \quad S = \frac{1}{2} E \times H^*$

$$\begin{aligned}
 S = \frac{1}{2} \bar{E} \bar{X} H^* &= \frac{1}{2} \hat{a}_y \bar{E}_y \times (\hat{a}_x H_x^* - \hat{a}_z H_z^*) = \frac{1}{2} (-\hat{a}_z \bar{E}_y H_x^* - \hat{a}_x \bar{E}_y H_z^*) \\
 &= \frac{1}{2} \left[ \hat{a}_x \frac{|\bar{E}_0|}{\sqrt{\omega \mu_0}} \left( \frac{\pi}{a} \right) \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi z}{c}\right) \sin^2\left(\frac{\pi z}{c}\right) \right. \\
 &\quad \left. + \hat{a}_z \frac{|\bar{E}_0|^2}{\sqrt{\omega \mu_0}} \left( \frac{\pi}{c} \right) \sin^2\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi z}{c}\right) \cos\left(\frac{\pi z}{c}\right) \right]
 \end{aligned}$$

Contributions to  $P_e$  from different walls:

$$\text{left \& right: } \int_0^c \int_0^b (\hat{a}_x S_x)_{x=0} \cdot (-\hat{a}_x dy dz) = 0$$

$$\int_0^c \int_0^b (\hat{a}_x S_x)_{x=0} \cdot (\hat{a}_x dy dz) = 0$$

$$\text{Back \& front: } \int_0^b \int_0^a (\hat{a}_z S_z)_{z=0} \cdot (-\hat{a}_z dx dy) = 0$$

$$\int_0^b \int_0^a (\hat{a}_z S_z)_{z=0} \cdot (\hat{a}_z dx dy) = 0.$$

Top \& Bottom: Since there are no  $y$  components of power density, there is no contribution from top \& bottom walls.

$$\therefore P_e = 0 + 0 + 0 + 0 + 0 = 0.$$

d.  $P_d = \frac{1}{2} \iiint_V |\bar{E}|^2 dV = 0.$

e.  $W_m = \frac{\mu_0}{4} \iiint_V |H|^2 dV = |\bar{E}_0|^2 \frac{\mu_0}{4} \frac{1}{(\omega \mu_0)^2} \left[ \left( \frac{\pi}{c} \right)^2 \int_0^a \int_0^b \int_0^c \sin^2\left(\frac{\pi x}{a}\right) \right. \\ \left. \cos^2\left(\frac{\pi z}{c}\right) dx dy dz + \left( \frac{\pi}{c} \right)^2 \int_b^c \int_0^c \int_0^a \cos^2\left(\frac{\pi x}{a}\right) \right]$

$$\begin{aligned}
 W_m &= |E_0|^2 \frac{\mu_0}{4} \frac{1}{(\omega \mu_0)^2} \left[ \left( \frac{\pi}{c} \right)^2 \frac{abc}{4} + \left( \frac{\pi}{a} \right)^2 \frac{abc}{4} \right] \\
 &= |E_0|^2 \frac{abc}{16} \frac{\epsilon_0}{\mu_0 \omega^2 \epsilon_0} \left[ \left( \frac{\pi}{a} \right)^2 + \left( \frac{\pi}{c} \right)^2 \right] \\
 &= |E_0|^2 \frac{abc}{16} \epsilon_0.
 \end{aligned}$$

$$\begin{aligned}
 f. \quad W_e &= \frac{\epsilon_0}{4} \iiint_V |E|^2 dv = |E_0|^2 \frac{\epsilon_0}{4} \int_0^a \int_0^b \int_0^c \sin^2\left(\frac{\pi}{a}x\right) \sin^2\left(\frac{\pi}{c}z\right) dx dy dz \\
 &= |E_0|^2 \frac{abc}{4} \frac{\epsilon_0}{4} = |E_0|^2 \frac{abc}{16} \epsilon_0
 \end{aligned}$$

Ultimately  $P_s = P_e + P_d + 1/2 \omega (W_m - W_e)$

$$0 = 0 + 0 + 1/2 \omega \left( |E_0|^2 \frac{abc \epsilon_0}{16} - |E_0|^2 \frac{abc \epsilon_0}{16} \right)$$

LHS = RHS  
Hence proved.

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CHAPTER 2

Ans 2.4 a.  $E_0 (0.25 \times 10^{-3}) + E_1 (1 \times 10^{-3}) = 100 \text{ V}$

$$\epsilon_0 E_0 = \epsilon_1 E_1 \Rightarrow E_1 = \frac{\epsilon_0}{\epsilon_1} E_0$$

$$\text{Thus } E_0 (0.25 \times 10^{-3}) + E_0 \left( \frac{10^{-3}}{5} \right) = E_0 (0.25 + 0.2) \times 10^{-3}$$

$$= 0.45 \times 10^{-3} E_0 = 100$$

$$E_0 = \frac{100 \times 10^3}{0.45} = 2.222 \times 10^5 \text{ V/m.}$$

$$E_1 = \frac{\epsilon_0}{\epsilon_1} E_0 = \frac{2.222 \times 10^5}{5} = 0.44 \times 10^5 \text{ V/m.}$$

b.  $D_0 = \epsilon_0 E_0 = 8.854 \times 10^{-12} (2.22 \times 10^5) = 19.67 \times 10^{-7} = 1.967 \times 10^{-6} \text{ C/m}^2$   
 $D_1 = \epsilon_1 E_1 = 5 (8.854 \times 10^{-12}) (0.44 \times 10^5) = 1.967 \times 10^{-6} = D_0 \text{ C/m}^2$ .

c.  $q_s = 1.967 \times 10^{-6} \text{ C/m}^2$  in upper plate where  $V$  is +ve.

$$q_s = -1.967 \times 10^{-6} \text{ C/m}^2$$
 in lower plate where  $V$  is -ve.

d.  $Q = q_s A = 1.967 \times 10^{-6} (2 \times 10^{-2}) = 3.93 \times 10^{-8} \text{ C}$  in upper plate  
 $Q = q_s A = -1.967 \times 10^{-6} (2 \times 10^{-2}) = -3.93 \times 10^{-8} \text{ C}$  in lower plate

e.  $C_0 = \frac{Q}{V_0}$ ,  $V_0 = E_0 (0.25 \times 10^{-3}) = 2.222 \times 10^5 / (0.25 \times 10^{-3}) = 55.55 \text{ V}$

$$C_0 = \frac{3.93 \times 10^{-8}}{55.55} = 7.08 \times 10^{-10}$$

$$Q = \frac{Q}{V_1}, V_1 = E_1 (1 \times 10^{-3}) = 44.44 \text{ V}$$

$$G = \frac{3.93 \times 10^{-8}}{44.44} = 8.85 \times 10^{-10}$$

$$C_t = \frac{Q}{V} = \frac{3.93 \times 10^{-8}}{100} = 3.93 \times 10^{-10} = \frac{\epsilon_0 G_1}{\epsilon_0 + G_1} = \frac{7.07(8.86)}{7.07 + 8.86} \times 10^{-10}$$

f.  $W_{e0} = \frac{1}{2} \epsilon_0 V_0^2 = \frac{1}{2} (7.08 \times 10^{-10}) (55.55)^2 = 1.09 \times 10^{-6} J$

$$W_{e1} = \frac{1}{2} G V_1^2 = \frac{1}{2} (8.85 \times 10^{-10}) (44.44)^2 = 0.87 \times 10^{-6} J$$

$$W_{eT} = W_{e0} + W_{e1} = 1.96 \times 10^{-6} J$$

Also,  $W_{eT} = \frac{1}{2} C_t V^2 = \frac{1}{2} (3.93 \times 10^{-10}) (100^2) = 1.96 \times 10^{-6} J$

Ans 2.6 Before the insertion of slab

$$E = \frac{V}{4 \times 10^{-2}} = \frac{8}{4 \times 10^{-2}} = 200 V/m.$$

$$D = \epsilon_0 E = 8.854 \times 10^{-12} (200) = 17.708 \times 10^{-10} = 1.77 \times 10^{-8} C/m^2$$

a.  $q_s = \pm D = \pm 1.7708 \times 10^{-9} C/m^2$

$$Q = \pm q_s A = \pm 1.7708 \times 10^{-9} (64 \times 10^{-4}) = \pm 113.33 \times 10^{-13} C$$

b.  $E = 200 V/m.$

c.  $D = 1.7708 \times 10^{-9} C/m^2$

d.  $C = \epsilon_0 \frac{A}{d} = 8.854 \times 10^{-12} \times \frac{64 \times 10^{-4}}{4 \times 10^{-2}} = 1.41 \times 10^{-12}$

$$e. \quad W_e = \frac{1}{2} C V^2 = \frac{1}{2} (1.4166 \times 10^{-12}) (8^2) = 45.3326 \times 10^{-12} J$$

After the insertion of the slab,  $E$  remains same:

$$\epsilon_0 = \epsilon_1 = 200 \text{ V/m.}$$

$$D_0 = \epsilon_0 E_0 = 8.85 \times 10^{-12} (200) = 1.77 \times 10^{-9} \Rightarrow q_{s0} = 1.77 \times 10^{-9}$$

$$D_1 = \epsilon_1 E_1 = 2.56 D_0 = 4.533 \times 10^{-9} \Rightarrow q_{s1} = 4.533 \times 10^{-9}.$$

$$f. \quad Q_0 = \pm q_{s0} A = \pm 1.77 \times 10^{-9} (32 \times 10^{-4}) = \pm 5.66 \times 10^{-12} C$$

(in free space).

$$Q_1 = \pm q_{s1} A = \pm 4.533 \times 10^{-9} (32 \times 10^{-4}) = \pm 14.506 \times 10^{-12} C$$

(in dielectric)

$$g. \quad \epsilon_0 = \epsilon_1 = 200 \text{ V/m.}$$

$$h. \quad D_0 = 1.77 \times 10^{-9} \text{ C/m}^2 \text{ (in free space)}$$

$$D_1 = 4.53 \times 10^{-9} \text{ C/m}^2 \text{ (in dielectric).}$$

$$i. \quad C_0 = \frac{Q_0}{V} = \frac{5.66 \times 10^{-12}}{8} = 0.708 \times 10^{-12} \text{ (in free space)}$$

$$C_1 = \frac{Q_1}{V} = \frac{14.506 \times 10^{-12}}{8} = 1.813 \times 10^{-12} \text{ (in dielectric).}$$

$$j. \quad C_t = \frac{Q_t}{V} = \frac{(5.66 + 14.506) \times 10^{-12}}{8} = \frac{20.17 \times 10^{-12}}{8} = 2.521 \times 10^{-12} C$$

$$k. \quad W_{e0} = \frac{1}{2} C_0 V^2 = \frac{1}{2} (0.708 \times 10^{-12}) (8)^2 = 22.66 \times 10^{-12} \text{ J (in free space)}$$

$$l. \quad W_{e1} = \frac{1}{2} C_1 V^2 = \frac{1}{2} (1.813 \times 10^{-12}) (8)^2 = 58.025 \times 10^{-12} \text{ J (in dielectric)}$$

$$l. \quad W_{et} = \frac{1}{2} CtV^2 = \frac{1}{2} (2.52 \times 10^{-12})(8^2) = 80.691 \times 10^{-12} J.$$

$$W_{et} = W_{eo} + W_{ei}$$

$$\text{Ans 2.15} \quad M = \partial_z 1.245 \times 10^{-6} A/m, \quad H = \partial_z 5 \times 10^3 A/m.$$

$$a. \quad \bar{T}_{ms} = M \times \hat{n}$$

$$y=0: \quad \bar{T}_{ms} = \partial_z M_z \times (-\partial_y) = \partial_x M_z = \partial_x 1.245 \times 10^6$$

$$x=6 \text{ cm}: \quad \bar{T}_{ms} = \partial_z M_z \times (-\partial_x) = \partial_y M_z = \partial_y 1.245 \times 10^6$$

$$y=4 \text{ cm}: \quad \bar{T}_{ms} = \partial_z M_z \times \partial_y = -\partial_x M_z = -\partial_x 1.245 \times 10^6$$

$$x=0: \quad \bar{T}_{ms} = \partial_z M_z \times (-\partial_x) = -\partial_y M_z = -\partial_y 1.245 \times 10^6$$

$$z=0: \quad \bar{T}_{ms} = \partial_z M_z \times (-\partial_z) = 0$$

$$z=1 \text{ cm}: \quad \bar{T}_{ms} = \partial_z M_z \times \partial_z = 0$$

$$b. \quad \bar{T}_m = \nabla \times M = 0$$

$$c. \quad \bar{J}_m = \oint_S \bar{T}_m \cdot d\hat{s} = \iiint_V (\nabla \cdot \bar{T}_m) dv = 0$$

$$d. \quad \chi_m = \frac{M}{H} = \frac{1.245 \times 10^{-6}}{5 \times 10^3} = 249.$$

$$M_r = 1 + \chi_m \Rightarrow M_r = 250.$$

$$\text{Ans 2.20} \quad \sigma = 5.76 \times 10^7 \text{ S/m}, \quad \epsilon = 8.854 \times 10^{-12} \text{ f/m}$$

$$T_d = \frac{\epsilon}{\sigma} = \frac{8.854 \times 10^{-12}}{5.76 \times 10^7} = 1.5372 \times 10^{-19} \text{ s}$$

For Microwave region ( $f = 1-10 \text{ GHz}$ ), the period is

$$f = 1 \text{ GHz}: \quad T = \frac{1}{f} = \frac{1}{10^9} = 10^{-9} >> T_d = 1.53 \times 10^{-19}$$

$$f = 26 \text{ Hz} : T = \frac{1}{f} = \frac{1}{10^{10}} = 10^{-10} \gg T_\lambda = 1.537 \times 10^{-19}$$

For x-rays [ $\lambda = (1-10) \times 10^{-8} \text{ cm}$ ] the period is

$$\lambda = 1 \times 10^{-8} \text{ cm} : f = \frac{v}{\lambda} = \frac{3 \times 10^{10}}{10^{-8}} = 3 \times 10^{18}$$

$$T = \frac{1}{f} = \frac{1}{3 \times 10^{18}} = 0.33 \times 10^{-18}$$

$$\lambda = 10 \times 10^{-8} \text{ cm} : f = \frac{v}{\lambda} = \frac{3 \times 10^{10}}{10 \times 10^{-8}} = 3 \times 10^{17}$$

$$T = \frac{1}{f} = \frac{1}{3 \times 10^{17}} = 0.33 \times 10^{-17}$$

which are comparable to relaxation time constant.

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CHAPTER 3

Ans 3.1  $\nabla \times E = -M_i - j\omega M H$ ,  $\nabla \times H = J_i + \sigma E + j\omega \mu E$

$$\nabla \times \nabla \times E = -\nabla \times M_i - j\omega M \nabla \times H$$

$$\nabla \times \nabla \times H = \nabla \times J_i + -\nabla \times E + j\omega \epsilon \nabla \times E$$

Using Maxwell's equations from above & the vector identity of

$$\nabla \times \nabla \times F = \nabla(\nabla \cdot F) - \nabla^2 F$$

We can write,

$$\begin{aligned} \nabla(\nabla \cdot E) - \nabla^2 E &= -\nabla \times M_i - j\omega M [J_i + \sigma E + j\omega \mu E] \\ &= -\nabla \times M_i - j\omega M J_i - j\omega \mu \sigma E + \omega^2 \mu \epsilon E \end{aligned}$$

$$\text{Since } \nabla \cdot D = \nabla \cdot (\epsilon E) = \epsilon \nabla \cdot E = q/v_r \Rightarrow \nabla \cdot E = \frac{q/v_r}{\epsilon}$$

$$\therefore \nabla^2 E = \nabla \times M_i + j\omega M J_i + \frac{1}{\epsilon} \nabla q/v_r + j\omega \mu \sigma E - \omega^2 \mu \epsilon E$$

which is an uncoupled second order differential equation.

Using the equation for the magnetic field from above along with Maxwell's equations and the vector identity,

$$\nabla(\nabla \cdot H) - \nabla^2 H = \nabla \times J_i + (\sigma + j\omega \epsilon) \nabla \times E = \nabla \times J_i + (\sigma + j\omega \epsilon) (-M_i - j\omega M H)$$

$$\text{Since } \nabla \cdot B = \nabla \cdot (M_H) = \mu \nabla \cdot H = q/v_m \Rightarrow \nabla \cdot H = \frac{1}{\mu} q/v_m$$

$$\text{Then, } \nabla^2 H = -\nabla \times J_i + \sigma M_i + \frac{1}{\mu} \nabla q/v_m + j\omega \epsilon M_i + j\omega \mu H - \omega^2 \mu \epsilon H$$

which also is an uncoupled second order diff. eqn.

Ans 3.2  $\frac{d^2 f}{dx^2} = -\beta_n^2 f, \quad f = f_1 = A_1 e^{-i\beta_n x} + B_1 e^{+i\beta_n x}$

Using  $f = f_1 = A_1 e^{-i\beta_n x}$ , then

$$(-1\beta_n)^2 A_1 e^{-i\beta_n x} = -\beta_n^2 A_1 e^{-i\beta_n x} = -\beta_n A_1 e^{-i\beta_n x}$$

The same can be shown by letting  $f = f_1 = B_1 e^{+i\beta_n x}$

Now let us try the sinusoidal solutions -

$$\text{let } f = f_2 = C \cos(\beta_n x)$$

Substituting this into the differential eq leads to

$$-\beta_n^2 C \cos(\beta_n x) = -\beta_n^2 C \cos(\beta_n x)$$

The same can be shown by letting  $f = f_2 = D_1 \sin(\beta_n x)$

Ans 3.3 Ex  $(x, y, z, t) = [C_1 \cos(\beta_n x) + D_1 \sin(\beta_n x)] [C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)]$   
 $B_3 \cos(\omega t + \beta_z z)$

To follow a point ZP for different values of t, we must maintain constant the amplitude of the cosine term. This is accomplished by letting

$$\omega t + \beta_z z = Q = \text{constant}$$

Taking derivative of both sides with respect to time, we can write

$$\omega(1) + \beta_z \frac{dz}{dt} = 0 \Rightarrow \omega + \beta_z v_p = 0 \Rightarrow v_p = \frac{-\omega}{\beta_z}$$

which indicates that the wave is moving in z-direction.

$$\text{Ans 3.4} \quad \nabla^2 E_n - \gamma^2 E_n = 0 = \frac{\partial^2 E_n}{\partial x^2} + \frac{\partial^2 E_n}{\partial y^2} + \frac{\partial^2 E_n}{\partial z^2} - \gamma^2 E_n$$

Letting  $E_n(x, y, z) = f(x) g(y) h(z)$  & substituting above gives -

$$g h \frac{d^2 f}{dx^2} + f h \frac{d^2 g}{dy^2} + f g \frac{d^2 h}{dz^2} - \gamma^2 fgh = 0$$

Dividing both sides by  $fgh$ , we can write

$$\frac{1}{f} \frac{d^2 f}{dx^2} + \frac{1}{g} \frac{d^2 g}{dy^2} + \frac{1}{h} \frac{d^2 h}{dz^2} = \gamma^2.$$

By letting each term on the left equal to a constant leads to

$$\frac{1}{f} \frac{d^2 f}{dx^2} = \gamma_x^2 \Rightarrow f = f_1 = A_1 e^{-\gamma_x x} + B_1 e^{+\gamma_x x}, \quad f = f_2 = G \cosh(\gamma_x x)$$

$$\frac{1}{g} \frac{d^2 g}{dy^2} = \gamma_y^2 \Rightarrow g = g_1 = A_2 e^{-\gamma_y y} + B_2 e^{+\gamma_y y} + D_1 \sinh(\gamma_y y)$$

$$g = g_2 = C_2 \cosh(\gamma_y y) + D_2 \sinh(\gamma_y y)$$

$$\frac{1}{h} \frac{d^2 h}{dz^2} = -\gamma_z^2 \Rightarrow h = h_1 = A_3 e^{-\gamma_z z} + B_3 e^{+\gamma_z z}$$

$$h = h_2 = C_3 \cosh(\gamma_z z) + D_3 \sinh(\gamma_z z)$$

provided that  $\gamma_x^2 + \gamma_y^2 + \gamma_z^2 = \gamma^2$

$$\text{Ans 3.5} \quad \nabla(\nabla \cdot \vec{E}) - \nabla \times \nabla \times \vec{E} = -\beta^2 \vec{E}, \quad \vec{E} = \hat{a}_r E_r + \hat{a}_\theta E_\theta + \hat{a}_z E_z$$

Using cylindrical coordinates

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z}$$

$$\nabla(\nabla \cdot \vec{E}) = \nabla \left[ \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z} \right]$$

$$= \hat{a}_r \left\{ \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z} \right] \right\}$$

$$+ \hat{a}_\theta \frac{1}{r} \left\{ \frac{\partial}{\partial \theta} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z} \right] \right\}$$

$$+ \hat{a}_z \left\{ \frac{\partial}{\partial z} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_z}{\partial z} \right] \right\}$$

$$= \hat{a}_r \left[ \frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r} \frac{\partial E_r}{\partial r} - \frac{E_r}{r^2} + \frac{1}{r} \frac{\partial^2 E_\theta}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial E_\theta}{\partial \theta} \right.$$

$$\left. + \frac{\partial^2 E_z}{\partial r \partial z} \right] + \hat{a}_\theta \left[ \frac{1}{r} \frac{\partial^2 E_r}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial E_r}{\partial \theta} \right.$$

$$\left. + \frac{1}{r^2} \frac{\partial^2 E_\theta}{\partial \theta^2} + \frac{1}{r} \frac{\partial^2 E_z}{\partial \theta \partial z} \right] + \hat{a}_z \left[ \frac{\partial^2 E_r}{\partial r \partial z} \right.$$

$$\left. + \frac{1}{r} \frac{\partial E_r}{\partial z} + \frac{1}{r} \frac{\partial^2 E_\theta}{\partial \theta \partial z} + \frac{\partial^2 E_z}{\partial z^2} \right]$$

$$\nabla \times \vec{E} = \hat{a}_r \left[ \frac{1}{r} \frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} \right] + \hat{a}_\theta \left[ \frac{\partial E_z}{\partial z} - \frac{\partial E_r}{\partial z} \right]$$

$$+ \hat{a}_z \left[ \frac{\partial E_r}{\partial \theta} + \frac{E_\theta}{r} - \frac{1}{r} \frac{\partial E_r}{\partial \theta} \right]$$

$$\nabla \times \nabla \times \vec{E} = \partial_s \left\{ \frac{1}{s} \frac{\partial^2 \bar{E}_\phi}{\partial s \partial q} + \frac{1}{s^2} \frac{\partial \bar{E}_\phi}{\partial q} - \frac{1}{s^2} \frac{\partial^2 \bar{E}_\phi}{\partial q^2} - \frac{\partial^2 \bar{E}_s}{\partial z^2} \right. \\ \left. + \frac{\partial^2 \bar{E}_z}{\partial s \partial z} \right\} + \partial_\phi \left\{ \frac{1}{s} \frac{\partial^2 \bar{E}_z}{\partial \phi \partial z} - \frac{\partial^2 \bar{E}_\phi}{\partial z^2} - \frac{\partial^2 \bar{E}_\phi}{\partial s^2} \right. \\ \left. - \frac{1}{s} \frac{\partial \bar{E}_\phi}{\partial s} + \frac{\bar{E}_\phi}{s^2} + \frac{1}{s} \frac{\partial^2 \bar{E}_s}{\partial s \partial \phi} - \frac{1}{s^2} \frac{\partial \bar{E}_s}{\partial \phi} \right\} \\ + \partial_z \left\{ \frac{\partial^2 \bar{E}_s}{\partial s \partial z} + \frac{1}{s} \frac{\partial \bar{E}_s}{\partial z} - \frac{\partial^2 \bar{E}_z}{\partial s^2} - \frac{1}{s^2} \frac{\partial^2 \bar{E}_z}{\partial q^2} - \frac{1}{s} \frac{\partial \bar{E}_z}{\partial s} \right. \\ \left. + \frac{1}{s} \frac{\partial^2 \bar{E}_\phi}{\partial q \partial z} \right\}$$

Substituting all of these into the wave equation and equating identical components from left and right hand side leads to:

$$S \text{ component: } \left[ \frac{\partial^2 \bar{E}_s}{\partial s^2} + \frac{1}{s} \frac{\partial \bar{E}_s}{\partial s} + \frac{1}{s^2} \frac{\partial^2 \bar{E}_s}{\partial \phi^2} + \frac{\partial^2 \bar{E}_s}{\partial z^2} \right] - \frac{\bar{E}_s}{s^2} - \frac{2}{s^2} \frac{\partial \bar{E}_\phi}{\partial \phi} \\ = -\beta^2 \bar{E}_s$$

$$\text{or } \nabla^2 \bar{E}_s + \left( -\frac{\bar{E}_s}{s^2} - \frac{2}{s^2} \frac{\partial \bar{E}_\phi}{\partial \phi} \right) = -\beta^2 \bar{E}_s$$

$$\phi \text{ component: } \left[ \frac{\partial^2 \bar{E}_\phi}{\partial s^2} + \frac{1}{s} \frac{\partial \bar{E}_\phi}{\partial s} + \frac{1}{s^2} \frac{\partial^2 \bar{E}_\phi}{\partial \phi^2} + \frac{\partial^2 \bar{E}_\phi}{\partial z^2} \right] - \frac{1}{s^2} + \frac{2}{s^2} \frac{\partial \bar{E}_s}{\partial \phi} \\ = -\beta^2 \bar{E}_\phi$$

$$\text{or } \nabla^2 \bar{E}_\phi + \left[ -\frac{\bar{E}_\phi}{s^2} + \frac{2}{s^2} \frac{\partial \bar{E}_s}{\partial \phi} \right] = -\beta^2 \bar{E}_\phi$$

$$z \text{ component: } \frac{\partial^2 \bar{E}_z}{\partial s^2} + \frac{1}{s} \frac{\partial \bar{E}_z}{\partial s} + \frac{1}{s^2} \frac{\partial^2 \bar{E}_z}{\partial \phi^2} + \frac{\partial^2 \bar{E}_z}{\partial z^2} \\ = -\beta^2 \bar{E}_z$$

$$\text{or } \nabla^2 \bar{E}_z = -\beta^2 \bar{E}_z$$

FIELDS AND WAVES  
CHAPTER 4

$$\text{Ans 4.3} \quad H = \frac{1}{120\pi} (\hat{a}_x - 2\hat{a}_y) e^{-i\beta_0 z}$$

$$a. \quad E = \frac{1}{120\pi} (-\hat{a}_y - 2\hat{a}_x) e^{-i\beta_0 z} = -(2\hat{a}_x + \hat{a}_y) e^{-i\beta_0 z}$$

$$b. \quad E = \operatorname{Re}[E e^{i\omega t}] = -(2\hat{a}_x + \hat{a}_y) \cos(\omega t - \beta_0 z)$$

$$H = \operatorname{Re}[H e^{i\omega t}] = \frac{1}{120\pi} (\hat{a}_x - 2\hat{a}_y) \cos(\omega t - \beta_0 z)$$

$$S = \vec{E} \times \vec{H} = \frac{-1}{120\pi} (2\hat{a}_x + \hat{a}_y) \times (\hat{a}_x - 2\hat{a}_y) \cos^2(\omega t - \beta_0 z) = \hat{a}_z \frac{5}{120\pi} \cos^2(\omega t - \beta_0 z)$$

$$= \hat{a}_z 1.326 \times 10^{-2} \cos^2(\omega t - \beta_0 z)$$

$$c. \quad S_{\text{avg}} = \frac{1}{2} \operatorname{Re}[(\vec{E} \times \vec{H})] = \frac{1}{2} \operatorname{Re}\left[-(2\hat{a}_x + \hat{a}_y) e^{-i\beta_0 z} \times \frac{1}{120\pi} (\hat{a}_x - 2\hat{a}_y) e^{i\beta_0 z}\right]$$

$$= \hat{a}_z \frac{5}{240\pi} = \hat{a}_z 6.63 \times 10^{-3} \text{ W/m}^2.$$

$$\text{Ans 4.8} \quad E = \hat{a}_x 4 \times 10^{-3} e^{+i\beta_0 z}, \quad \beta_0 = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi \times 3 \times 10^8 \frac{1}{3 \times 10^8} = 2\pi \text{ rad/m.}$$

$$a. \quad H = -\hat{a}_y \frac{4 \times 10^{-3}}{\eta_0} e^{i\beta_0 z} = -\hat{a}_y 10.61 \times 10^{-6} e^{+i\beta_0 z}$$

$$b. \quad E = \operatorname{Re}[E e^{i\omega t}] = \hat{a}_x 4 \times 10^{-3} \cos(\omega t + \beta_0 z)$$

$$H = \operatorname{Re}[H e^{i\omega t}] = -\hat{a}_y 10.61 \times 10^{-6} \cos(\omega t + \beta_0 z)$$

$$c. \quad S_{\text{avg}} = \frac{1}{2} \operatorname{Re}(\vec{E} \times \vec{H}^*) = -\hat{a}_z 2.22 \times 10^{-9}$$

$$S = \vec{E} \times \vec{H} = -\hat{a}_z 42.44 \times 10^{-9} \cos^2(\omega t + \beta_0 z)$$

$$d. \overline{W} = \frac{1}{4} \epsilon_0 |E|^2 = \frac{1}{4} (8.85 \times 10^{-12}) (16 \times 10^{-6})^2 = 35.416 \times 10^{-18} \text{ J/m}^3$$

$$\overline{W_m} = \frac{1}{4} M_0 |H|^2 = \frac{1}{4} (4\pi \times 10^{-7}) (10.61 \times 10^{-6})^2 = 353.656 \times 10^{-19} \text{ J/m}^3$$

$$W_e = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} (8.85 \times 10^{-12}) (16 \times 10^{-6}) \cos^2(\omega t + \beta_0 z) \\ = 70.83 \times 10^{-18} \cos^2(\omega t + \beta_0 z)$$

$$W_{mm} = \frac{1}{2} M_0 H^2 = \frac{1}{2} (4\pi \times 10^{-7}) (10.61 \times 10^{-6})^2 = 70.731 \times 10^{-18} \cos^2(\omega t + \beta_0 z)$$

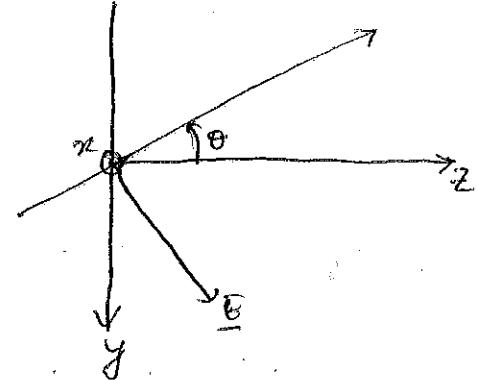
Ans 4.15  $E = 5 \times 10^{-3} (4\hat{a}_y + 3\hat{a}_z) e^{j(8y - 8z)}$

$$E = 5 \times 10^{-3} (s) \left( \frac{4}{5} \hat{a}_y + \frac{3}{5} \hat{a}_z \right) e^{j(10(0.6y - 0.8z))} \\ = 25 \times 10^{-3} (0.8\hat{a}_y + 0.6\hat{a}_z) e^{j(10(0.6y - 0.8z))}$$

a.  $E = 25 \times 10^{-3} (\cos \theta \hat{a}_y + \sin \theta \hat{a}_z) e^{j\beta_0 (\sin \theta y - \cos \theta z)}$   
 $\cos \theta = 0.8 \Rightarrow \theta = 36.87^\circ$   
 $\sin \theta = 0.6 \Rightarrow \theta = 36.87^\circ$ .

b.  $E = 25 \times 10^{-3} (\cos \theta \hat{a}_y + \sin \theta \hat{a}_z) e^{j(\beta_y y - \beta_z z)}$

$$\beta_x = 10 \text{ rad/m}, \beta_y = 6 \text{ rad/m}, \beta_z = 8 \text{ rad/m}$$



c.  $\lambda_x = \frac{2\pi}{\beta_x} = \frac{2\pi}{10} = 0.6283 \text{ m}; \lambda_y = \frac{2\pi}{\beta_y} = \frac{2\pi}{6} = 1.047 \text{ m}; \lambda_z = \frac{2\pi}{\beta_z} = \frac{2\pi}{8} = 0.785 \text{ m}$

d.  $V_{ph} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$

$$\cos\theta = \frac{V_x}{V_z} \Rightarrow V_z = \frac{V_x}{\cos\theta} = \frac{3 \times 10^8}{0.8} = 3.75 \times 10^8 \text{ m/s}$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{V_y}{V_z} \Rightarrow V_y = \frac{V_z}{\cos\left(\frac{\pi}{2} - \theta\right)} = \frac{V_z}{\sin\theta} = \frac{3 \times 10^8}{0.6} = 5 \times 10^8 \text{ m/s}$$

e  $V_{ex} = 3 \times 10^8 \text{ m/s}$

$$V_{ez} = V_{ex}\cos\theta = 3 \times 10^8 (0.8) = 2.4 \times 10^8 \text{ m/s}$$

$$V_{ey} = V_{ex}\sin\theta = 3 \times 10^8 (0.6) = 1.8 \times 10^8 \text{ m/s}$$

f  $\beta_0 = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi f \sqrt{\mu_0 \epsilon_0} = 10 \Rightarrow f = \frac{10}{2\pi \sqrt{\mu_0 \epsilon_0}} = \frac{15 \times 10^8}{\pi} = 4.77 \times 10^8$

g  $H = -\hat{a}_x \frac{25 \times 10^3}{377} e^{j(10(0.6y - 0.8z))} = -\hat{a}_x 66.313 \times 10^{-6} e^{j(6y - 8z)}$

Ans 4.19  $E^+ = E_0^+ (\hat{a}_x (\cos\theta_i - \hat{a}_z \sin\theta_i) e^{-j\beta(x \sin\theta_i + z \cos\theta_i)}$

$$H^+ = \hat{a}_y \frac{E_0^+}{\eta} e^{-j\beta(x \sin\theta_i + z \cos\theta_i)}$$

$$E^+ = \operatorname{Re}[E^+ e^{j\omega t}] = E_0^+ (\hat{a}_x (\cos\theta_i - \hat{a}_z \sin\theta_i) \cos[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)])$$

$$H^+ = \operatorname{Re}[H^+ e^{j\omega t}] = \hat{a}_y \frac{E_0^+}{\eta} \cos[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)]$$

$$S^+ = E^+ \times H^+ = \frac{(E_0^+)^2}{\eta} (\hat{a}_x (\cos\theta_i - \hat{a}_z \sin\theta_i) \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)])$$

$$Sz^+ = \frac{(E_0^+)^2}{\eta} \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)] \cos\theta_i = \frac{(E_0^+)^2}{\eta} (\cos\theta_i) \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)]$$

$$Mee = \frac{1}{2} E |E|^2 = \frac{(E_0^+)^2}{2} E \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)]$$

$$Um = \frac{1}{2} M |H|^2 = \frac{\mu}{2\eta^2} (E_0^+)^2 \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)] = \frac{E_0^2}{2} \frac{\epsilon \cos^2[\omega t - \beta(x \sin\theta_i + z \cos\theta_i)]}{\eta^2}$$

$$Vet = \frac{Sz^+}{We + Um} = \frac{[(E_0^+)^2 / \eta]}{\frac{(E_0^+)^2 \epsilon}{2} + \frac{(E_0^+)^2}{2} \epsilon} = \frac{\cos\theta_i}{\eta \epsilon} = \frac{1}{\sqrt{M\epsilon}} \cos\theta_i = V \cos\theta_i \leq V$$

$$\text{Ans 4.20} \quad f = 36 \text{ Hz}, \sigma = 5.76 \times 10^7 \text{ S/m}, \epsilon = \epsilon_0, \mu = \mu_0$$

$$a. \text{ Since } \frac{\sigma}{\omega_{ho}} = \frac{5.76 \times 10^7}{2\pi(3 \times 10^9) \left(\frac{10^{-9}}{36\pi}\right)} = 6(5.76) \times 10^7 > 1$$

$$\text{then } \eta = \sqrt{\frac{\omega h}{2\pi}} (1+j) = \sqrt{\frac{\omega h}{\sigma}} (1+j) = \sqrt{\frac{2\pi(3 \times 10^9)(4\pi \times 10^{-7})}{2(5.76 \times 10^7)}} (1+j) = 2\pi \sqrt{\frac{30}{5.76}} \times 10^{-3} (1+j)$$

$$\approx 14.33(1+j) \times 10^{-3} = 14.33 \times 10^{-3} (1+j)$$

$$b. \quad S = \frac{1}{\alpha} \approx \sqrt{\frac{2}{\omega \mu_0}} = \sqrt{\frac{2}{2\pi(3 \times 10^9)(4\pi \times 10^{-7})(5.76 \times 10^7)}} = \frac{1 \times 10^{-4}}{2\pi \sqrt{30}(5.76)} = 1.21 \times 10^{-6} \text{ m}$$

$$\text{Ans 4.22} \quad \sigma = 4 \text{ S/m}, \epsilon_0 = 81, \mu_0 = 1, f = 10^4 \text{ Hz}$$

$$\frac{\sigma}{\omega \epsilon} = \frac{4}{2\pi \times 10^4 \times \frac{81 \times 10^{-4}}{36\pi}} = \frac{4(18)}{81} \times 10^5 = \frac{8}{9} \times 10^5 > 1$$

$$a. \quad \alpha = \beta \approx \sqrt{\frac{\omega \mu_0 \sigma}{2}} = \sqrt{\frac{2\pi \times 10^4 (4\pi \times 10^{-7}) 4}{2}} = 4\pi \times 10^2 \sqrt{10} = 0.3973$$

$$\Rightarrow \gamma = \alpha + j\beta = 0.3973(1+j)$$

$$b. \quad \beta = \frac{\omega}{V} \Rightarrow V = \frac{\omega}{\beta} = \frac{2\pi \times 10^4}{0.3973} = 15.811 \times 10^4 - 1.5811 \times 10^5 \sqrt{\frac{2\omega}{\mu_0}}$$

$$c. \quad \lambda = \frac{2\pi}{\beta} \approx \frac{2\pi}{0.3973} = 15.811 \text{ m.}$$

$$d. \quad \alpha = 0.3973 \text{ N/m}$$

$$e. \quad S = \frac{1}{\alpha} = \frac{1}{0.3973} = 2.516 \text{ m.}$$

$$\text{Ans 4.28} \quad H = i(\hat{a}_y - j\hat{a}_z)\frac{E_0}{\eta_0} e^{+i\beta_0 x}$$

- a. Circular polarization because of two equal components and  $90^\circ$  time phase difference.
- b. CCW because the  $y$  component leads the  $z$  component.

c.  ~~$E = E_0 (-\hat{a}_y + j\hat{a}_z) e^{+i\beta_0 x}$~~

$$S_{\text{avg}} = \frac{1}{2} \operatorname{Re}(E_x H^*) = -\hat{a}\pi \frac{1}{2\eta_0} |E|^2 = -\hat{a}\pi \frac{2|E_0|^2}{2(3\pi)} = -\hat{a}\pi 2.6525 \times 10^{-3}$$

d.  ~~$S_{\text{av}} = \frac{1}{2} \operatorname{Re}(E_x H^*) = -\hat{a}\pi \frac{1}{2\eta_0} |E|^2 = -\hat{a}\pi \frac{2|E_0|^2}{2}$~~

d.  $\gamma = \tan^{-1} \left[ \frac{1}{1} \right] = \frac{\pi}{4} \Rightarrow 2\gamma = 90^\circ$

$$S = \Phi_y - \Phi_z = -180 - 90 = -270^\circ = +90^\circ$$

} North pole.

FIELDS AND WAVES  
CHAPTER 5

Ans 5)  $E^i = \hat{a}_y 2 \times 10^{-3} e^{-i\beta z}$

a.  $H^i = -\hat{a}_x \frac{2 \times 10^{-3}}{\eta} e^{-i\beta z} = -\hat{a}_x \frac{2 \times 10^{-3}}{376.73/2} e^{-i\beta z} = -\hat{a}_x 1.06 \times 10^{-3} e^{-i\beta z}$

b.  $\eta_0 = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.73, \eta = \sqrt{\frac{\mu_0}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{376.73}{2} = 188.37$

$$\Gamma^b = \frac{\eta_0 - \eta}{\eta_0 + \eta} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}} - \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{4\epsilon_0}} + \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$T^b = \frac{2\eta_0}{\eta_0 + \eta} = \frac{2 \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{4\epsilon_0}} + \frac{1}{2} \sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{2}{3/2} = \frac{4}{3} = 1.333$$

c.  $E^s = \hat{a}_y \Gamma^b E_0 e^{+i\beta z} = \hat{a}_y \frac{2 \times 10^{-3}}{3} e^{+i\beta z} = \hat{a}_y 0.667 \times 10^{-3} e^{+i\beta z}$

$$H^s = \hat{a}_x \frac{\Gamma^b E_0}{\eta} e^{+i\beta z} = \hat{a}_x \frac{2 \times 10^{-3}}{3(188.37)} e^{+i\beta z} = \hat{a}_x 3.54 \times 10^{-6} e^{+i\beta z}$$

$$E^t = \hat{a}_y T^b E_0 e^{-i\beta z} = \hat{a}_y \frac{4}{3} (2 \times 10^{-3}) e^{-i\beta z} = \hat{a}_y 2.667 \times 10^{-3} e^{-i\beta z}$$

$$H^t = \hat{a}_x \frac{T^b E_0}{\eta_0} e^{-i\beta z} = -\hat{a}_x \frac{2 \cdot 667 \times 10^{-3}}{376.73} e^{-i\beta z} = -\hat{a}_x 7.08 \times 10^{-6} e^{-i\beta z}$$

d.  $S_{av}^i = \frac{1}{2} \operatorname{Re}(E^i \times H^{i*}) = \hat{a}_z \frac{|E^i|^2}{2\eta} = \hat{a}_z \frac{|2 \times 10^{-3}|^2}{2(188.37)} = \hat{a}_z 1.06 \times 10^{-8} W/m^2$

$$S_{av}^s = \frac{1}{2} \operatorname{Re}(E^s \times H^{s*}) = -\hat{a}_z |\Gamma^b|^2 S_{av}^i = -\hat{a}_z \left(\frac{1}{3}\right)^2 (1.06 \times 10^{-8}) \\ = -\hat{a}_z 1.18 \times 10^{-9} W/m^2$$

$$\begin{aligned}
 S_{\text{av}}^t &= \frac{1}{2} \operatorname{Re} (E^t * H^{t*}) = \hat{a}Z (S^i - S_{\text{av}}^i) = \hat{a}Z (1 - |\Gamma^b|^2) S_{\text{av}}^i \\
 &= \hat{a}Z |\Gamma^b|^2 \frac{\eta}{\eta_0} S_{\text{av}}^i \\
 &= \hat{a}Z 9.44 \times 10^{-9} \text{ W/m}^2
 \end{aligned}$$

Ans 5.2

$$\Gamma^b = \frac{n_1 - n_0}{n_1 + n_0} = \frac{\sqrt{\frac{\mu_0}{\epsilon_0}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{1-9}{1+9} = \frac{-8}{10} = -0.8$$

$$S_{\text{av}}^{\text{ref}} = |\Gamma^b|^2 S_{\text{av}}^i$$

$$\frac{S_{\text{av}}^{\text{ref}}}{S_{\text{av}}} = |\Gamma^b|^2 = | -0.8 |^2 = 0.64 \text{ or } 64\%.$$

$$\begin{aligned}
 S_{\text{av}}^t &= (1 - |\Gamma^b|^2) S_{\text{av}}^i \\
 \frac{S_{\text{av}}^t}{S_{\text{av}}^i} &= (1 - |\Gamma^b|^2) = 1 - 0.64 = 0.36 \text{ or } 36\%.
 \end{aligned}$$

Ans 5.7 In the three different regions, we can work the corresponding fields as:

$$\left. \begin{aligned}
 E_1 &= E_1^+ e^{-i\beta_0 z} + E_1^- e^{+i\beta_0 z} \\
 H_1 &= \frac{E_1^+}{\eta_0} e^{-i\beta_0 z} - \frac{E_1^-}{\eta_0} e^{+i\beta_0 z}
 \end{aligned} \right\} z \leq 0$$

$$\left. \begin{aligned}
 E_2 &= E_2^+ e^{-i\beta_0 z} + E_2^- e^{+i\beta_0 z} \\
 H_2 &= \frac{E_2^+}{\eta_1} e^{-i\beta_0 z} - \frac{E_2^-}{\eta_1} e^{+i\beta_0 z}
 \end{aligned} \right\} 0 \leq z \leq l$$

$$\left. \begin{aligned}
 E_3 &= E_3^+ e^{-i\beta_0 (z-l)} \\
 H_3 &= \frac{E_3^+}{\eta_0} e^{-i\beta_0 (z-l)}
 \end{aligned} \right\} z \geq l$$

We assume that the electric fields are x-polarized while the magnetic fields are y-polarized.

Applying the continuity of the tangential electric and magnetic fields at the interface at  $z=0$ , we have

$$E_1^+ + E_1^- = E_2^+ + E_2^- \Rightarrow E_1^+ - E_1^- = E_2^+ - E_2^- \quad (1)$$

$$\frac{E_1^+ - E_1^-}{n_0} = \frac{E_2^+ - E_2^-}{n_1} \Rightarrow E_1^+ - E_1^- = \frac{n_0}{n_1} (E_2^+ - E_2^-) \quad (2)$$

Solving (1) and (2), we get

$$E_1^+ = \frac{E_2^+}{2} \left( 1 + \frac{n_0}{n_1} \right) + \frac{E_2^-}{2} \left( 1 - \frac{n_0}{n_1} \right) = E_2^+ \left( \frac{n_1 + n_0}{2n_1} \right) + E_2^- \left( \frac{n_1 - n_0}{2n_1} \right) \quad (3)$$

$$E_1^- = E_2^+ \left( \frac{n_1 - n_0}{2n_1} \right) + E_2^- \left( \frac{n_1 + n_0}{2n_1} \right) \quad (4)$$

Now apply the boundary conditions at  $z=t$ , we get

$$E_2^+ e^{-i\beta_1 t} + E_2^- e^{+i\beta_1 t} = E_3^+ \Rightarrow E_2^+ e^{-i\beta_1 t} + E_2^- e^{+i\beta_1 t} = E_3^+ \quad (5)$$

$$\frac{E_2^+}{n_1} e^{-i\beta_1 t} - \frac{E_2^-}{n_1} e^{+i\beta_1 t} = \frac{E_3^+}{n_0} \Rightarrow E_2^+ e^{-i\beta_1 t} - E_2^- e^{+i\beta_1 t} = \frac{n_1}{n_0} E_3^+ \quad (6)$$

Solving (5) & (6) for  $E_2^+$  &  $E_2^-$ , we get

$$E_2^+ = \frac{E_3^+ e^{+i\beta_1 t}}{2} \left( \frac{n_1 + n_0}{n_0} \right) = E_3^+ e^{+i\beta_1 t} \left( \frac{n_1 + n_0}{2n_0} \right) = E_3^+ e^{+i\beta_1 t} \left( \frac{n_0 + n_1}{2n_0} \right) \quad (7)$$

$$E_2^- = E_3^+ e^{-i\beta_1 t} \left( \frac{n_0 - n_1}{2n_0} \right) = -E_3^+ e^{-i\beta_1 t} \left( \frac{n_1 - n_0}{2n_0} \right) \quad (8)$$

Substituting (7) & (8) into (3), we get

$$E_1^+ = \frac{E_3^+ e^{+i\beta_1 t}}{n_0 n_1} \left( \frac{n_0 + n_1}{2} \right)^2 - \frac{E_3^+ e^{-i\beta_1 t}}{n_0 n_1} \left( \frac{n_1 - n_0}{2} \right)^2 = \frac{E_3^+}{4n_0 n_1} \left[ e^{+i\beta_1 t} (n_0 + n_1)^2 - e^{-i\beta_1 t} (n_1 - n_0)^2 \right]$$

$$\frac{E_3^+}{E_1^+} = \Gamma^b = \frac{\frac{4n_0 n_1}{(n_1+n_0)^2 e^{+i\beta_1 t} - (n_1-n_0)^2 e^{-i\beta_1 t}}}{\frac{4n_0 n_1}{(n_1+n_0)^2 e^{-i\beta_1 t} - (n_1-n_0)^2 e^{+i\beta_1 t}}} = \frac{4n_0 n_1 e^{-i\beta_1 t}}{(n_1+n_0)^2 - (n_1-n_0)^2 e^{-i2\beta_1 t}}$$

To find the reflection coefficient, we substitute (7) & (8) into (3) and (4) and then take the ratio of (4) and (3). We get;

$$\frac{E_1^-}{E_1^+} = \Gamma^b = \frac{\frac{e^{i\beta_1 t}}{4n_0 n_1} (n_1-n_0)(n_1+n_0) - \frac{e^{-i\beta_1 t}}{4n_0 n_1} (n_1+n_0)(n_1-n_0)}{\frac{e^{i\beta_1 t}}{4n_0 n_1} (n_1+n_0)(n_1+n_0) - \frac{e^{-i\beta_1 t}}{4n_0 n_1} (n_1-n_0)(n_1-n_0)}$$

$$\frac{E_1^-}{E_1^+} = \Gamma = \frac{(n_1+n_0)(n_1-n_0)(1-e^{-i2\beta_1 t})}{(n_1+n_0)^2 - (n_1-n_0)^2 e^{-i2\beta_1 t}}$$

Ans 5.9 For the prism with dielectric constant of 2.25,  $\theta_c$  is -

$$\theta_c = \sin^{-1} \sqrt{\frac{\epsilon_0}{\epsilon_1 \epsilon_2}} = \sin^{-1} \sqrt{\frac{1}{2.25}} = \sin^{-1} \sqrt{0.44} = \sin^{-1}(0.66) = 41.81^\circ.$$

Therefore at the hypotenuse the reflection coefficient  $|\Gamma|=1$  since the incident angle of  $45^\circ$  is greater than the critical angle of  $41.81^\circ$ .

$$\beta_{20} = \frac{n_2 - n_0}{n_2 + n_0} = \frac{\frac{1}{1.5} n_0 - n_0}{\frac{1}{1.5} n_0 + n_0} = \frac{1 - 1.5}{1 + 1.5} = -0.2$$

$$\begin{aligned} \Gamma_{02} &= -\beta_{20} = +0.2 \\ \text{Say } \frac{\epsilon_0}{\epsilon_2} &= (1 - (\beta_{20})^2) (r^2) (1 - |\Gamma_{02}|^2) = (1 - |\beta_{20}|^2)^2 = [1 - (0.2)^2]^2 = (0.96)^2 \\ &= 0.9216 \text{ or } 92.16\% \end{aligned}$$

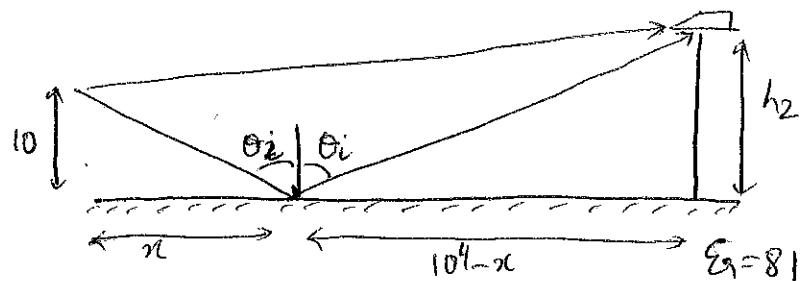
$$\text{Ans 5.17} \quad \theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right), \quad \theta_C = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$(a) \quad \theta_B = \tan^{-1}\left(\sqrt{\frac{1}{81}}\right) = \tan^{-1}\left(\frac{1}{9}\right) = 6.34^\circ \quad \theta_C = \sin^{-1}\left(\sqrt{\frac{1}{81}}\right) = 6.379^\circ.$$

$$(b) \quad \theta_B = \tan^{-1}\sqrt{81} = 83.64^\circ \quad \theta_C = \sin^{-1}(\sqrt{81}) = \text{Does not exist}$$

$$(c) \quad \theta_B = \tan^{-1}\left(\sqrt{\frac{1}{9}}\right) = 18.435^\circ \quad \theta_C = \sin^{-1}\left(\sqrt{\frac{1}{9}}\right) = 19.47^\circ.$$

$$\text{Ans 5.19} \quad \theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$$



For the reflected wave not to possess a parallel polarized component, then the incident angle must be equal to the Brewster angle.

$$\theta_i = \theta_B = \tan^{-1}\left(\frac{n}{10}\right) = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{81}\right) = \tan^{-1}(9) = 83.66^\circ.$$

$$n = 90 \text{ m.}$$

Thus

$$\tan^{-1}\left(\frac{104-90}{h_2}\right) = \tan^{-1}\left(\sqrt{81}\right)$$

$$\Rightarrow \frac{104-90}{9} = h_2 \quad \Rightarrow \quad h_2 = 1101.11 \text{ m.}$$

FIELDS AND WAVES  
CHAPTER 7

Ans 7.1 a.  $H_\phi = \frac{E_0}{l} = j \frac{\beta_0 l e^{-j\beta_0 s}}{4\pi} \sin\theta [2\cos(\beta_0 \cos\theta)]$

b.  $S_{av} = \frac{1}{2} \operatorname{Re} [ExH^*] = \frac{1}{2} \operatorname{Re} [\hat{a}_\theta E_0 \times \hat{a}_\phi H_\phi^*] = \frac{\alpha_s}{2} \operatorname{Re} [E_0 H_\phi^*]$

$$= \alpha_s \frac{1}{2} \operatorname{Re} \left[ E_0 \frac{E_0^*}{n} \right] = \alpha_s \frac{1}{2} \frac{|I_0|^2}{\eta}$$

$$S_{av} = \alpha_s \frac{n}{2} \left| \frac{\beta_0 l}{4\pi} \right|^2 \sin^2\theta [2\cos(\beta_0 \cos\theta)]^2$$

$$= \alpha_s 2\eta \left| \frac{\beta_0 l}{4\pi} \right|^2 \sin^2\theta \cos^2(\beta_0 \cos\theta)$$

c.  $P_{av} = \oint_S S_{av} \cdot ds = \int_0^{2\pi} \int_0^{\pi/2} \alpha_s S_{av} \cdot d\vec{s} \cdot r^2 \sin\theta d\theta d\phi$

$$= \int_0^{2\pi} \int_0^{\pi/2} S_{av} r^2 \sin\theta d\theta d\phi$$

$$P_{av} = 4\pi\eta \left| \frac{\beta_0 l}{4\pi} \right|^2 \int_0^{\pi/2} \sin^3\theta \cos^2(\beta_0 \cos\theta) d\theta$$

$$= \pi\eta \left| \frac{I_0 l}{\pi} \right|^2 \int_0^{\pi/2} \sin^3\theta \cos^2(\beta_0 \cos\theta) d\theta$$

$$P_{rad} = P_{av} = \pi\eta \left| \frac{I_0 l}{\pi} \right|^2 I$$

$$\text{where } I = I_1 + I_2 = \int_0^{\pi/2} \sin^3\theta \cos^2(\beta_0 \cos\theta) d\theta = \int_0^{\pi/2} \sin^3\theta \left[ 1 + \frac{1}{2} \frac{(2\beta_0 \cos\theta)^2}{2} \right] d\theta$$

$$I = I_1 + I_2 = \frac{1}{2} \int_0^{\pi/2} \sin^3\theta d\theta + \frac{1}{2} \int_0^{\pi/2} \sin^3\theta \cos(2\beta_0 \cos\theta) d\theta$$

$$I_1 = \frac{1}{2} \int_0^{\pi/2} \sin^3\theta d\theta = -\frac{1}{6} \cos\theta (\sin^2\theta + 2) \Big|_0^{\pi/2} = \frac{1}{3}$$

$$I_2 = \frac{1}{2} \int_0^{\pi/2} \sin^3 \theta \cos(2\beta h \cos \theta) d\theta = \frac{1}{2} \int_0^{\pi/2} \sin^3 \theta \cos(2\beta h \cos \theta) \sin \theta d\theta$$

$$\text{Let } u = \sin^2 \theta \quad dv = -\frac{\cos(2\beta h \cos \theta)}{2\beta h} d(2\beta h \cos \theta)$$

$$du = 2 \sin \theta \cos \theta d\theta$$

$$V = \frac{-1}{2\beta h} \sin(2\beta h \cos \theta)$$

$$I_2 = \frac{-\sin^2 \theta}{4\beta h} \sin(2\beta h \cos \theta) \Big|_0^{\pi/2} + \frac{1}{2\beta h} \int_0^{\pi/2} \cos \theta \sin(2\beta h \cos \theta) \sin \theta d\theta$$

↓

$$\text{Let } u = \cos \theta \quad dv = \frac{-1}{2\beta h} \sin(2\beta h \cos \theta) d(2\beta h \cos \theta)$$

$$du = -\sin \theta d\theta$$

$$I_2 = 0 + \frac{1}{2\beta h} \left\{ \frac{\cos \theta}{2\beta h} \cos(2\beta h \cos \theta) \Big|_0^{\pi/2} + \frac{1}{2\beta h} \int_0^{\pi/2} \cos(2\beta h \cos \theta) \sin \theta d\theta \right\}$$

$$I_2 = \frac{1}{2\beta h} \left\{ \frac{-1}{2\beta h} \cos(2\beta h) - \frac{1}{(2\beta h)^2} \sin(2\beta h \cos \theta) \Big|_0^{\pi/2} \right\} = \left\{ -\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3} \right\}$$

Therefore

$$P_{\text{rad}} = \pi n \left| \frac{2\beta l}{\lambda} \right|^2 \left[ \frac{1}{3} - \frac{\cos 2\beta h}{(2\beta h)^2} + \frac{\sin 2\beta h}{(2\beta h)^3} \right]$$

Ans 7.2 a From 7.1, b

$$S_{\text{av}} = 2\pi \left| \frac{\beta \beta l}{4\pi \lambda} \right|^2 \sin^2 \theta \cos^2(\beta h \cos \theta) = \frac{\pi}{2} \left| \frac{2\beta l}{\lambda} \right|^2 \sin^2 \theta \cos^2(\beta h \cos \theta)$$

$$U = g^2 S_{\text{av}} = \frac{\pi}{2} \left| \frac{2\beta l}{\lambda} \right|^2 \sin^2 \theta \cos^2(\beta h \cos \theta)$$

$$\text{b. } D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}, \quad M_{\text{max}} = M_{\text{max}} = \frac{\pi}{2} \left| \frac{2\beta l}{\lambda} \right|^2$$

From 7-1c

$$\begin{aligned} P_{\text{road}} &= \pi \eta \left| \frac{g_0 l}{d} \right|^2 \left[ \frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3} \right] \\ &= \pi \eta \left| \frac{g_0 l}{d} \right|^2 F(\beta h) \end{aligned}$$

$$\text{where } F(\beta h) = \left[ \frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3} \right]$$

Thus

$$\frac{D_0 = 4\pi \frac{\eta}{2} \left| \frac{g_0 l}{d} \right|^2}{\pi \eta \left| \frac{g_0 l}{d} \right|^2 F(\beta h)} = \frac{2}{F(\beta h)}$$

C.  $R_n = \frac{2P_{\text{road}}}{(J_0)^2} = 2\pi \eta \left| \frac{l}{d} \right|^2 F(\beta h).$

FIELDS AND WAVES  
CHAPTER 8

Ans 8.1  $V_g = V \cos \theta, \cos \theta = \frac{\beta_2}{\beta} = \sqrt{1 - (\frac{f_c}{f})^2}$

$$(f_c)_{10} = \frac{1}{2a\sqrt{\mu_0\epsilon_0}} = \frac{3 \times 10^{10}}{2(2.54)(0.9)} = 6.56 \times 10^9, \quad (\frac{f_c}{f})_{10} = \frac{6.56}{10} = 0.656$$

$$\cos \theta = \sqrt{1 - (0.656)^2} = \sqrt{1 - 0.43} = \sqrt{0.57} = 0.755$$

$$V_g = 3 \times 10^8 (0.755) = 2.265 \times 10^8 \text{ m/s.}$$

$$d = \text{delay} \times \text{velocity} = (2 \times 10^{-6}) (2.265 \times 10^8) = 4.53 \times 10^2 = 453 \text{ m.}$$

Ans 8.2a.  $(f_c)_{10} = \frac{1}{2a\sqrt{\mu_0\epsilon_0}} = \frac{1}{2a\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_0}} = \frac{V_0}{2a\sqrt{\epsilon_0}} = \frac{30 \times 10^9}{2(2.286)\sqrt{2.56}} = 4.1016 \text{ Hz}$

b.  $\alpha_g = \frac{1}{\sqrt{1 - (\frac{f_c}{f})^2}} = \frac{20/\sqrt{\epsilon_0}}{\sqrt{1 - (4.101/10)^2}} = \frac{30 \times 10^9 / 10 \times 10^9 \sqrt{2.56}}{\sqrt{1 - (0.4101)^2}} = \frac{1.875}{0.912} = 2.056 \text{ cm}$

c.  $Z_w = \frac{\eta}{\sqrt{1 - (\frac{f_c}{f})^2}} = \frac{377 / \sqrt{2.56}}{0.912} = 258.36 \Omega$

d.  $V_p = \frac{V}{\sqrt{1 - (\frac{f_c}{f})^2}} = \frac{V_0 / \sqrt{\epsilon_0}}{0.912} = \frac{3 \times 10^8 / \sqrt{2.56}}{0.912} = \frac{1.875 \times 10^8}{0.912} = 2.056 \times 10^8 \text{ m/sec.}$

e.  $V_g = V \sqrt{1 - (\frac{f_c}{f})^2} = \frac{V_0}{\sqrt{\epsilon_0}} \sqrt{1 - (\frac{f_c}{f})^2} = \frac{3 \times 10^8}{\sqrt{2.56}} (0.912) = 1.71 \times 10^8 \text{ m/s.}$