Chapter 2

2.1 - 1

2.1 - 6

2.1 - 7

2.2 - 1

2.3 - 2

2.4 - 3

2.6 - 1

2.8 - 2

2.8 - 3

2.9 - 1

Chapter 2

2.1-1 Both $\varphi(t)$ and $w_0(t)$ are periodic.

The average power of $\varphi(t)$ is $P_g = \frac{1}{T}\int_0^T \varphi^2(t)\,dt = \frac{1}{\pi}\int_0^\pi \left(e^{-t/2}\right)^2 dt = \frac{1-e^{-\pi}}{\pi}$.

The average power of $w_0(t)$ is $P_g = \frac{1}{T_0}\int_o^{T_0} w_o^2(t)\,dt = \frac{1}{T_0}\int_0^{T_0} 1\cdot dt = 1$.

2.1-2

(a) Since $x(t)$ is a real signal, $E_x = \int_0^2 x^2(t)\,dt$.

Solving for Fig. S2.1-2(a), we have

$E_x = \int_0^2 (1)^2\,dt = 2, \quad E_y = \int_0^1 (1)^2\,dt + \int_1^2 (-1)^2\,dt = 2$

$E_{x+y} = \int_0^1 (2)^2\,dt = 4, \quad E_{x-y} = \int_1^2 (2)^2\,dt = 4$

Therefore, $E_{x\pm y} = E_x + E_y$.

Solving for Fig. S2.1-2(b), we have

$E_x = \int_0^\pi (1)^2\,dt + \int_\pi^{2\pi} (-1)^2\,dt = 2\pi, \quad E_y = \int_0^{\pi/2} (1)^2\,dt + \int_{\pi/2}^{\pi} (-1)^2\,dt + \int_\pi^{3\pi/2} (1)^2\,dt + \int_{3\pi/2}^{2\pi} (-1)^2\,dt = 2\pi$

$E_{x+y} = \int_0^{\pi/2} (2)^2\,dt + \int_{\pi/2}^{3\pi/2} (0)^2\,dt + \int_{3\pi/2}^{2\pi} (-2)^2\,dt = 4\pi$

$E_{x-y} = \int_0^{\pi/2} (0)^2\,dt + \int_{\pi/2}^{\pi} (2)^2\,dt + \int_{\pi/2}^{3\pi/2} (-2)^2\,dt + \int_{3\pi/2}^{2\pi} (0)^2\,dt = 4\pi$.

Therefore, $E_{x\pm y} = E_x + E_y$.

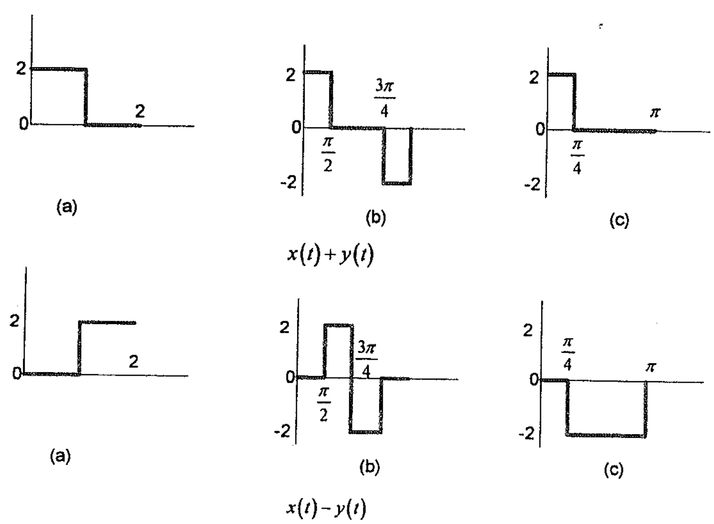

Fig. S2.1-2

(b)

$E_x = \int_0^{\pi/4} (1)^2\,dt + \int_{\pi/4}^{\pi} (-1)^2\,dt = \pi, \quad E_y = \int_0^\pi (1)^2\,dt = \pi$

$E_{x+y} = \int_0^{\pi/4} (2)^2\,dt + \int_{\pi/4}^{\pi} (0)^2\,dt = \pi$, $\quad E_{x-y} = \int_0^{\pi/4} (0)^2\,dt + \int_{\pi/4}^{\pi} (-2)^2\,dt = 3\pi$ Therefore, $E_{x\pm y} \neq E_x + E_y$, and $E_{\hat{x}\pm\hat{y}} = E_{\hat{x}} \pm E_{\hat{y}}$ are not true in general.

2.1-3

$$\begin{aligned} P_g &= \frac{1}{T_0}\int_0^{T_0} C^2\cos^2(\omega_0 t + \theta)\,dt = \frac{C^2}{2T_0}\int_0^{T_0} [1 + \cos(2\omega_0 t + 2\theta)]\,dt \\ &= \frac{C^2}{2T_0}\left[\int_0^{T_0} dt + \int_0^{T_0} \cos(2\omega_0 t + 2\theta)\,dt\right] = \frac{C^2}{2T_0}[T_0 + 0] = \frac{C^2}{2} \end{aligned}$$

2.1-4 If $\omega_1 = \omega_2$, then

$$\begin{aligned} g^2(t) &= (C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_1 t + \theta_2))^2 \\ &= C_1^2\cos^2(\omega_1 t + \theta_1) + C_2^2\cos^2(\omega_1 t + \theta_2) + 2C_1C_2\cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2) \end{aligned}$$

$$\begin{aligned} P_g &= \lim_{T_0\to\infty} \frac{1}{T_0}\int_0^{T_0} (C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_1 t + \theta_2))^2\,dt \\ &= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty} 2C_1C_2\frac{1}{T_0}\int_0^{T_0} \cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2)\,dt \\ &= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty} 2C_1C_2\frac{1}{T_0}\int_0^{T_0} \frac{1}{2}[\cos(2\omega_1 t + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)]\,dt \\ &= \frac{C_1^2}{2} + \frac{C_2^2}{2} + 0 + \frac{2C_1C_2}{2}\cos(\theta_1 - \theta_2) \\ &= \frac{C_1^2 + C_1^2 + 2C_1C_2\cos(\theta_1 - \theta_2)}{2} \end{aligned}$$

2.1-5

$$\begin{aligned} P_g &= \frac{1}{4}\int_{-2}^{2} (t^3)^2\,dt = 64/7 \\ \textbf{(a) } P_{-g} &= \frac{1}{4}\int_{-2}^{2} (-t^3)^2\,dt = 64/7 \\ \textbf{(b) } P_{2g} &= \frac{1}{4}\int_{-2}^{2} (2t^3)^2\,dt = 4(64/7) = 256/7 \\ \textbf{(c) } P_{cg} &= \frac{1}{4}\int_{-2}^{2} (ct^3)^2\,dt = 64c^2/7 \end{aligned}$$

Changing the sign of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor of c^2.

2.1-6 Let us denote the signal in question by $g(t)$ and its energy by E_g.

(a),(b) For parts **(a)** and **(b)**, we write

$$E_g = \int_0^{2\pi} \sin^2 t\,dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt = \pi + 0 = \pi$$

(c)

$$E_g = \int_{2\pi}^{4\pi} \sin^2 t\, dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\, dt = \pi + 0 = \pi$$

(d)

$$E_g = \int_0^{2\pi} (2\sin t)^2\, dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\, dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way, we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-7

$$\begin{aligned} P_g &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g(t)g^*(t)\, dt \\ &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n}\sum_{r=m}^{n} D_k D^*{}_r e^{j(\omega_k-\omega_r)t}\, dt \\ &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n}\sum_{r=m,r\neq k}^{n} D_k D^*{}_r e^{j(\omega_k-\omega_r)t}\, dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n} |D_k|^2\, dt \end{aligned}$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands (functions to be integrated) are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n} |D_k|^2\, dt = \sum_{k=m}^{n} |D_k|^2$$

2.1-8

(a) From Eq. (2.5a), the power of a signal of amplitude C is $P_g = \frac{C^2}{2}$, regardless of phase and frequency; therefore, $P_g = 100/2 = 50$; the rms value is $\sqrt{P_g} = 5\sqrt{2}$.

(b) From Eq. (2.5b), the power of the sum of two sinusoids of different frequencies is the sum of the power of individual sinusoids, regardless of the phase, $\frac{C_1^2}{2} + \frac{C_2^2}{2}$, therefore, $P_g = 100/2 + 256/2 = 50 + 128 = 178$; the rms value is $\sqrt{P_g} = \sqrt{178}$.

(c) $g(t) = (10 + 2\sin(3t))\cos(10t) = 10\cos(10t) + 2\sin(3t)\cos(10t) = 10\cos(10t) + \sin(13t) - \cos(7t)$
Therefore, $P_g = 100/2 + 1/2 + 1/2 = 50 + 0.5 + 0.5 = 51$; the rms value is $\sqrt{P_g} = \sqrt{51}$.

(d) $g(t) = 10\cos(5t)\cos(10t) = \frac{10(\cos(15t)+\cos(5t))}{2} = 5\cos(15t) + 5\cos(5t)$
Therefore, $P_g = 25/2 + 25/2 = 25$; the rms value is $\sqrt{P_g} = 5$.

(e) $g(t) = 10\sin(5t)\cos(10t) = 5(\cos(15t) - \cos(5t)) = 5\cos(15t) - 5\cos(5t)$
Therefore, $P_g = 25/2 + 25/2 = 25$; the rms value is $\sqrt{P_g} = 5$.

(f) $|g(t)|^2 = \cos^2(\omega_0 t)$
Therefore, $P_g = 1/2 = 0.5$; the rms value is $\sqrt{P_g} = \sqrt{0.5}$

2.1-9

(a) Power $P_g = \frac{1}{4}\int_0^4 1 \cdot dt = 1$, and the rms value is $\sqrt{P_g} = \sqrt{1} = 1$

(b) Power $P_g = \frac{1}{10\pi}\left[\int_0^{\pi} 1 \cdot dt + \int_{\pi}^{9\pi} 0 \cdot dt + \int_{9\pi}^{10\pi} 1 \cdot dt\right] = \frac{1}{10\pi}\left[\pi + 0 + \pi\right] = \frac{1}{5}$;

and the rms value is $\sqrt{1/5}$.

(c) Power

$$\begin{aligned} P_g &= \frac{1}{T}\int_0^T g^2(t)\,dt = \frac{1}{6}\left[\int_0^1 g^2(t)\,dt + \int_1^2 g^2(t)\,dt + \int_2^4 g^2(t)\,dt + \int_4^5 g^2(t)\,dt + \int_5^6 g^2(t)\,dt\right] \\ &= \frac{1}{6}\left[1 + \int_1^2 (-t+2)^2\,dt + 0 + \int_4^5 (t-4)^2\,dt + 1\right] \\ &= \frac{1}{6}\left[1 + \frac{1}{3} + 0 + \frac{1}{3} + 1\right] = \frac{4}{9} \end{aligned}$$

and the rms value is $\sqrt{4/9} = 2/3$

2.2-1 If a is complex with real part 0, $a = i\alpha$; then, $g(t) = e^{-i\alpha t}$ and $|g(t)|^2 = 1$

$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1 \cdot dt = \lim_{T\to\infty} \frac{1}{T} T = 1$. Hence it is a power signal. It is not an energy signal since $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \cdot dt = \infty$. If a is real, then both $E_g = \int_{-\infty}^{\infty} |e^{-at}|^2 \cdot dt = \infty$ and $P_g = \infty$.

2.2-2 Let $c = a + jb$, where a, b are real valued. Therefore, $|e^{-ct}| = |e^{-(a+jb)t}| = |e^{-at} \cdot e^{-jbt}| = |e^{-at}| \cdot |e^{-jbt}| = |e^{-at}| \cdot 1 = |e^{-at}|$

$$\begin{aligned} E_g &= \int_{-\infty}^{\infty} |e^{-ct}|^2 \cdot dt = \int_{-\infty}^{\infty} e^{-2at} \cdot dt = \infty \\ P_g &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |e^{-ct}|^2 \cdot dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} e^{-2at} \cdot dt = \infty \end{aligned}$$

Therefore, e^{-ct} is neither energy nor a power signal for a complex value of c with nonzero real part.

2.3-1

$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiplying by 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2} - 1)$.

2.3-2

(a) See Fig. S2.3-2a.

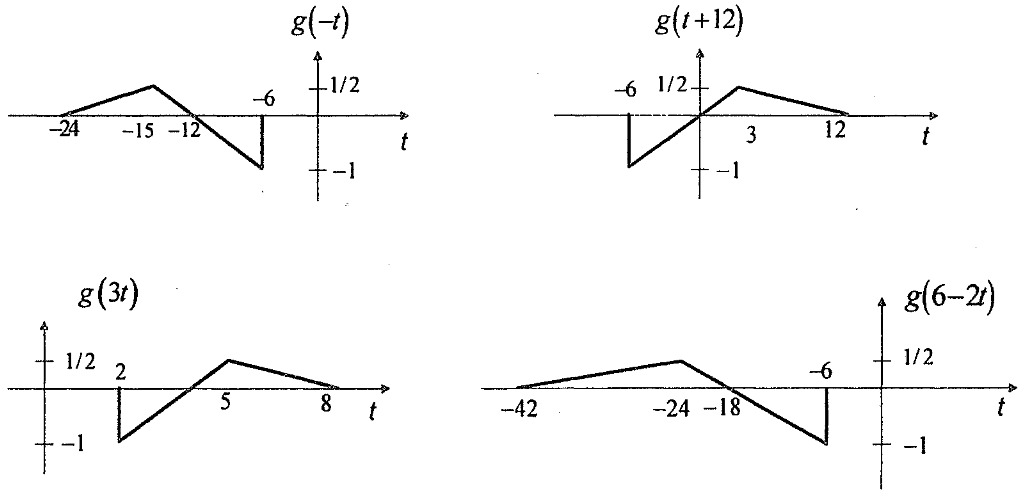

Fig. S2.3-2a

(b) Energy of $g(t)$,

$$E_g = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_6^{15}\left[\frac{1}{6}(t-12)\right]^2 dt + \int_{15}^{24}\left[-\frac{1}{18}(t-24)\right]^2 dt = \frac{9}{4} + \frac{3}{4} = 3$$

Since [**see Problem 2.3-5**], time shifting or time inversion does not change the signal energy,

$$E_{g(-t)} = E_{g(t+12)} = E_{g(t)} = 3$$

On the other hand, a scaling of $g(at)$ will change the signal energy to E_g/a [Problem **2.3-5**], the energy of $g(3t)$ is 1 and energy of $g(6-2t)$ is $\frac{3}{2}$.

2.3-3 See Fig. S2.3-3.

2.3-4 Denote $g(at) = f(t)$. Since $g(t)$ is periodic with period T,

$$\begin{aligned} g(t) &= g(t+T) \\ g(a\,t) &= g(a\,t+T) = g\left(a(t+\frac{T}{a})\right) \\ f(t) &= f\left(t+\frac{T}{a}\right) \end{aligned}$$

Therefore, $g(at)$ is periodic with period T/a.

The average power of $g(a\,t)$ is

$$P_{g(a\,t)} = \lim_{T\to\infty}\frac{a}{T}\int_{-T/2a}^{T/2a} g^2(at)\,dt = \lim_{T\to\infty}\frac{a}{T}\int_{-T/2}^{T/2} g^2(z)\,\frac{dz}{a} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g^2(z)\,dz = P_g$$

Therefore, the average power remains the same.

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\,dt = \frac{1}{|a|}\phi(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Therefore,

$$\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$$

2.4-3 Using the fact that $\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$, and the equality $\int_a^b \phi(t)\delta(t-T)\,dt = \phi(T)$, we get

(a) $\int_{-\infty}^{\infty} g(\tau + a)\delta(t-\tau)\,d\tau = g(t+a)$

(b) $\int_{-\infty}^{\infty} \delta(\tau)g(t-\tau)\,d\tau = g(t)$

(c) 1

(d) 0

(e) 0

(f) 5

(g) $g(-1)$

(h)

$$\begin{aligned}\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(x-5)\delta(2x-3)dx &= \int_{-\infty}^{\infty} \cos\frac{\pi}{2}(x-5)\delta\left(2(x-\frac{3}{2})\right)dx \\ &= \frac{1}{2}\int_{-\infty}^{\infty} \cos\frac{\pi}{2}(x-5)\delta\left(x-\frac{3}{2}\right)dx = \frac{1}{2}\cos\frac{\pi}{2}\left(\frac{3}{2}-5\right) = \sqrt{3}/4\end{aligned}$$

Here, we used the fact $\delta(at) = \frac{1}{|a|}\delta(t)$(see Problem **2.3-2**)

2.5-1

$$|\mathbf{e}|^2 = |\mathbf{g}|^2 + c^2|\mathbf{x}|^2 - 2c\mathbf{g}\cdot\mathbf{x}$$

To minimize error, set $\frac{d|\mathbf{e}|^2}{dc} = 0$:

$$2c|\mathbf{x}|^2 - 2\mathbf{g}\cdot\mathbf{x} = 0$$

$$c = \frac{\mathbf{g}\cdot\mathbf{x}}{|\mathbf{x}|^2} = \frac{<\mathbf{g},\mathbf{x}>}{|\mathbf{x}|^2}$$

2.5-2

(a) In this case $E_x = \int_0^1 dt = 1$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\,dt = \frac{1}{1}\int_0^1 t\,dt = 0.5$$

(a) If $x(t)$ and $y(t)$ are orthogonal, then we can show that the energy of $x(t) \pm y(t)$ is $E_x + E_y$.

$$\begin{aligned}\int |x(t) \pm y(t)|^2\,dt &= \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt \pm \int_{-\infty}^{\infty} x(t)y^*(t)\,dt \pm \int_{-\infty}^{\infty} x^*(t)y(t)\,dt \\ &= \int_{-\infty}^{\infty} |x(t)|^2\,dt + \int_{-\infty}^{\infty} |y(t)|^2\,dt\end{aligned}$$

The last result follows from the fact that because of orthogonality, the two integrals of the cross products $x(t)y^*(t)$ and $x^*(t)y(t)$ are zero [see Eq. (2.40)]. Thus the energy of $x(t) + y(t)$ is equal to that of $x(t) - y(t)$ if $x(t)$ and $y(t)$ are orthogonal.

(b) We can use a similar argument to show that the energy of $c_1x(t) + c_2y(t)$ is equal to that of $c_1x(t) - c_2y(t)$ if $x(t)$ and $y(t)$ are orthogonal. This energy is given by $|c_1|^2E_x + |c_2|^2E_y$.

(c) If $z(t) = x(t) \pm y(t)$, then it follows from **part (a)** in the preceding derivation that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

2.6-1 We shall use Eq. (2.51) to compute ρ_n for each of the four cases. Let us first compute the energies of all the signals:

$$E_x = \int_0^1 \sin^2 2\pi t\,dt = 0.5$$

In the same way, we find $E_{g_1} = E_{g_2} = E_{g_3} = E_{g_4} = 0.5$.

From Eq. (2.51), the correlation coefficients for four cases are found as follows:

(1) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 \sin 2\pi t \sin 4\pi t\,dt = 0$

(2) $\frac{1}{\sqrt{(0.5)(0.5)}} \int (\sin 2\pi t)(-\sin 2\pi t)\,dt = -1$

(3) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 0.707 \sin 2\pi t\,dt = 0$

(4) $\frac{1}{\sqrt{(0.5)(0.5)}} \left[\int_0^{0.5} 0.707 \sin 2\pi t\,dt - \int_{0.5}^1 0.707 \sin 2\pi t\,dt\right] = 2.828/\pi = 0.9$

Signals $x(t)$ and $g_2(t)$ provide the maximum protection against noise.

2.6-2 Since

$$g(t) = u(t) - u(t-2)$$

it is a rectangular function that exists only from $t = 0$ to $t = 2$. Thus, if $\tau > 2$ or if $\tau < -2$, then $g(t+\tau)$ and $g(t)$ does not overlap. Thus,

$$\psi_g(\tau) = \int 0 \cdot dt = 0 \qquad |\tau| \geq 2.$$

If $0 \leq \tau < 2$, then

$$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)\,dt = \int_0^{2-\tau} 1 \cdot dt = 2 - \tau \qquad 0 \leq \tau < 2$$

Because $\psi_g(-\tau) = \psi_g(\tau)$, we have

$$\psi_g(\tau) = \begin{cases} 2 - |\tau| & |\tau| < 2 \\ 0 & |\tau| \geq 2 \end{cases}$$

Similarly, if $g(t)$ is an odd function of t, then $g(t)\cos n\omega_0 t$ is an odd function of t and $g(t)\sin n\omega_0 t$ is an even function of t. Therefore

$$\begin{aligned} a_0 &= a_n = 0 \\ b_n &= \frac{4}{T_0}\int_0^{T_0/2} g(t)\sin n\omega_0 t\, dt \end{aligned}$$

Observe that because of symmetry, the integration required to compute the coefficients need be performed over only half the period.

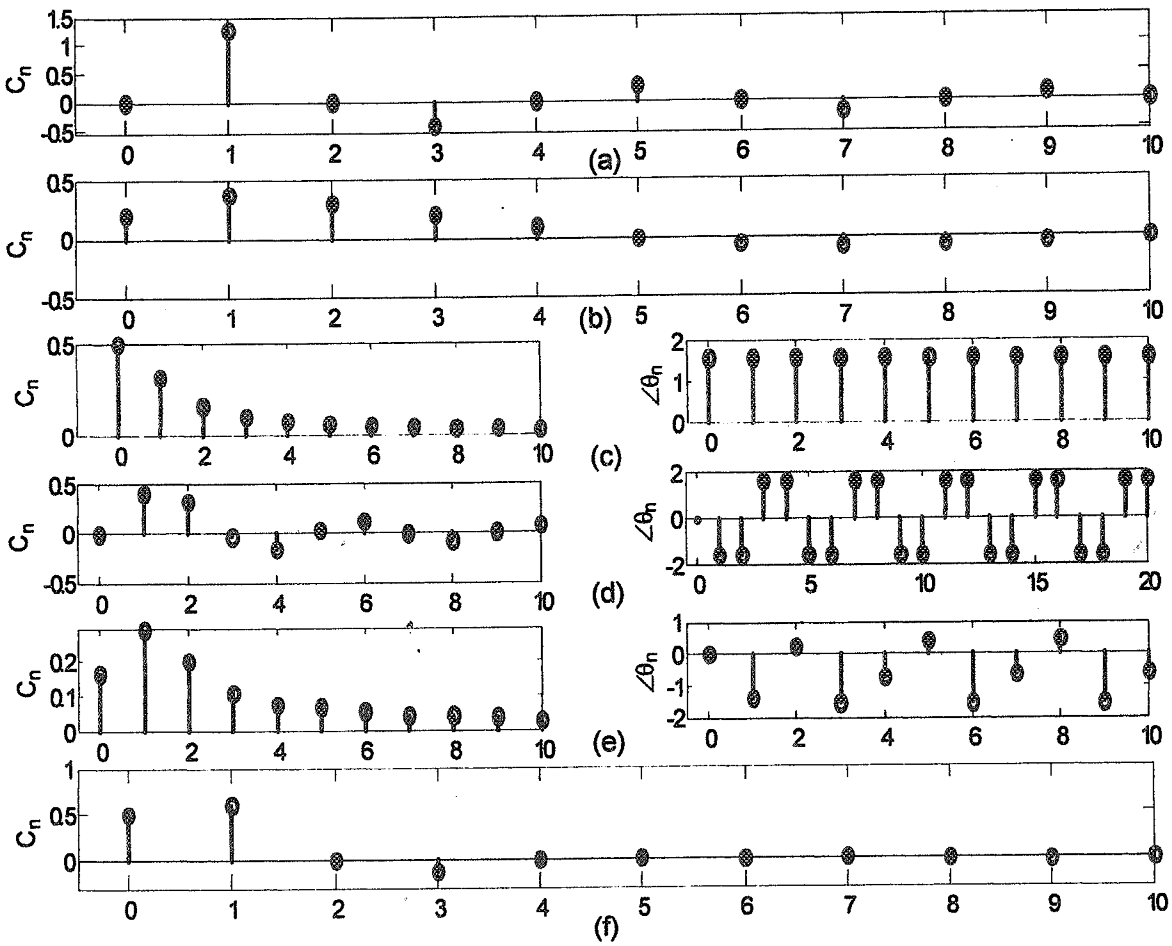

Fig. S2.8-2

2.8-2

(a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

$$\begin{aligned} g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2}t\right) \\ a_0 &= 0 \text{ (by inspection of its lack of dc)} \\ a_n &= \frac{4}{4}\left[\int_0^1 \cos\left(\frac{n\pi}{2}t\right)\, dt - \int_1^2 \cos\left(\frac{n\pi}{2}t\right)\, dt\right] \\ &= \frac{4}{n\pi}\sin\frac{n\pi}{2} \end{aligned}$$

Therefore, the Fourier series for $g(t)$ is

$$g(t) = \frac{4}{\pi}\left(\cos\frac{\pi t}{2} - \frac{1}{3}\cos\frac{3\pi t}{2} + \frac{1}{5}\cos\frac{5\pi t}{2} - \frac{1}{7}\cos\frac{7\pi t}{2} + \cdots\right)$$

Here $b_n = 0$, and we allow C_n to take negative values. Figure S2.8-2(a) shows the plot of C_n.

(b) $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$\begin{aligned}
g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right) \\
a_0 &= \frac{1}{5} \qquad \text{(by inspection)}. \\
a_n &= \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right)\,dt \\
&= \frac{1}{5\pi}\left(\frac{5}{n}\right)\sin\left(\frac{n}{5}t\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\left(\frac{n\pi}{5}\right) \\
b_n &= \frac{2}{10\pi}\int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right)\,dt \\
&= 0 \qquad \text{(integrand is an odd function of } t\text{)}
\end{aligned}$$

Here $b_n = 0$, and we allow C_n to take negative values. Note that $C_n = a_n$ for $n = 0, 1, 2, 3, \ldots$. Fig. S2.8-2(b) shows the plot of C_n.

(c) $T_0 = 2\pi$, $\omega_0 = 1$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

with

$$a_0 = 0.5 \quad \textbf{(by inspection of the dc or average)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos nt\,dt = 0, \qquad \text{and} \qquad b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\sin nt\,dt = -\frac{1}{\pi n}$$

and

$$\begin{aligned}
g(t) &= 0.5 - \frac{1}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \frac{1}{4}\sin 4t + \cdots\right) \\
&= 0.5 + \frac{1}{\pi}\left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t + \frac{\pi}{2}\right) + \cdots\right]
\end{aligned}$$

The cosine terms vanish because when 0.5 (the dc component) is subtracted from $g(t)$, the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-2(c) shows the plots of C_n and θ_n.

(d) $T_0 = \pi$, $\omega_0 = 2$ and $g(t) = \frac{4}{\pi}t$.

$a_0 = 0$ (by inspection)

$a_n = 0$ $(n > 0)$ (because of odd symmetry)

$$b_n = \frac{4}{\pi}\int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt\, dt = \frac{2}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

$$\begin{aligned} g(t) &= \frac{4}{\pi^2}\sin 2t + \frac{1}{\pi}\sin 4t - \frac{4}{9\pi^2}\sin 6t - \frac{1}{2\pi}\sin 8t + \cdots \\ &= \frac{4}{\pi^2}\cos\left(2t - \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(4t - \frac{\pi}{2}\right) + \frac{4}{9\pi^2}\cos\left(6t + \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(8t + \frac{\pi}{2}\right) + \cdots \end{aligned}$$

Figure S2.8-2(d) shows the plots of C_n and θ_n.

(e) $T_0 = 3$, $\omega_0 = 2\pi/3$, and

$$a_0 = \frac{1}{3}\int_0^1 t\, dt = \frac{1}{6}$$

$$a_n = \frac{2}{3}\int_0^1 t\cos\frac{2n\pi}{3}t\, dt = \frac{3}{2\pi^2 n^2}\left[\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3}\right]$$

$$b_n = \frac{2}{3}\int_0^1 t\sin\frac{2n\pi}{3}t\, dt = \frac{3}{2\pi^2 n^2}\left[\sin\frac{2\pi n}{3} - \frac{2\pi n}{3}\cos\frac{2\pi n}{3}\right]$$

Therefore, $C_0 = \frac{1}{6}$ and

$$C_n = \frac{3}{2\pi^2 n^2}\left[\sqrt{2 + \frac{4\pi^2 n^2}{9} - 2\cos\frac{2\pi n}{3} - \frac{4\pi n}{3}\sin\frac{2\pi n}{3}}\right]$$

and

$$\theta_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

Figure S2.8-2(e) shows the plots of C_n and θ_n.

(f) $T_0 = 6$, $\omega_0 = \pi/3$, $a_0 = 0.5$ (by inspection of the dc value). There is even symmetry, and $b_n = 0$.

$$a_n = \frac{4}{6}\int_0^3 g(t)\cos\frac{n\pi}{3}\, dt = \frac{2}{3}\left[\int_0^1 \cos\frac{n\pi}{3}\, dt + \int_1^2 (2-t)\cos\frac{n\pi}{3}t\, dt\right] = \frac{6}{\pi^2 n^2}\left[\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3}\right]$$

$$g(t) = 0.5 + \frac{6}{\pi^2}\left(\cos\frac{\pi}{3}t - \frac{2}{9}\cos\pi t + \frac{1}{25}\cos\frac{5\pi}{3}t + \frac{1}{49}\cos\frac{7\pi}{3}t + \cdots\right)$$

Observe that even harmonics vanish. This is because if the dc (0.5) is subtracted from $g(t)$, the resulting function has half-wave symmetry. Figure S2.8-2(f) shows the plot of C_n.

2.8-3

(a) An even function $g_e(t)$ and an odd function $g_o(t)$ have the properties that

$$g_e(t) = g_e(-t)$$

and

$$g_o(t) = -g_o(-t) \tag{1}$$

Every signal $g(t)$ can be expressed as a sum of even and odd components because

$$g(t) = \underbrace{\tfrac{1}{2}\left[g(t) + g(-t)\right]}_{\textbf{even}} + \underbrace{\tfrac{1}{2}\left[g(t) - g(-t)\right]}_{\textbf{odd}}$$

From the definitions of the Eq. (1), it can be seen that the first component on the right-hand side is an even function, while the second component is odd. This is readily seen from the fact that replacing t by $-t$ in the first component yields the same function. The same maneuver in the second component yields the negative of that component.

(b)(i) To find the odd and the even components of $g(t) = u(t)$, we have

$$g(t) = g_e(t) + g_o(t)$$

where from Eq. (1), we write

$$\begin{aligned} g_e(t) &= \tfrac{1}{2}\left[u(t) + u(-t)\right] = \frac{1}{2} \\ g_o(t) &= \tfrac{1}{2}\left[u(t) - u(-t)\right] = \frac{1}{2}\text{sgn}(t) \end{aligned}$$

The even and odd components of function $u(t)$ are shown in Fig. S2.8-3(a).

(ii) Similarly, to find the odd and the even components of $g(t) = e^{-at}u(t)$, we have

$$g(t) = g_e(t) + g_o(t)$$

where

$$\begin{aligned} g_e(t) &= \tfrac{1}{2}\left[e^{-at}u(t) + e^{at}u(-t)\right] \\ g_o(t) &= \tfrac{1}{2}\left[e^{-at}u(t) - e^{at}u(-t)\right] \end{aligned}$$

The even and odd components of function $e^{-at}u(t)$ are shown in Fig. S2.8-3(b).

(iii) For $g(t) = e^{jt}$, we have

$$e^{jt} = g_e(t) + g_o(t)$$

where

$$g_e(t) = \tfrac{1}{2}\left[e^{jt} + e^{-jt}\right] = \cos t$$

and

$$g_o(t) = \tfrac{1}{2}\left[e^{jt} - e^{-jt}\right] = j \sin t$$

The even and odd components of function e^{jt} are shown in Fig. S2.8-3(c).

2.8-4

(f) $\omega_0 = 1$, therefore, $T_0 = \frac{2\pi}{\omega_0} = 2\pi$. We can use

$$g(t) = \begin{cases} t, & 0 \le t \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le t \le \pi \\ -t+\pi, & \pi \le t \le \frac{3\pi}{2} \\ 0, & \frac{3\pi}{2} \le t \le 2\pi \end{cases}$$

Using formula from 2.8-4 (b)(i), we get

$$\begin{aligned} a_n &= \frac{4}{T_0}\int_0^{T_0/2} g(t)\cos nt\,dt \\ &= \frac{2}{\pi}\int_0^{\pi/2} t\cos nt\,dt \\ &= \frac{2}{\pi}\frac{\cos\frac{n\pi}{2} + \frac{n\pi}{2}\sin\frac{n\pi}{2}}{n^2} - \frac{2}{\pi}\frac{1}{n^2} \\ &= \begin{cases} 0, & n \to \text{even} \\ \frac{2}{\pi n^2}\left(\frac{n\pi}{2}\sin\frac{n\pi}{2} - 1\right), & n \to \text{odd} \end{cases} \end{aligned}$$

Similarly,

$$\begin{aligned} b_n &= \frac{4}{T_0}\int_0^{T_0/2} g(t)\sin nt\,dt \\ &= \frac{2}{\pi}\int_0^{\pi/2} t\sin nt\,dt \\ &= \frac{2}{\pi}\left[\frac{\sin\frac{n\pi}{2} - \frac{n\pi}{2}\cos\frac{n\pi}{2}}{n^2}\right] \\ &= \begin{cases} \frac{2}{n^2\pi}\sin\left(\frac{n\pi}{2}\right), & n \to \text{odd} \\ 0, & n \to \text{even} \end{cases} \end{aligned}$$

We now have the Fourier series

$$f(t) = \frac{\pi}{2} + \sum_{n\ \text{odd}}\left[\frac{2}{\pi n^2}\left(\frac{n\pi}{2}\sin\frac{n\pi}{2} - 1\right)\cos\left(\frac{\pi}{2}nt\right) + \frac{2}{n^2\pi}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{\pi}{2}nt\right)\right]$$

2.9-1 See Fig. S2.9-1.

(a) $T_0 = 4, \omega_0 = \pi/2$. Also $D_0 = 0$ (by inspection):

$$D_n = \frac{1}{2\pi}\int_{-1}^{1} e^{-j(n\pi/2)t}\,dt - \int_1^3 e^{-j(n\pi/2)t}\,dt = \frac{2}{\pi n}\sin\frac{n\pi}{2}, \qquad |n| \ge 1$$

(b) $T_0 = 10\pi$, $\omega_0 = 2\pi/10\pi = 1/5$. Also $D_0 = 1/5$ (by inspection):

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t}$$

where

$$D_n = \frac{1}{10\pi}\int_{\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt = \frac{j}{2\pi n}\left(-2j\sin\frac{n\pi}{5}\right) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

(c)

$$g(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt}$$

where, by inspection,

$$D_0 = 0.5$$

$$D_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi} e^{-jnt}\, dt = \frac{j}{2\pi n}$$

so that

$$|D_n| = \frac{1}{2\pi n}$$

and

$$\angle D_n = \begin{cases} \frac{\pi}{2}, & n > 0 \\ -\frac{\pi}{2}, & n < 0 \end{cases}$$

(d) $T_0 = \pi$, $\omega_0 = 2$ and $D_n = 0$,

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt}$$

where

$$D_n = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j2nt}\, dt = \frac{-j}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

(e) $T_0 = 3, \omega_0 = \frac{2\pi}{3}$. Also, $D_0 = 1/6$ (by inspection):

$$g(t) = \sum_{n=\infty}^{\infty} D_n\, e^{j2\pi nt/3}$$

where

$$D_n = \frac{1}{3}\int_0^1 t\, e^{-j2\pi nt/3}\, dt = \frac{3}{4\pi^2 n^2}\left[e^{-j2\pi n/3}\left(\frac{j2\pi n}{3} + 1\right) - 1\right]$$

Therefore

$$|D_n| = \frac{3}{4\pi^2 n^2}\left[\sqrt{2 + \frac{4\pi^2 n^2}{9} - 2\cos\frac{2\pi n}{3} - \frac{4\pi n}{3}\sin\frac{2\pi n}{3}}\right]$$

and

$$\angle D_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

(f) $T_0 = 6$, $\omega_0 = \pi/3$ $D_0 = 0.5$

$$g(t) = 0.5 + \sum_{n=-\infty}^{\infty} D_n e^{j\pi nt/3}$$

$$D_n = \frac{1}{6}\left[\int_{-2}^{-1} (t+2) e^{-j\pi nt/3}\, dt + \int_{-1}^{1} e^{-j\pi nt/3}\, dt + \int_1^2 (-t+2) e^{-j\pi nt/3}\, dt\right] = \frac{3}{\pi^2 n^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2\pi n}{3}\right)$$

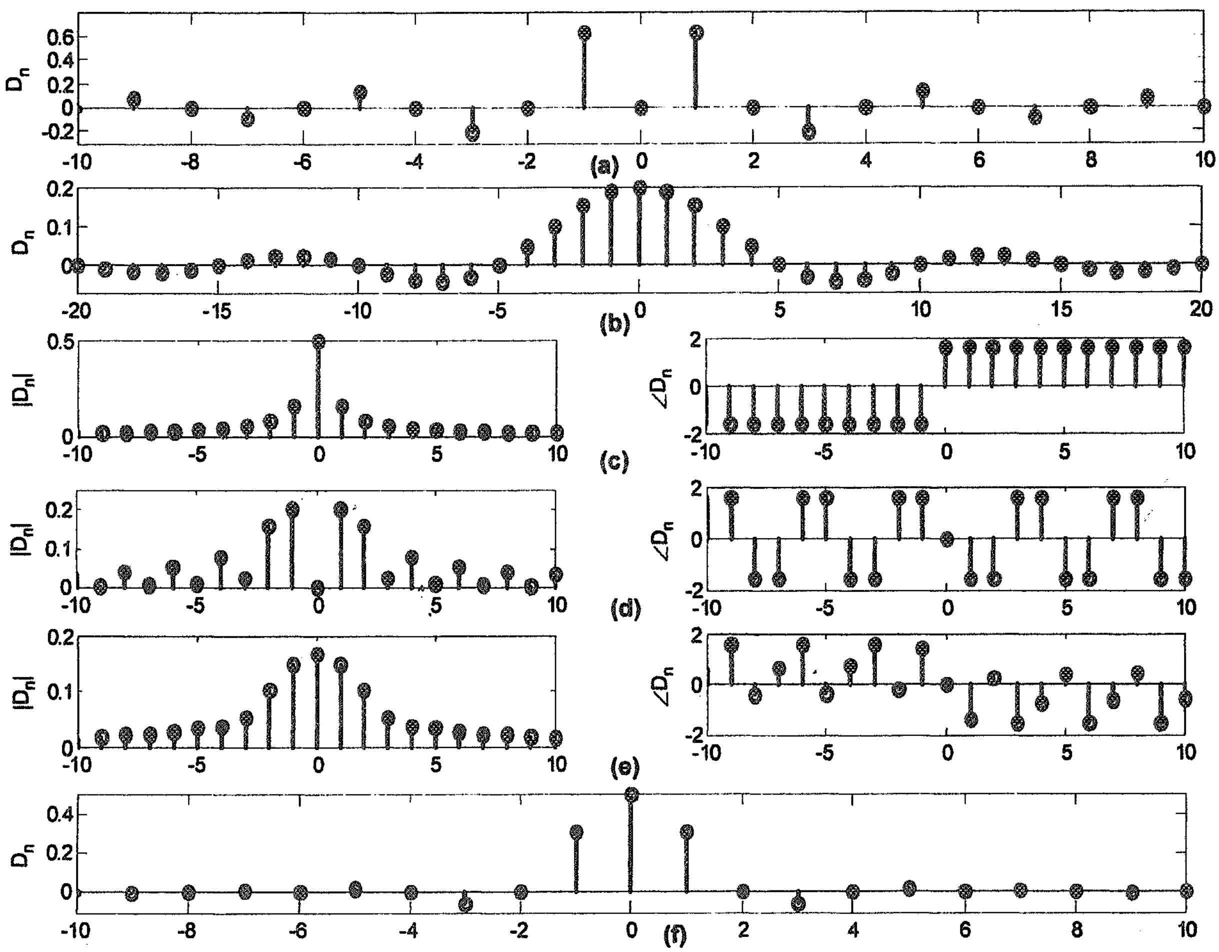

Fig. S2.9-1