Chapter 6

6.1-2

6.1-6

6.2-1

6.2-4

6.2-7

Chapter 6

6.1-1 The bandwidths of $g_1(t)$ and $g_2(t)$ are 5 and 12 kHz, respectively. Therefore the Nyquist sampling rate is 10 kHz for $g_1(t)$ and is 24 kHz for $g_2(t)$.

Also

$$g_1{}^2(t) \Longleftrightarrow \frac{1}{2\pi} g_1(\omega) * g_1(\omega)$$

and from the width property of convolution the bandwidth of $g_1{}^2(t)$ is twice the bandwidth of $g_1(t)$ and that of $g_2{}^m(t)$ is m times the bandwidth of $g_2(t)$. Similarly the bandwidth of $g_1(t)g_2(t)$ is the sum of the bandwidth of $g_1(t)$ and $g_2(t)$. Therefore the Nyquist rate is 20 for $g_1{}^2(t)$ kHz, $24m$ kHz for $g_2{}^m(t)$, and 34 kHz for $g_1(t) \cdot g_2(t)$.

6.1-2

(a)

$$\text{sinc}\,(2100\pi t) \Longleftrightarrow \frac{1}{2100} \Pi\,(f/2100)$$

The bandwidth of this signal is 2100 π rad/s or 1050 Hz. The Nyquist rate is 2100 Hz (samples/s).

(b)

$$5\,\text{sinc}^2\,(200\pi t) \Longleftrightarrow 0.025\Delta\,(f/400)$$

The bandwidth of this signal is 400 π rad/s or 200 Hz. The Nyquist rate is 400 Hz (samples/s).

(c)

$$\text{sinc}\,(2100\pi t) + \text{sinc}^2\,(200\pi t) \Longleftrightarrow \frac{1}{2100} \Pi\,(f/2100) + 0.005\Delta(f/400)$$

The bandwidth of the first term on the right-hand side is 1050 Hz, and the bandwidth of the second term is 200 Hz. Clearly the bandwidth of the composite signal is the higher of the two, that is, 1050 Hz. The Nyquist rate is 2100 Hz (samples/s).

(d)

$$\text{sinc}\,(200\pi t) \Longleftrightarrow 0.005\,\Pi\,(f/200)$$
$$\text{sinc}\,(2100\pi t) \Longleftrightarrow \frac{1}{2100}\,\Pi(f/2100)$$

The two signals have bandwidths 100 Hz, and 1050 Hz, respectively. The spectrum of the product of two signals is $1/2\pi$ times the convolution of their spectra. From width property of the convolution, the width of the convoluted signal is the sum of the widths of the signals convolved. Therefore, the bandwidth of $\text{sinc}\,(200\pi t)\text{sinc}\,(2100\pi t)$ is $100 + 1050 = 1150$ Hz. The Nyquist rate is 2300 Hz.

(a) When the input to this filter is $\delta(t)$, the output of the summer is $\delta(t) - \delta(t - T_s)$. This acts as the input t the integrator. And, $h(t)$, the output of the integrator is

$$h(t) = \int_0^t [\delta(\tau) - \delta(\tau - T_s)]\, d\tau = u(t) - u(t - T_s) = \Pi\left(\frac{t - T_s/2}{T_s}\right)$$

(b) The transfer function of this circuit is

$$H(f) = T_s \text{sinc}\,(\pi f T_s)\, e^{-j\pi f T_s}$$

and

$$|H(f)| = T_s\, |\text{sinc}\,(\pi f T_s)|$$

The amplitude response of the filter is shown in Fig. S6.1-6. Observe that the filter is a lowpass filter of bandwidt $2\pi/T_s$ rad/s or $1/T_s$ Hz.

(c) The impulse response of the circuit is a rectangular pulse. When a sampled signal is applied at the input each sample generates, at the output, a rectangular pulse proportional to the corresponding sample value.

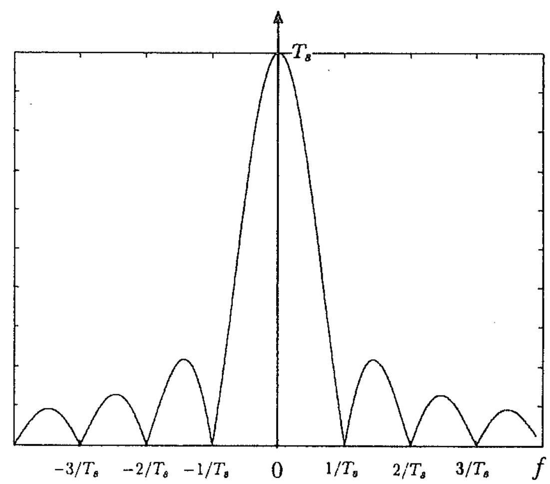

Fig. S6.1-6

6.1-7

(a) From Eq.(6.10), the output of this circuit is given by

$$g(t) = \sum_k g(kT_s) h(t - kT_s) = \sum_k g(kT_s) \Delta\left(\frac{t - kT_s}{2T_s}\right)$$

For $mT_s \leq t \leq (m+1)T_s$, the above equation becomes

$$\begin{aligned} g(t) &= g(mT_s)\left(1 - 2\frac{t - mT_s}{2T_s}\right) + g((m+1)T_s)\left(1 + 2\frac{t - (m+1)T_s}{2T_s}\right) \\ &= g(mT_s) + \frac{g((m+1)T_s) - g(mT_s)}{T_s}(t - mT_s) \end{aligned}$$

Obviously, the resulting signal consists of straight line segments joining the sample tops.

Define

$$F_l(f) = \frac{1}{8}\sum_n Q_n \text{sinc}\left[\pi f + (n+l)\pi f_s T_p\right]$$

Let $p(t)$ be the practical reconstruction pulse. Then the equalizer should satisfy the following constraints as given in

$$E(f)P(f)F_0(f) = \begin{cases} 1, & |f| \le B \\ \text{flexible}, & B < |f| < (1/T_s - B) \\ 0, & |f| \ge (1/T_s - B) \end{cases}$$

See Figure S6.1-10 for the block diagram of this reconstruction system.

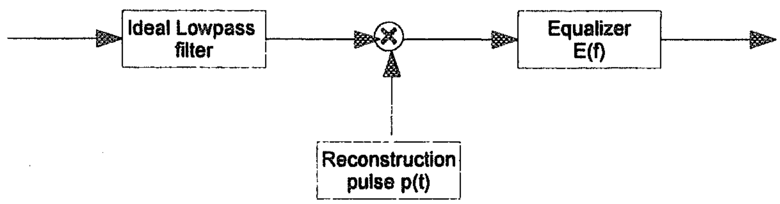

Fig. S6.1-10

6.2-1

(a) Since $65536 = 2^{16}$, 16 binary digits are needed to encode each sample.

(b) Given $m_p = 1$ and $P_m = \overline{m^2(t)} = 0.1$, $L = 65536$. Thus

$$\text{SQNR} = 3L^2\frac{P_m}{m_p^2} = 91.1\,\text{dB}$$

(c) The bandwidth is 15 kHz. The Nyquist rate is 30 kHz. We have $30000 \times 16 = 480000$ bits/s.

(d) $44100 \times 16 = 705600$ bits/s. The minimal bandwidth is $705600/2 = 352800$ Hz.

6.2-2

(a) The Nyquist rate is $2 \times 4.5 = 9$ MHz. The actual sampling rate is $1.2 \times 9 = 10.8$ MHz.

(b) Since $1024 = 2^{10}$, 10 bits or binary pulses are needed to encode each sample.

(c) $10.8 \times 10 = 108$ Mbit/s. The minimal bandwidth is $108/2 = 54$ MHz.

6.2-3

(a) If m_p is the peak sample amplitude, then

$$\text{quantization error} \leq \frac{0.25 m_p}{100} = \frac{m_p}{400}$$

Because the maximum quantization error is $\frac{\Delta v}{2} = \frac{2m_p}{2L} = \frac{m_p}{L}$, it follows that

$$\frac{m_p}{L} = \frac{m_p}{400} \Longrightarrow L = 400$$

Because L should be a power of 2, we choose $L = 512 = 2^9$. This requires a 9-bit binary code per sample.

(b) $(15 \times 2 \times (1+0.2) \times 9) \times 2 \times 128 = 82944\,\text{kbit/s} = 82.944\,\text{Mbit/s}$. The minimum bandwidth is 82.944/2= 41.472 MHz.

(c) In this case, the minimum bandwidth is $82.944 \times (1+0.05)/2 = 43.57185$ MHz

6.2-4 From Eq. (6.34)

$$\frac{S_0}{N_0} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} = 0.06L^2 = 10^{4.3} = 19952.623$$

Thus $L = \sqrt{19952.623/0.06} = 576.7$. Then the minimum number of bits required to code the uniform quantizer is 10.

The SNR obtained with this quantizer is given by

$$10\log_{10}\left(3L^2 \frac{\overline{m^2(t)}}{m_p^2}\right) = 10\log_{10}\left(3 \times 1024^2 \times 0.02\right) = 47.9875\,\text{dB}$$

6.2-5 The power of the signal is 1/6 and the peak amplitude is 1. From Eq. (6.34)

$$\frac{S_0}{N_0} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} = \frac{1}{2}L^2 = 10^{4.3} = 19952.623$$

Thus $L = \sqrt{19952.623 \times 2} = 199.76$. Then the minimum number of bits required to code the uniform quantizer is 8.

The SNR obtained with this quantizer is given by

$$10\log_{10}\left(3L^2 \frac{\overline{m^2(t)}}{m_p^2}\right) = 10\log_{10}\left(3 \times 256^2 \times \frac{1}{6}\right) = 45.15\,\text{dB}$$

6.2-6 The power of the signal is

$$\frac{5\left(1 - e^{-\pi/5}\right)}{\pi}$$

and the peak amplitude is 1. From Eq. (6.34),

$$\frac{S_0}{N_0} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} = \frac{15\left(1 - e^{-\pi/5}\right)}{\pi} L^2 = 10^{4.3} = 19952.623$$

Thus $L = 94.65$. Then the minimum number of bits required to code the uniform quantizer is 7.

The SNR obtained with this quantizer is given by

$$10\log_{10}\left(3L^2 \frac{\overline{m^2(t)}}{m_p^2}\right) = 10\log_{10}\left(\frac{15\left(1 - e^{-\pi/5}\right)}{\pi} \times 128^2\right) = 45.6\,\text{dB}$$

6.2-7 The peak amplitude is 1. From Eq. (6.36)

$$\frac{S_0}{N_0} = \frac{3L^2}{[\ln(1+\mu)]^2} = 0.14L^2 = 10^{4.3} = 19952.623$$

Thus $L = 376.4$. Then the minimum number of bits required to code the uniform quantizer is 9.

The SNR obtained with this quantizer is given by

$$10\log_{10}\left(\frac{3L^2}{[\ln(1+\mu)]^2}\right) = 10\log_{10}\left(\frac{3\times 512^2}{[\ln(1+100)]^2}\right) = 45.67\,\text{dB}$$

6.2-8 Here $\mu = 100$ and the SNR= 45 dB=31622.77. From Eq. (6.36)

$$\frac{S_0}{N_0} = \frac{3L^2}{(\ln(101))^2} = 31622.7 \Longrightarrow L = 473.83$$

Because L is a power of 2, we select $L = 512 = 2^9$. The SNR for this value of L is

$$\frac{S_0}{N_0} = \frac{3\times 512^2}{(\ln(101))^2} = 36922.84 = 45.67\,\text{dB}$$

6.2-9 The minimum possible data rate is

$$(240\times 2)\times(1+0.2)\times(1+0.005)\times 9\times 5 = 26049.6\,\text{kbit/s} = 26.0496\,\text{Mbit/s}$$

The minimal bandwidth is 13.0248 MHz

6.2-10 **(a)** Nyquist rate = 2 MHz. The actual sampling rate is $1.5\times 2 = 3$ MHz. Moreover, $L = 256$ and $\mu = 255$. From Eq. (6.36),

$$\frac{S_0}{N_0} = \frac{3L^2}{[\ln(\mu+1)]^2} = \frac{3\times 256^2}{(\ln(256))^2} = 6394 = 38.06\,\text{dB}$$

If we reduce the sampling rate and increase the value of L so that the same data rate is maintained, we can improve the SNR with the same bandwidth. In part **(a)** the sampling rate is 3 MHz, and each sample is encoded by 8 bits. Hence the transmission rate is 24 Mbit/s.

If we reduce the sampling rate to 2.4 MHz (20% above the Nyquist rate), then for the same transmission rate, we can have $(24/2.4) = 10$. This results in $L = 2^{10} = 1024$. Hence the new SNR is

$$\frac{S_0}{N_0} = \frac{3L^2}{[\ln(\mu+1)]^2} = \frac{3\times 1024^2}{(\ln(256))^2} = 102300 = 50.1\,\text{dB}$$

Clearly, the SNR is increased by more than 10 dB.

6.2-11 Equation (6.41) shows that increasing n by one bit increases the SNR by 6 dB. Hence, an increase in the SNR by 12 dB (from 30 to 42) can be accomplished by increasing n from 13 to 15, that is increasing by 15.4%.

6.7-1