Chapter 4

4.2-1

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4.3-2

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4.4-1

4.4-2

4.4-5

4.4-6

4.2 DSB AM
SC

4.3 AM

4.4 - SSB
Hilbert tr.
QAM

4.5 VSB

(4.6 LO sync
(4.7 FDM

4.8 PLL

4.2-1

(i) $m(t) = \cos \omega_m t = \cos 2\pi f_m t = \cos 1000\pi t \rightarrow f_m = 500\text{Hz}$.
$M(f) = 0.5\delta(f-500) + 0.5\delta(f+500)$.
See Fig. S4.2-1a(i).

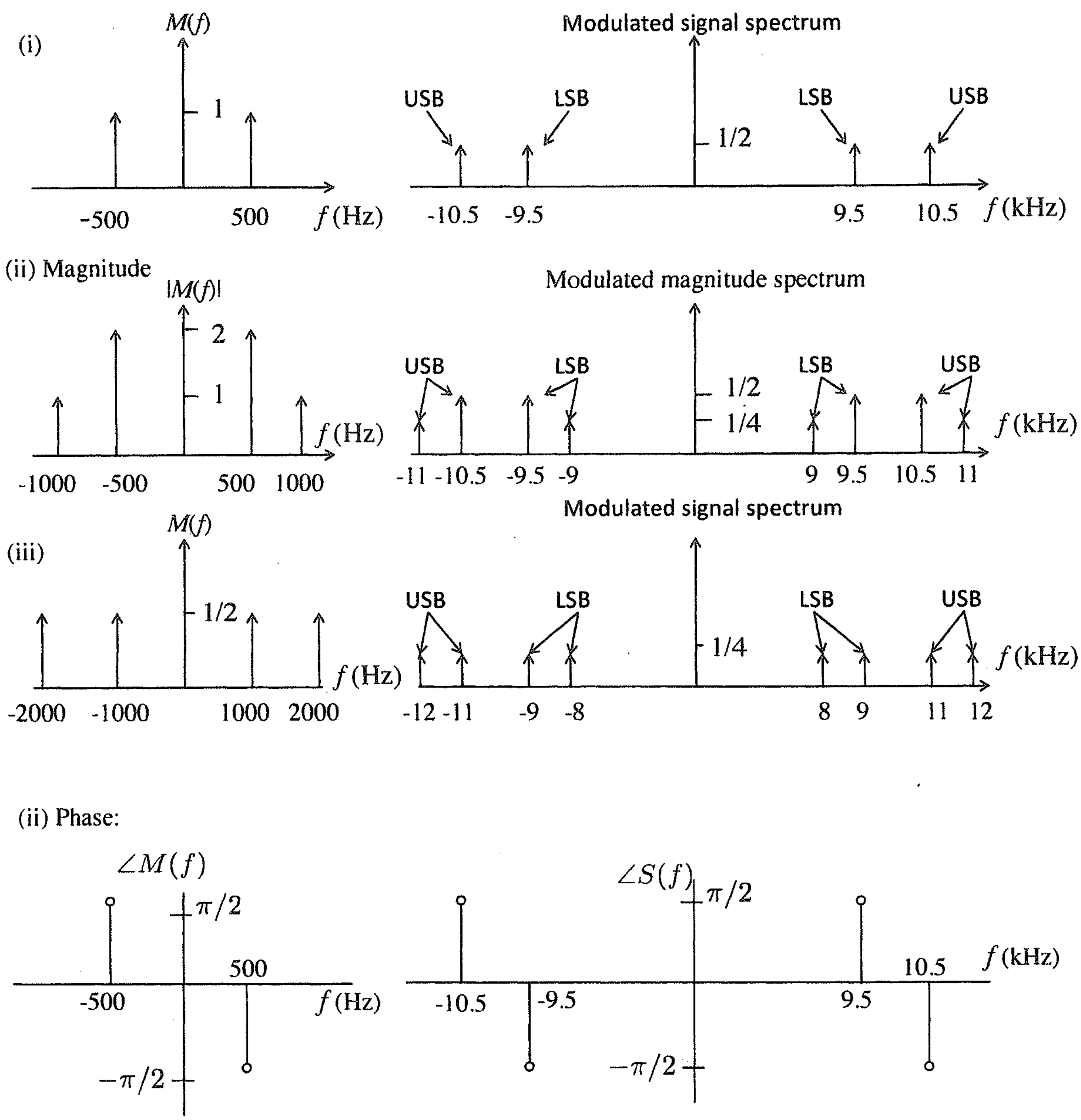

Fig. S4.2-1a

(ii) $m(t) = 2\cos \omega_{m,1} t + \sin \omega_{m,2} t = 2\cos 2\pi f_{m,1} t + \sin 2\pi f_{m,2} t = 2\cos 1000\pi t + \sin 2000\pi t$
$\rightarrow M(f) = \delta(f-1000) + \delta(f+1000) - 0.5j\delta(f-500) + 0.5j\delta(f+500)$
$|M(f)| = \delta(f-1000) + \delta(f+1000) + 0.5\delta(f-500) + 0.5\delta(f+500)$

$$\angle M(f) = \begin{cases} -\pi/2, & f = 500 \\ \pi/2, & f = -500 \\ 0, & \text{else} \end{cases}$$

See Fig. S4.2-1a(ii) for its magnitude plot.

(iii) $m(t) = \cos\omega_{m,1}t \cdot \cos\omega_{m,2}t = \cos 1000\pi t \cdot \cos 3000\pi t = \frac{1}{2}(\cos 2\pi f_{m,1}t + \cos 2\pi f_{m,2}t) = \frac{1}{2}(\cos 2000\pi t + \cos 4000\pi t) \rightarrow$ $f_{m,1} = 1000\text{Hz}$ $f_{m,2} = 2000\text{Hz}$

See Fig. S4.2-1a(iii) for the graphical results.

(iv) Since $m(t) = e^{-10|t|}$, we have

$$\mathcal{F}(m(t)) = M(f) = \frac{20}{100 + 4\pi^2 f^2}$$

See Fig. S4.2-1b.

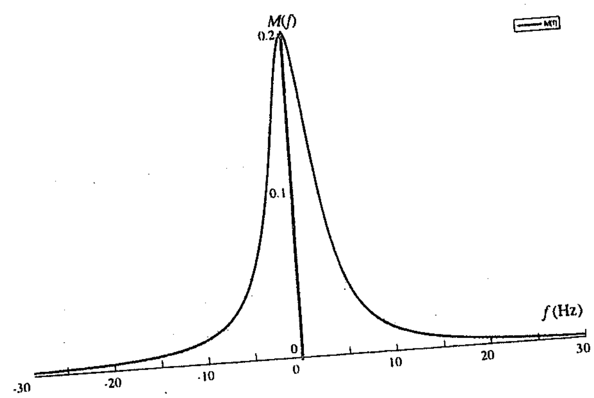

Fig. S4.2-1b

(v) Using frequency shift property on the result of part **(iv)**, we have $\mathcal{F}(m(t)) = M(f) = \frac{10}{100+4\pi^2(f-f_c)^2} + \frac{10}{100+4\pi^2(f+f_c)^2}$. See Fig. S4.2-1c.

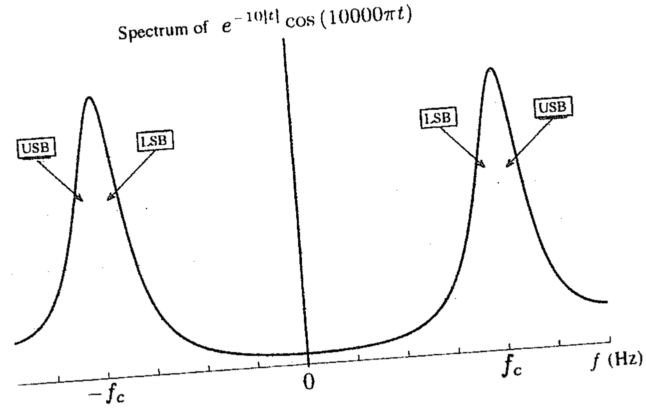

Fig. S4.2-1c

(c) If we pass the output $e_o(t)$ through a bandpass filter (centered at ω_c), the filter suppresses the signal $m(t)$ and $m(t)\cos(n\omega_c t)$ for all $n \neq 1$, leaving intact only the modulated term

$$\frac{4R}{\pi(R+r)} m(t)\cos(\omega_c t)$$

Hence, the system acts as a DSB-SC modulator.

(d) The same circuit can be used as a demodulator if we use a basepass filter at the output. In this case, the input is $\phi(t) = m(t)\cos(\omega_c t)$ and the output is

$$\frac{2R}{\pi(R+r)} m(t).$$

4.2-6 From the results in Problem **4.2-5**, the output

$$e_o(t) = km(t)\cos(\omega_c t) \ + \text{other terms}$$

where $k = 4R/\pi(R+r)$. In the present case, $m(t) = \sin(\omega_c t + \theta)$. Hence, the output is

$$\begin{aligned} e_o(t) &= k\sin(\omega_c t + \theta)\cos(\omega_c t) \ + \text{other terms} \\ &= \frac{k}{2}[\sin\theta + \sin(2\omega_c t + \theta)] \ + \text{other sinusoidal terms of angular frequencies } n\omega_c \end{aligned}$$

The other terms now all have frequency higher than ω_c. The lowpass filter suppresses the sinusoid of frequencies $\omega_c, 2\omega_c, \ldots$, and it transmits only the dc term $k/2 \cdot \sin\theta$.

4.2-7

(a) Figure S4.2-7 shows the signals at points a,b and c.

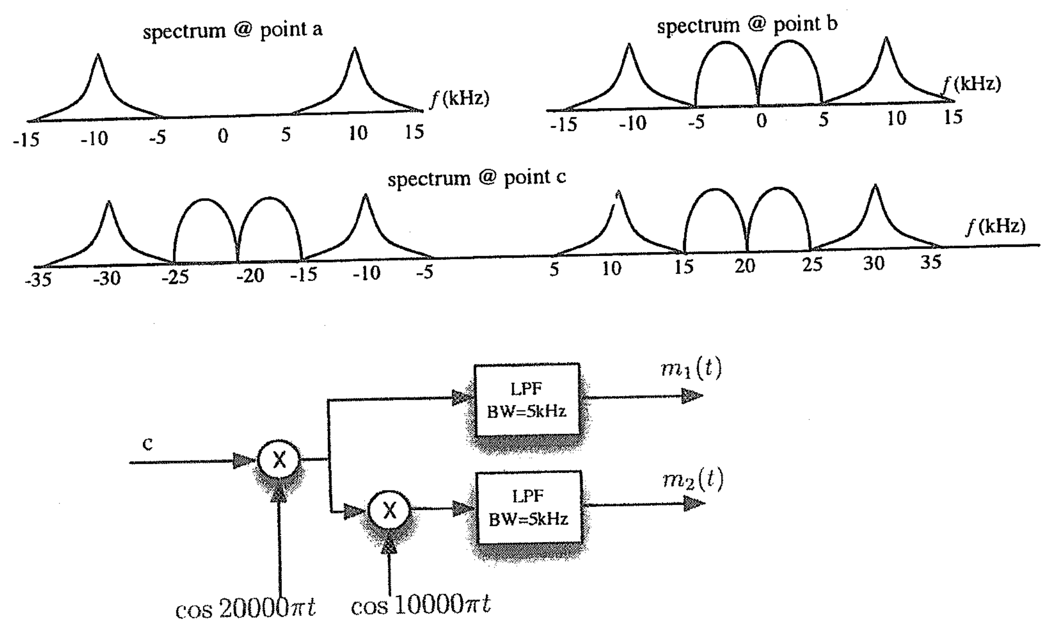

Fig. S4.2-7

(b) From the spectrum at point c, it is clear that the channel bandwidth must be at least 30000 Hz (from 5000 Hz to 35000 Hz).

(c) Figure S4.2-7 shows the receiver diagram to recover both $m_1(t)$ and $m_2(t)$from the modulated signal at point c.

4.2-8

(a) Figure S4.2-8 shows the output signal spectrum $Y(f)$ after the scrambler processing.

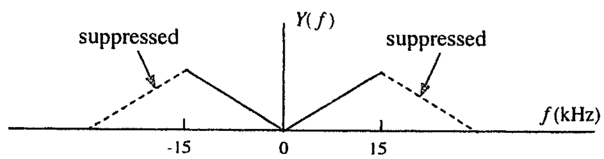

Fig. S4.2-8

(b) Observe that $Y(\omega)$ is the same as $M(\omega)$ with the frequency spectrum inverted, that is, the high frequencies are shifted to lower frequencies and vice versa. Thus, the scrambler in Fig. P4.2-8 inverts the frequency spectrum. To get back to the original spectrum $M(\omega)$, we need to invert the spectrum $Y(\omega)$ once again. This can be done by passing the scrambled signal $y(t)$ through the same scrambler in Fig. S4.2-8.

4.2-9

(a) See the frequency responses in Figure S4.2-9a.

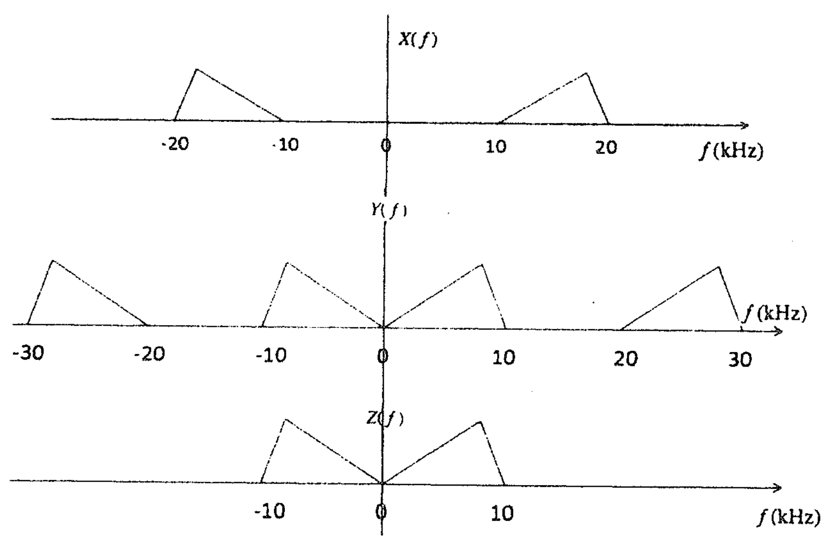

Fig. S4.2-9a

(b) See Fig. S4.2-9b1 and S4.2-9b2

(c) See Fig. S4.2-9c

In case (b) the signal $z(t)$ cannot be unscrambled to recover $m(t)$.

4.3-1

$$\begin{aligned} s_{AM}(t) &= 2\left[b + \tfrac{1}{2}m(t)\right]\cos(\omega_c t) \\ &= (2b + m(t))\cos(\omega_c t) \end{aligned} \qquad \text{with} \qquad \omega_c = 20000\pi$$

(a) $P_c = \frac{4b^2}{2} = 2b^2$ Because

$$\overline{m^2(t)} = \frac{2}{T}\int_0^{\frac{T}{2}} m^2(t)\,dt = \frac{2}{0.1}\int_0^{\frac{0.1}{2}} m^2(t)\,dt = \frac{4}{3}$$

we have $P_s = \frac{1}{2}\overline{m^2(t)} = \frac{2}{3}$ and $P_c + P_s = 2b^2 + \frac{2}{3}$.

(b) $b = 1$

$\eta = \frac{P_s}{P_c + P_s} = \frac{\frac{2}{3}}{2 + \frac{2}{3}} \cdot 100\% = 25\%$

(c) Assume $b = 1$ See Fig. S4.3-1

(d) If $b = 0.5$, $P_s = \frac{2}{3}$, $P_{\hat{c}} + P_{\hat{s}} = 2b^2 + \frac{2}{3} = \frac{7}{6}$, $\eta = \frac{P_{\hat{s}}}{P_{\hat{s}} + P_{\hat{c}}} = \frac{\frac{2}{3}}{\frac{7}{6}} 100\% = 57\%$

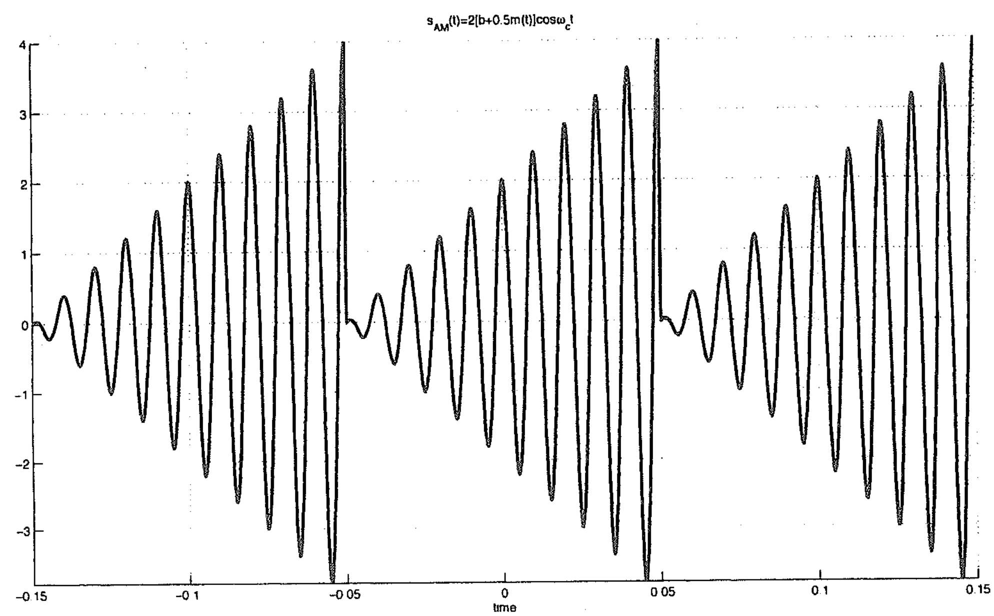

Fig. S4.3-1

4.3-2 In the SPECIAL case of $m_{\min} = -m_p$ and $m_{max} = m_p$, we have $mu = m_p/A$.

(a) $\mu = \frac{m_p}{A} = 0.5 \rightarrow A = 4$

(b) $\mu = \frac{m_p}{A} = 1.0 \rightarrow A = 2$

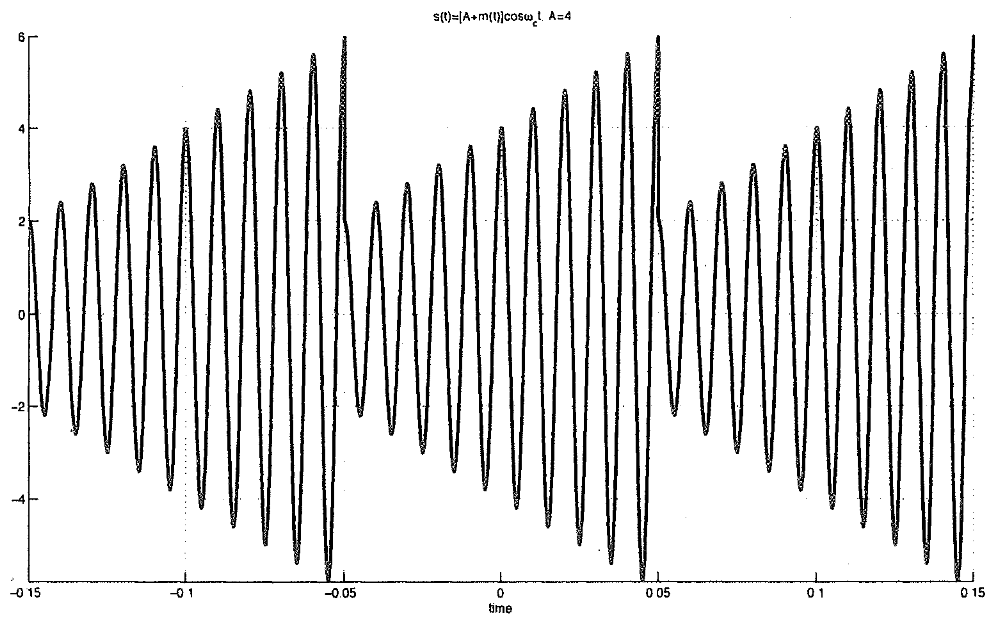

Fig. S4.3-2a

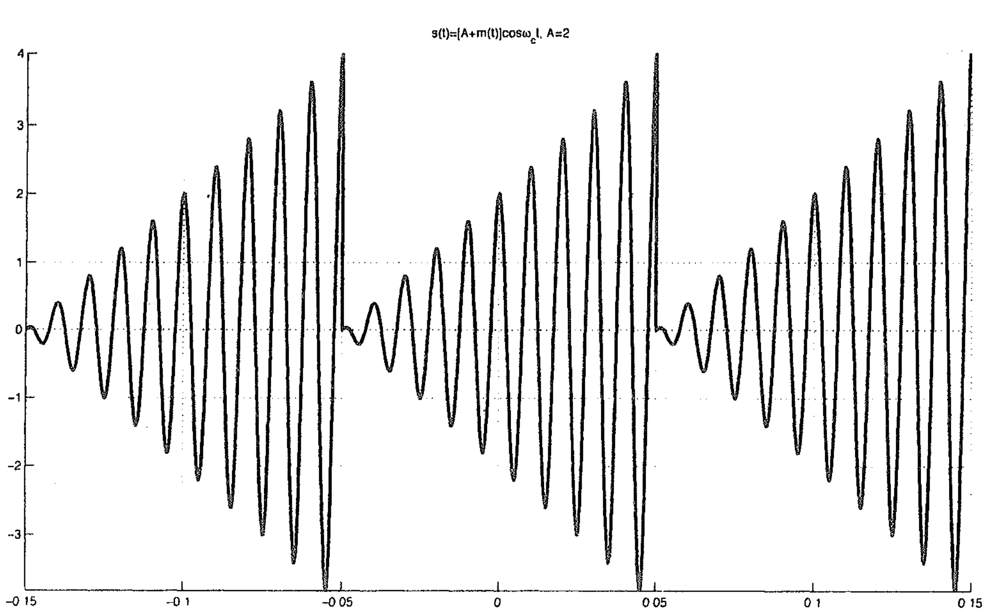

Fig. S4.3-2b

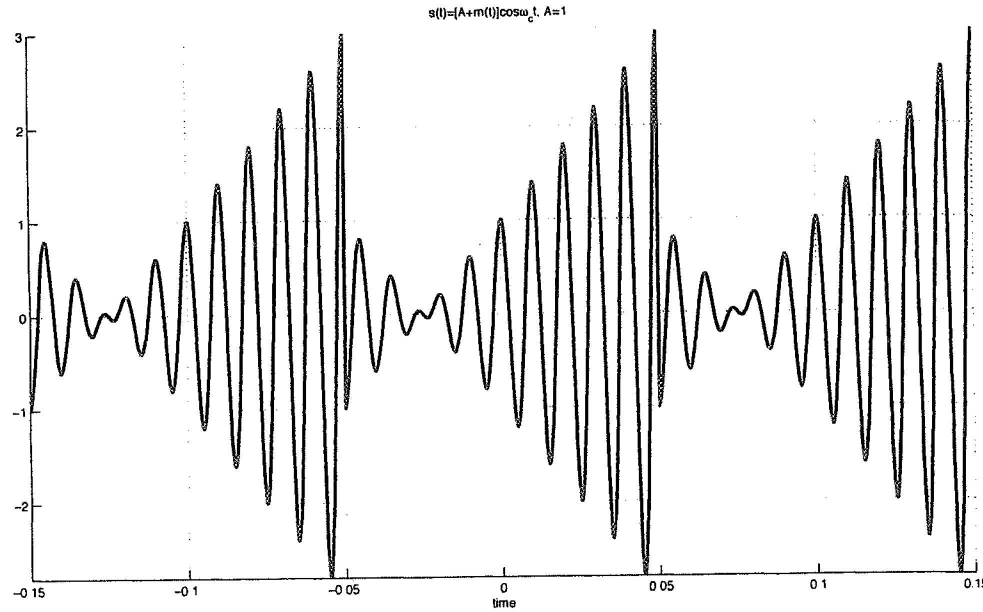

Fig. S4.3-2c

(c) $\mu = \frac{m_p}{A} = 2.0 \rightarrow A = 1$

(d) $\mu = \frac{m_p}{A} = \infty \rightarrow A = 0$
This means that $\mu = \infty$ represents the DSB-SC case.

4.3-3

(a) According to Eq. (4.10a), the carrier amplitude is $A = \frac{m_p}{\mu} = \frac{10}{0.75} = 13.34$. The carrier power is $P_c = \frac{A^2}{2} =$ 88.89

(b) The sideband power is $\overline{m^2(t)}/2$. Because of symmetry of amplitude values every quarter cycle, the power of $m(t)$ may be computed by averaging the signal energy over a quarter cycle only. Over a quarter cycle $m(t)$ can be represented as $m(t) = 40t/T_0$ (see Fig. S4.3-3). Note that $T_0 = 10^{-3}$ Hence,

$$\overline{m^2(t)} = \frac{1}{T_0/4} \int_0^{T_0/4} \left[\frac{40t}{T_0}\right]^2 dt = 33.34$$

The sideband power is

$$P_s = \frac{\overline{m^2(t)}}{2} = 16.67$$

The efficiency is

$$\eta = \frac{P_s}{P_c + P_s} = \frac{16.67}{88.89 + 16.67} \times 100\% = 15.79\%$$

4.3-4

(a) For $\omega_c = 2\pi f_c$ in which $f_c = 10$ kHz, the DSB-SC signal is plotted in Fig. S4.3-4a:

(b) From Fig. S4.3-4 it is clear that the envelope of the signal $m(t)\cos(\omega_c t)$ is $|m(t)|$. The signal $[A+m(t)]\cos(\omega$ is identical to $m(t)\cos(\omega_c t)$ with $m(t)$ replaced by $A+m(t)$. Hence, using the previous argument, it is clear tha its envelope is $|A+m(t)|$. Now, if $A+m(t)>0$ for all t, then $A+m(t)=|A+m(t)|$. Therefore, the conditio for demodulating an AM signal using envelope detector is

$$A+m(t)>0 \qquad \text{for all } t$$

4.3-5

(a) $m(t)=e^{-3t}u(t-2)$

$$M(f)=\int_{-\infty}^{+\infty} e^{-3t}u(t-2)\,e^{-j2\pi ft}dt=\int_{2}^{+\infty} e^{-3t}e^{-j2\pi ft}dt=\frac{1}{j2\pi f+3}e^{-j4\pi f-6}$$

The magnitude of $M(f)$ is shown in Fig. S4.3-5a as $|M(f)$

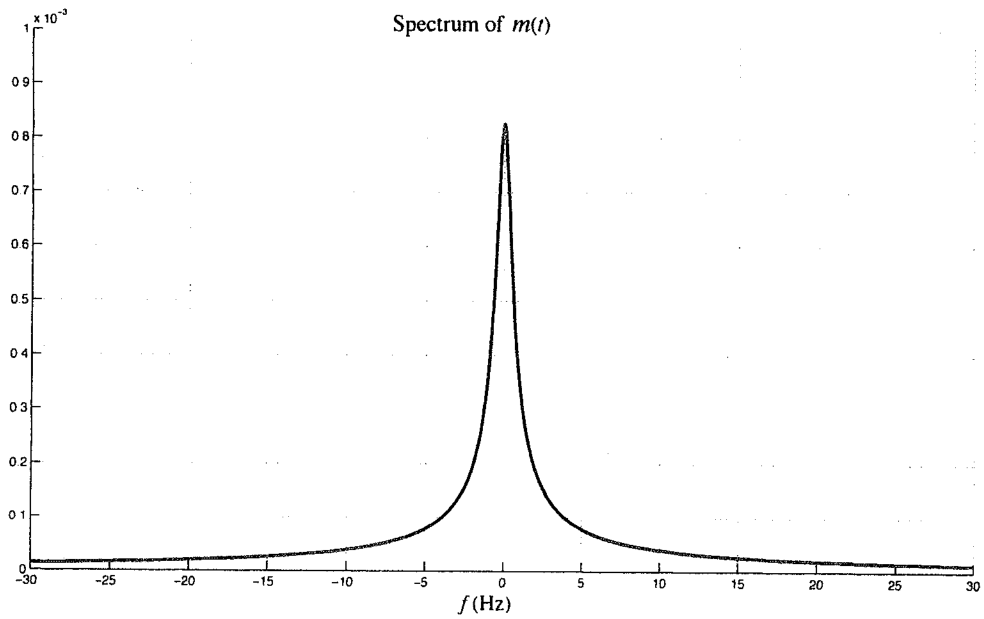

Fig. S4.3-5a

(b) $f_c = 1000\,\text{Hz}$. See Fig. S4.3-5b for the modulation effect.

4.3-6 When an input to a DSB-SC generator is $m(t)$, and the corresponding output $m(t)\cos(\omega_c t)$. Clearly, if the input is $A+m(t)$, the corresponding output will be $[A+m(t)]\cos(\omega_c t)$, the corresponding output will be $A+m(t)$. This is precisely the AM signal. Thus, by adding a dc of value A to the baseband signal $m(t)$, we can use a DSB-SC generator to generate AM signals.

The converse is generally not true. However, we can use two AM generators to generate DSB-SC signals if we use two identical AM generators in a balanced scheme as shown in Fig. S4.3-6 to remove the carrier component.

4.3-7 Observe that $m^2(t) = A^2$ for all t. Hence, the time average of $m^2(t)$ is also A^2. Thus:

$$\begin{cases} \overline{m^2(t)} = A^2 \\ P_s = \frac{\overline{m^2(t)}}{2} = \frac{A^2}{2} \end{cases}$$

The carrier amplitude is $A = \frac{m_p}{\mu} = m_p = A$. Hence $P_c = \frac{A^2}{2}$. The total power is $P_t = P_c + P_s = A^2$. The power efficiency is:

$$\eta = \frac{P_s}{P_t} \times 100\% = \frac{\frac{A^2}{2}}{A^2} \times 100\% = 0.5$$

The AM signal for $\mu = 1$ is shown in Fig. S4.3-7.

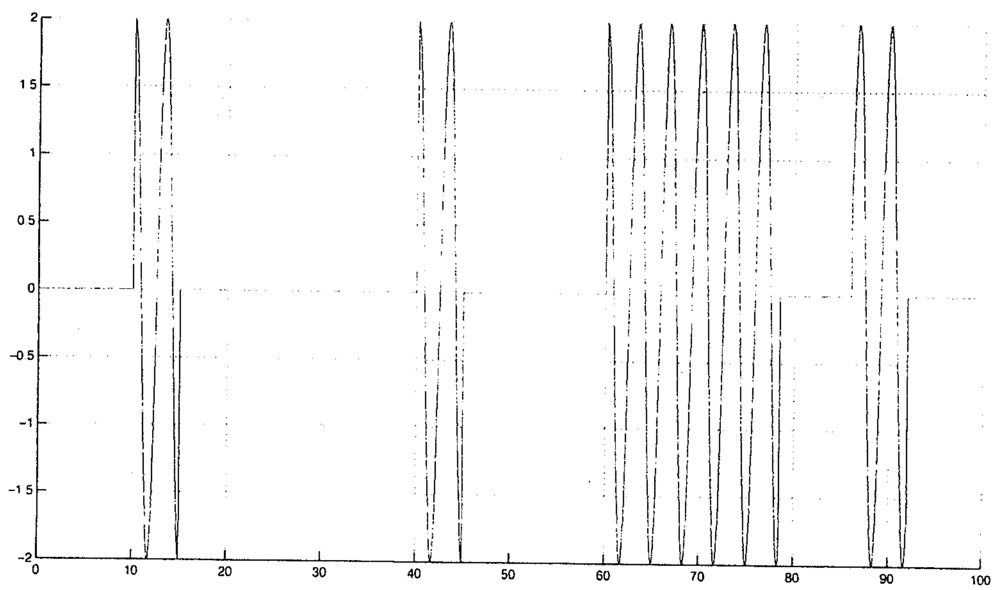

Fig. S4.3-7

4.3-8 The signal at point **a** is $[A + m(t)]\cos(\omega_c t)$. The signal at point **b** is:

$$x(t) = [A + m(t)]^2 \cos^2(\omega_c t) = \frac{A^2 + 2Am(t) + m^2(t)}{2}(1 + \cos(2\omega_c t))$$

The lowpass filter suppresses the term containing $\cos(2\omega_c t)$. Hence, the signal at point **c** is:

$$w(t) = \frac{A^2 + 2Am(t) + m^2(t)}{2} = \frac{A^2}{2}\left[1 + \frac{2m(t)}{A} + \left(\frac{m(t)}{A}\right)^2\right]$$

Usually, $m(t)/A \ll 1$ for most of the time. This condition is violated only when $m(t)$ is near its peak, Hence, the output at point d is:

$$y(t) \approx \frac{A^2}{2} + Am(t)$$

A blocking capacitor will suppress the dc term $A^2/2$, yielding the output $m(t)$. From the signal $w(t)$, we see that the distortion component is $m^2(t)/2$.

4.3-9 $\phi(t) = m(t)\cos(\omega_c t)$

Define:

$$\phi_1(t) = (m(t)\cos(\omega_c t) + \cos(\omega_c t))^2 = \phi^2(t) + 2\phi(t)\cos(\omega_c t) + \cos^2(\omega_c t)$$

$$\phi_2(t) = (m(t)\cos(\omega_c t) - \cos(\omega_c t))^2 = \phi^2(t) - 2\phi(t)\cos(\omega_c t) + \cos^2(\omega_c t)$$

Then;

$$\phi_1(t) - \phi_2(t) = 4\phi(t)\cos(\omega_c t)$$

The block diagram is shown in Fig. S4.3-9

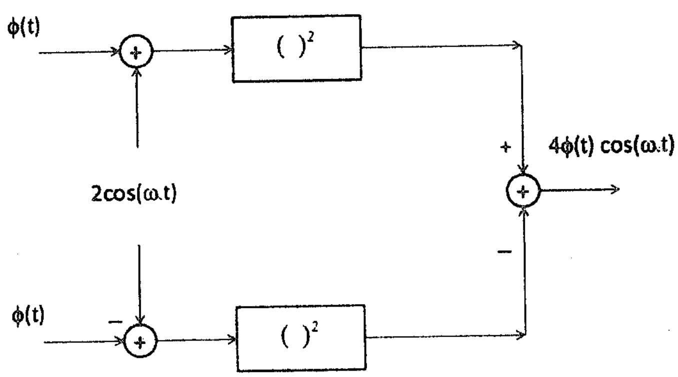

Fig. S4.3-9

4.4-1 In Fig. 4.14, when the carrier is $\cos[(\Delta\omega)t + \delta]$ or $\sin[(\Delta\omega)t + \delta]$, we have:

$$\begin{aligned} x_1(t) &= 2[m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)]\cos[(\omega_c + \Delta\omega)t + \delta] \\ &= 2m_1(t)\cos(\omega_c t)\cos[(\omega_c + \Delta\omega)t + \delta] + 2m_2(t)\sin(\omega_c t)\cos[(\omega_c + \Delta\omega)t + \delta] \\ &= m_1(t)\{\cos[(\Delta\omega)t + \delta] + \cos[(2\omega_c + \Delta\omega)t + \delta]\} + m_2(t)\{\sin[(2\omega_c + \Delta\omega)t + \delta] - \sin[(\Delta\omega)t + \delta]\} \end{aligned}$$

Similarly

$$x_2(t) = m_1(t)\{\sin[(2\omega_c + \Delta\omega)t + \delta] + \sin[(\Delta\omega)t + \delta]\} + m_2(t)\{\cos[(\Delta\omega)t + \delta] - \cos[(2\omega_c + \Delta\omega)t + \delta]\}$$

After $x_1(t)$ and $x_2(t)$ are passed through low-pass filter, the outputs are

$$m_1'(t) = m_1(t)\cos[(\Delta\omega)t + \delta] - m_2(t)\sin[(\Delta\omega)t + \delta]$$

$$m_2'(t) = m_2(t)\sin[(\Delta\omega)t + \delta] + m_2(t)\cos[(\Delta\omega)t + \delta]$$

4.4-2 To generate a DSB-SC signal from $m(t)$, we multiply $m(t)$ by $\cos(\omega_c t)$. However, to generate the SSB signals of the same relative magnitude, it is convenient to multiply $m(t)$ by $2\cos(\omega_c t)$. This also avoids the nuisance of the fractions 1/2, and yields the DSB-SC spectrum

$$M(\omega - \omega_c) + M(\omega + \omega_c)$$

We suppress the USB spectrum (above ω_c and below $-\omega_c$) to obtain the LSB spectrum. Similarly, to obtain the USB spectrum, we suppress the LSB spectrum (between $-\omega_c$ and ω_c) from the DSB-SC spectrum. Figures S4.4-2a and S4.4-2b show the three cases.

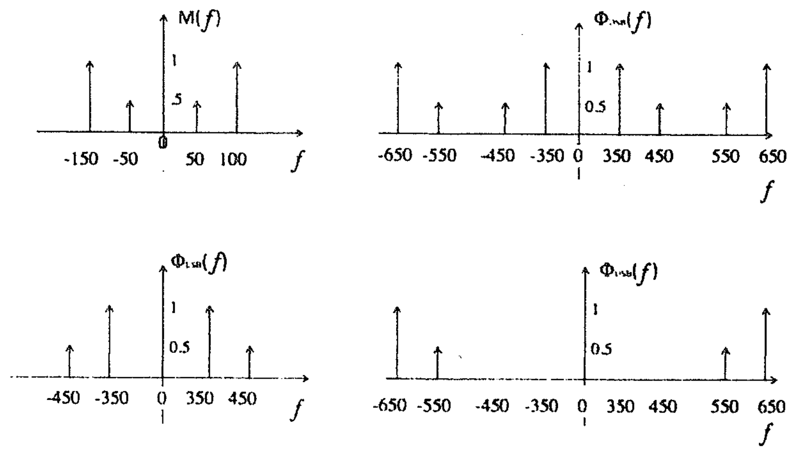

Fig. S4.4-2a

(a) From Fig. S4.4-2a, we can express $\phi_{LSB}(t) = 2\cos(700\pi t) + \cos(900\pi t)$ and $\phi_{USB}(t) = \cos(1100\pi t) + 2\cos(1300\pi t)$.

(b) From Fig. S4.4-2b, we can express:
$\phi_{LSB}(t) = \frac{1}{2}[\cos(400\pi t) + \cos(600\pi t)]$ and $\phi_{USB}(t) = \frac{1}{2}[\cos(1400\pi t) + \cos(1600\pi t)]$.

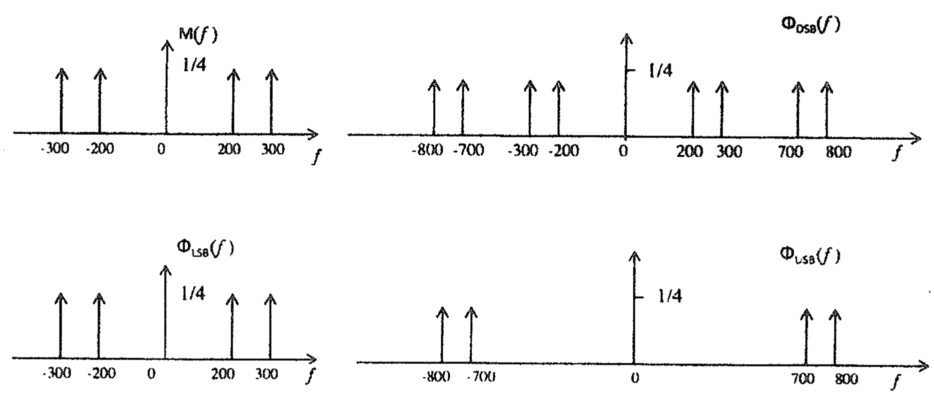

Fig. S4.4-2b

4.4-3 $\phi_{USB}(t) = m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)$ and $\phi_{LSB}(t) = m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)$

(a) $m(t) = \cos(100\pi t) + 2\cos(300\pi t)$ and $m_h(t) = \sin(100\pi t) + 2\sin(300\pi t)$. The carrier is $\cos 1000t$. Hence,

$$\begin{aligned}
\phi_{LSB}(t) &= [\cos(100\pi t) + 2\cos(300\pi t)]\cos(1000t) + [\sin(100\pi t) + 2\sin(300\pi t)]\sin(1000t) \\
&= \cos(1000 - 100\pi)t + 2\cos(1000 - 300\pi)t \\
\phi_{USB}(t) &= [\cos(100\pi t) + 2\cos(300\pi t)]\cos(1000t) - (\sin(100\pi t) + 2\sin(300\pi t))\sin(1000t) \\
&= \cos(1000 + 100\pi)t + 2\cos(1000 + 300\pi)t
\end{aligned}$$

$$\begin{aligned}\varphi_{USB}(t) &= \int_{-f_c-2\pi B}^{-f_c} \tfrac{1}{4\pi B}(f+f_c+2\pi B)\,e^{j2\pi ft}df + \int_{f_c}^{f_c+2\pi B} -\tfrac{1}{4\pi B}(f-fc-2\pi B)\,e^{j2\pi ft}df \\ &= \tfrac{1}{2\pi B}\tfrac{1}{(2\pi t)^2}\left[\cos(2\pi f_c t) - \cos(2\pi(f_c+2\pi B)t)\right] - \tfrac{1}{2\pi t}\sin(2\pi f_c t)\end{aligned}$$

4.4-5 Because $M_h(f) = -jM(f)\,\mathrm{sgn}(f)$, the transfer function of a Hilbert transformer is:

$$H(f) = -j\ \mathrm{sgn}(f)$$

If we apply $m_h(t)$ at the input of the Hilbert transformer; $Y(f)$, the spectrum of the output signal $y(t)$ is

$$Y(f) = M_h(f)\,H(f) = [-jM(f)\ \mathrm{sgn}(f)]\,[-j\ \mathrm{sgn}(f)] = -M(f)$$

This shows that the Hilbert transform of $m_h(t)$ is $-m(t)$. To show that the energies of $m(t)$ and $m_h(t)$ are equal, we have:

$$E_m = \int_{-\infty}^{+\infty} m^2(t)\,dt = \int_{-\infty}^{+\infty} |M(f)|^2 df$$

$$E_{m_h} = \int_{-\infty}^{+\infty} m_h^2(t)\,dt = \int_{-\infty}^{+\infty} |M_h(f)|^2 df = \int_{-\infty}^{+\infty} |M(f)|^2 |\ \mathrm{sgn}(f)|^2 df = \int_{-\infty}^{+\infty} |M(f)|^2 df = E_m$$

4.4-6 The incoming SSB signal at the receiver is given by [Eq. (4.17b)]

$$\varphi_{LSB}(t) = m(t)\cos[(\omega_c+\Delta\omega)t+\delta] + m_h(t)\sin[(\omega_c+\Delta\omega)t+\delta]$$

Let the local carrier be $\cos[\omega_c t]$. The product of the incoming signal and the local carrier is $e_d(t)$, given by

$$e_d(t) = \varphi_{LSB}(t)\cos[\omega_c t]$$

$$e_d(t) \quad = 2\left[m(t)\cos((\omega_c+\Delta\omega)t+\delta) + m_h(t)\sin((\omega_c+\Delta\omega)t+\delta)\right]\cos(\omega_c t)$$

The lowpass filter suppresses the sum frequency component centered at the frequency $(2\omega_c+\Delta\omega)$, and passes only the difference frequency component centered at the frequency $\Delta\omega$. Hence, the filter output $e_0(t)$ is given by

$$e_0(t) = m(t)\cos(\Delta\omega t+\delta) - m_h(t)\sin(\Delta\omega t+\delta)$$

Observe that if both $\Delta\omega$ and δ are zero, the output is given by

$$e_0(t) = m(t)$$

as expected. If only $\delta = 0$, then the output is given by:

$$e_0(t) = m(t)\cos(\Delta\omega t) - m_h(t)\sin(\Delta\omega t)$$

This is a USB signal corresponding to a small carrier frequency $\Delta\omega$. This spectrum is the same as the spectrum $M(\omega)$ with each frequency component shifted by a frequency $\Delta\omega$. This changes the sound of an audio signal slightly. For voices signals, the frequency shift within ±20Hz is considered tolerable. Most US systems, however, restrict the shift to ±2Hz.

(b) When only $\Delta\omega = 0$, the lowpass filter output is:

$$e_0(t) = m(t)\cos\delta - m_h(t)\sin\delta$$

We now show that this is a phase distortion, where each frequency component of $M(\omega)$ is shifted in phase by an amount δ. The Fourier transform of this equation yields:

$$E_0(\omega) = M(\omega)\cos\delta - M_h(\omega)\sin\delta$$

But from Eq. (4.14b)

$$M_h(\omega) = -j\ \text{sgn}(\omega)\, M(\omega) = \begin{cases} -jM(\omega) & \omega > 0 \\ M(\omega) & \omega < 0 \end{cases}$$

And

$$E_0(\omega) = \begin{cases} M(\omega)\, e^{j\delta} & \omega > 0 \\ M(\omega)\, e^{-j\delta} & \omega < 0 \end{cases}$$

It follows that the amplitude spectrum of $e_0(t)$ is $M(\omega)$, the same as that for $m(t)$. But the phase of each component is shifted by δ. Phase distortion generally is not a serious problem with voice signals, because the human ear is somewhat insensitive to phase distortion. Such distortion may change the quality of speech, but the voice is still intelligible. In video signals and data transmission, however, phase distortion may be intolerable.

4.4-7 We showed in Problem 4.4-5 that the Hilbert transform of $m_h(t)$ is the $-m(t)$. Hence, if $m_h(t)$ [instead of $m(t)$] is applied at the input in Fig. 4.20, the USB output is:

$$\begin{aligned} y(t) &= m_h(t)\cos(\omega_c t) - m(t)\sin(\omega_c t) \\ &= m(t)\cos\left(\omega_c t + \tfrac{\pi}{2}\right) + m_h(t)\sin\left(\omega_c t + \tfrac{\pi}{2}\right) \end{aligned}$$

Thus, if we apply $m_h(t)$ at the input of the Fig. 4.20, the USB output is an LSB signal corresponding to $m(t)$. The carrier also acquires a phase shift of $\pi/2$. Similarly, we can show that if we apply $m_h(t)$ at the input of the Fig. 4.20, the LSB output would be an USB signal corresponding to $m(t)$ (with a carrier phase shifted by $\frac{\pi}{2}$).

4.5-1 From Eq. (4.25)

$$H_0(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)} \qquad |f| \le 2\pi B$$

Figure S4.5-1 shows $H_i(f-f_c) + H_i(f+f_c)$ and the reciprocal, which is $H_0(f)$.

4.5-2 We use 1.5 MHz as the carrier frequency. Thus, the VSB uses all the lower sideband width until 1.496 MHz.

(a) Figure S4.5-2a shows the receiver block diagram. Without a receiver filter $H_R(f)$, the correction is performed solely by output filter $H_o(f)$ on

$$H_i(f) = H_T(f)\, H_R(f) = H_T(f)$$

(b) $B = (1501 - 1496) = 5$ kHz.

(c) Fig. S4.5-2c shows $H_i(f+f_c) + H_i(f-f_c)$ and the corresponding design of $H_0(f)$ spectrum.

4.5-3