Chapter 10

10.1-1

10.2-1

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10.4-1

Chapter 10

10.1-1 By

$$10^4 = \int_{-W}^{W} \frac{\|H_c(f)\|^2 S_m(f)}{S_n(f)} \mathrm{d}f,$$

we have

$$1 = \int_{-W}^{W} \frac{8}{4\pi^2 f^2 + (3000\pi)^2} \mathrm{d}f$$

Hence, it is possible to arrive

$$\frac{\arctan(2\pi f/3000\pi)}{3000\pi \times 3\pi} = \frac{1}{16},$$

or $f = 473.5$Hz.

10.1-2

$$S_{n_o}(\omega) = S_n(\omega)|H_d(\omega)|^2 = 10^{-10}\left(\frac{\omega^2+\alpha^2}{\alpha^2}\right) \qquad \alpha = 8000\pi$$

$$N_o = \frac{1}{\pi}\int_0^{\alpha} 10^{-8}\left(\frac{\omega^2+\alpha^2}{\alpha^2}\right) d\omega = \frac{32}{3} \times 10^{-5}$$

$$50 \text{ dB} = 10^5 = \frac{S_o}{N_o} = \frac{S_o}{\frac{32}{3} \times 10^{-5}} \to S_o = 32/3$$

But $s_\alpha(t) = \frac{10^{-3} m(t)}{\alpha}$. Hence,

$$S_o(t) = \frac{10^{-6}}{\alpha^2}\overline{m^2(t)} = \frac{32}{3} \to \overline{m^2(t)} = 6.738 \times 10^{15}$$

Also, $\overline{m^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \beta \, d\omega = \frac{\beta\alpha}{\pi} = 8000\beta = 6.738 \times 10^{15}$ Hence, $\beta = 8.422 \times 10^{11}$ and $S_m(\omega) = 8.422 \times 10^{11}\Pi(\frac{\omega}{2\alpha})$,

$$\begin{aligned}
S_i &= \frac{1}{\pi}\int_0^{\alpha} S_m(\omega)|H_c(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{\alpha} 8.422 \times 10^{11}\left(\frac{10^{-6}}{\omega^2+\alpha^2}\right) d\omega \\
&= \frac{8.422 \times 10^5}{4\alpha} = 83.775 \\
S_T &= \frac{1}{\pi}\int_0^{\alpha} S_m(\omega) d\omega = \frac{1}{\pi}\int_0^{\alpha} 8.422 \times 10^{11} \, d\omega = 8.422 \times 10^{11} \times 8000 = 6.738 \times 10^{15}
\end{aligned}$$

10.2-1

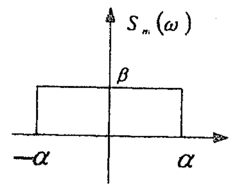
(a)

$$47 \text{ dB} = 50000 = \frac{S_o}{N_o} = \gamma = \frac{S_i}{NB} = \frac{S_i}{10^{-12} \times 5000} \to S_i = 2.5 \times 10^{-4}$$

(b) From Eq.(10.7), $N_o = NB = 10^{-12}(5000) = 5 \times 10^{-9}$

(c)

$$S_i = |H_c(\omega)|^2 S_T \text{ and } 10^{-6} S_T = 2.5 \times 10^{-4} \to S_T = 250$$



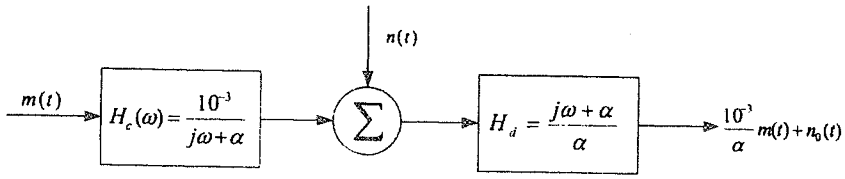


Fig. S10.1-2

10-2.2

(a)

$$\frac{S_o}{N_o} = 50000 = \frac{S_i}{NB} = \frac{S_i}{10^{-12} \times 5000} \rightarrow S_i = 2.5 \times 10^{-4}$$

(b)

$$N_o = NB = 10^{-12} \times 5000 = 5 \times 10^{-9}$$

(c)

$$S_i = |H_c(\omega)|^2 S_T = 10^{-6} S_T = 2.5 \times 10^{-4} \rightarrow S_T = 250$$

10.2-3

(a)

$$\mu = \frac{-[m(t)]\min}{A} = \frac{m_p}{A}$$

Hence, $m_p = \mu A$.

$$\frac{S_o}{N_o} = \frac{\overline{m^2}}{A^2 + \overline{m^2}}\gamma = \frac{\overline{m^2}}{\frac{m_p^2}{\mu^2} + \overline{m^2}}\gamma = \frac{\mu^2}{\kappa^2 + \mu^2}\gamma$$

where $\kappa^2 = \frac{m_p^2}{\overline{m^2}}$

(b) For tone modulation

$$\kappa = \frac{m_p^2}{m_p^2/2} = 2$$

and for

$$\mu = 1, \quad \frac{S_o}{N_o} = \frac{1}{2+1}\gamma = \frac{\gamma}{3}$$

(c) Ratio

$$\frac{S_T}{S_T'} = \frac{A^2 + \overline{m^2}}{\overline{m^2}} = \frac{m_p^2 + \overline{m^2}}{\overline{m^2}} = \kappa.$$

if $\kappa^2 \gg 1$.

10.2-6 See Figure S10.2-6. Let the signals $m_1(t)$ and $m_2(t)$ be transmitted over the same band by carriers of the same frequency (ω_c), but in phase quadrature. The two transmitted signals are $\sqrt{2}[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]$. The bandpass noise over the channel is $n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$. Hence, the received signal is

$$[\sqrt{2}m_1(5) + n_c(t)]\cos\omega_c t + [\sqrt{2}m_2(t) + n_s(t)]\sin\omega_c t$$

Eliminating the high frequency terms, we get the output of the upper lowpass filter as

$$m_1(t) + \frac{1}{\sqrt{2}}n_c(t)$$

Similarly, the output of the lower demodulator is

$$m_2(t) + \frac{1}{\sqrt{2}}n_s(t)$$

Hence, we have $\frac{S_o}{N_o} = \gamma$ for both QAM channels.

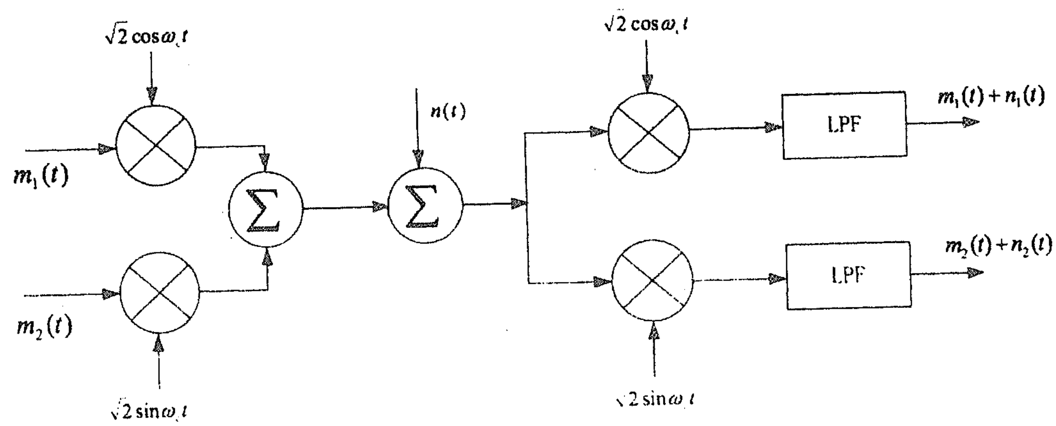


Fig. S10.2-6

10.3-1

$$\frac{S_o}{N_o} = 28 \text{ dB} = 631.$$

Hence,

$$\frac{S_o}{N_o} = 631 = 3\beta^2\gamma\frac{\overline{m^2(t)}}{m_p^2} = 45\gamma\frac{\sigma_m^2}{(3\sigma_m)^2} = 5\gamma$$

Therefore, $\gamma = 631/5 = 126.2$

(a) Also,

$$\gamma = \frac{S_i}{NB} \rightarrow S_i = \gamma NB = 126.2 \times 2 \times 10^{-10} \times 15000 = 3.786 \times 10^{-4}$$

(b)

$$\beta = \frac{\delta\omega}{2\pi B} = \frac{k_f m_p}{2\pi B} \rightarrow 5 = \frac{k_f(3\sigma_m)}{30,000\pi} \rightarrow k_f\sigma_m = 50,000\pi$$

$$S_o = \alpha^2 k_f^2\overline{m^2(t)} = \alpha^2 k_f^2\sigma_m^2 = (10^{-2})^2(50,000\pi)^2 = 250000\pi^2$$

(c) $N_o = \frac{S_o}{126.2} = 19552$

$m_p = B$, $m'_p = \frac{4B}{T_o}$, and bandwidth=$5/T_o$. Hence,

$$\frac{(S_0/N_0)_{\text{PM}}}{(S_0/N_0)_{\text{FM}}} = \frac{(2\pi \times 5/T_0)^2 B^2}{3(4B/T_0)^2} = \frac{(2\pi \times 5)^2}{3 \times 16} = 20.56$$

3 $m(t) = a_1 \cos \omega_1 t + a_2 \cos \omega_2 t$, $m_p = a_1 + a_2$
$= -(a_1\omega_1 \sin \omega_1 t + a_2\omega_2 \sin \omega_2 t)$, $m'_p = a_1\omega_1 + a_2\omega_2$

$$\frac{(S_o/N_o)_{\text{PM}}}{(S_o/N_o)_{\text{FM}}} = \frac{(2\pi B)^2 m_p^2}{3m_p'^2} = \frac{\omega_2^2 a_2^2 \left(1 + \frac{a_1}{a_2}\right)^2}{3\omega_2^2 a_2^2 \left(1 + \frac{a_1\omega_1}{a_2\omega_2}\right)^2} = \frac{(1+x)^2}{3(1+xy)^2}$$

fore the PM is superior to FM if $\frac{(1+x)^2}{3(1+xy)^2} > 1$, or, equivalently $(1+x)^2/3 > (1+xy)^2$.

-4 There is a typo in this problem. There should be $4\pi^2$ in the denominator of the answer (see below).

$$S_{\dot{m}}(\omega) = \omega^2 S_m(\omega)$$

ce,

$$\int_{-\infty}^{\infty} [\dot{m}(t)]^2 = \int_{-\infty}^{\infty} S_{\dot{m}}(2\pi f)df = \int_{-\infty}^{\infty} 4\pi^2 f^2 S_m(2\pi f)df$$

n Eq.(10.37)

$$\overline{B_m^2} = \frac{\int f^2 S_m(2\pi f)df}{\int S_m(2\pi f)df} = \frac{1}{4\pi^2} \frac{\int_{-\infty}^{\infty} [\dot{m}(t)]^2 \, dt}{\int_{-\infty}^{+\infty} m^2(t)dt}$$

se results are true for a waveform $m(t)$.

3-5

$$\begin{aligned}\overline{B_m^2} &= \frac{\int_{-\infty}^{\infty} \frac{f^2}{1+(f/f_0)^{2k}} df}{\int_{-\infty}^{\infty} \frac{1}{1+(f/f_o)^{2k}} df} = \frac{f_o^3 \int_{-\infty}^{\infty} \frac{x^2}{1+x^{2k}} dx}{f_0 \int_{-\infty}^{\infty} \frac{1}{1+x^{2k}}} \\ &= \frac{f_0^2 \left(\frac{\pi}{2k \sin(3\pi/2k)}\right)}{\frac{\pi}{2k \sin(\pi/2k)}} = f_o^2 \frac{\sin \frac{\pi}{2k}}{\sin\left(\frac{3\pi}{2k}\right)}\end{aligned}$$

definite integrals are found from integral tables. As $k \to \infty$,

$$\overline{B_m^2} = f_o^2 \frac{\sin \pi/2k}{\sin 3\pi/2k} \to f_o^2 \frac{(\pi/2k)}{3\pi/2k} = \frac{1}{3} f_o^2$$

3-6

$$S_m(\omega) = \frac{|\omega|}{\sigma^2} e^{-\omega^2/2\sigma^2}$$

$$\overline{m^2} = 2\int_0^{\infty} \frac{w}{2\sigma^2} e^{-\omega^2/2\sigma^2} = 2$$

ce, the normalized PSD's is

$$\frac{|\omega|}{2\sigma^2} e^{-\omega^2/2\sigma^2}$$

$= 2\pi B$, then $\overline{W^2} = \overline{(2\pi B)^2} = 2\int_0^{\infty} \frac{\omega^3}{2\sigma^2} e^{-\omega^2/2\sigma^2} d\omega = 2\sigma^2$.

If $p(W)$ is the power within the band $-W$ to W.

$$p(W) = 2\int_0^W \frac{\omega}{2\sigma^2} e^{-\omega^2/2\sigma^2} d\omega = 2\left[1 - e^{-W^2/2\sigma^2}\right]$$

$p(\infty) = 2$, and

$$\frac{p(W)}{p(\infty)} = 1 - e^{-\frac{W^2}{2\sigma^2}} = 0.99 \rightarrow W = 3.03\sigma, B = 0.482\sigma$$

$$\frac{p(W)}{p(\infty)} = 0.9 \rightarrow W = 2.15\sigma, \quad B = 0.342\sigma$$

$$\frac{p(W)}{p(\infty)} = 0.70 \rightarrow W = 1.55\sigma, \quad B = 0.247\sigma$$

$x = 0.99$, $\frac{W^2}{3} = 3.06\sigma^2 > \overline{W^2} \rightarrow$ PM superior.

$x = 0.9$, $\frac{W^2}{3} = 1.54\sigma^2 \leq \overline{W^2} \rightarrow$ FM superior.

$x = 0.7$, $\frac{W^2}{3} = 0.8\sigma^2 \leq \overline{W^2} \rightarrow$ FM superior.

10.3-7 From Eq.(10.35)

$$\beta^2 = \frac{1}{3}\left[\frac{1}{1 + \left(\frac{\overline{m^2}}{m_p^2}\right)}\right]$$

(a) Tone modulation $\beta^2 = \frac{1}{3}\left(\frac{1}{1+0.5}\right) \rightarrow \beta = 0.47$

For tone modulation,

$$\frac{\overline{m^2}}{m_p^2} = 0.5$$

(b) Gaussian with 3σ-loading

$$\beta^2 = \frac{1}{3}\left(\frac{1}{1+\frac{1}{9}}\right) \rightarrow \beta = 0.547$$

For Gaussian modulation with 3σ-loading,

$$\frac{\overline{m^2}}{m_p^2} = \frac{\sigma^2}{(3\sigma)^2} = \frac{1}{9}$$

(c) Gaussian with 4σ-loading

$$\beta^2 = \frac{1}{3}\left(\frac{1}{1+\frac{1}{16}}\right) \rightarrow \beta = 0.56$$

For Gaussian modulation with 4σ-loading,

$$\frac{\overline{m^2}}{m_p^2} = \frac{\sigma^2}{(4\sigma)^2} = \frac{1}{16}$$

that is, if

$$\frac{(a_1/a_2)^2(f_1/f_2)^2+1}{(a_1/a_2)^2+1} < \frac{1}{3}$$

or if

$$1 + x^2y^2 < \frac{1+x^2}{3}$$

10.4-1

$$\frac{S_o}{N_o} = 50 \text{ dB} = 100,000$$

For uniform distribution

$$\overline{m^2} = \frac{1}{2m_p}\int_{-m_p}^{m_p} m^2 dm = \frac{1}{3}m_p$$

(a)

$$\begin{aligned} 100000 &= 3(2)^{2n}\left(\frac{\overline{m^2}}{m_p^2}\right) \\ &= 3(2)^(2n)\left(\frac{1}{3}\right) \end{aligned}$$

$2n = 16.61$. Since n must be an integer, choose $n = 9$ and $L = 512$.

(b)

$$\frac{S_o}{N_o} = 3(2)^{18}\frac{1}{3} = 262144 = 54.18 \text{ dB}$$

$$B_{\text{PCM}} = 2nB = 81 \text{ MHz}$$

(Assuming bipolar signaling)

(c) To increase the SNR by 6 dB, we can increase n by 1, that is $n = 10$. Then the new bandwidth of transmissio is $20 \times 4.5 = 90$ MHz

10.4-2

(a)

$$L = M^n \rightarrow n = \log_M L$$

$$\frac{S_o}{N_o} = 3L^2\frac{\overline{m^2(t)}}{m_p^2} = 3M^{2n}\left(\frac{\overline{m^2}}{m_p^2}\right)$$

(b) Using 8-PAM signaling, we can save the bandwidth by a factor of 3. Hence, the new bandwidth requiremen is 27 MHz and 30 MHz, respectively.

10.4-3 First, we have $S_i = 2BnE_p = 1.28$, and $\gamma = S_i/(NB) = 2.56 \times 10^2$.

It is easy to arrive that

$$Q(\sqrt{\frac{\gamma}{n}}) = 7.569 \times 10^{-9}.$$

Hence, $B_m = nB = 8 \times 8,000 = 64$kHz

APPENDIX E

MISCELLANEOUS

E.1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)} \tag{E.1}$$

E.2 Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

E.3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \frac{\pi^2}{4}$$

$$Q(x) = \frac{e^{-x^2/2}}{x\sqrt{2\pi}}\left(1 - \frac{1}{x^2} + \frac{1\cdot 3}{x^4} - \frac{1\cdot 3\cdot 5}{x^6} + \cdots\right)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

E.4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

E.5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2+b^2}, \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

E.6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$$

$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

E.7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax \cos bx \, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} \, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax} \, dx = \frac{e^{ax}}{a^3}(a^2 x^2 - 2ax + 2)$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx)$$

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2} \, dx = \frac{1}{2} \ln (x^2 + a^2)$$