Chapter 4 Solutions

Problem 4.2-1

(i) The baseband signal which is also known as the signal delivered by the source is designated by $m(t)$.

$m(t) = \cos 1000\pi t$

The centre frequency of this signal can be found as shown below.

$$m(t) = \cos \omega_c t$$

$$= \cos 2\pi f_c t$$

$$\therefore \quad 2\pi f_c = 1000\pi$$

$$\therefore \quad f_c = 500 \text{ Hz}.$$

To plot the spectrum of the baseband signal we need the Fourier transform of $m(t)$.

$$F(m(t)) = \int_{-\infty}^{\infty} m(t)\, e^{-j2\pi f t}\, dt \qquad -(1)$$

We know from trigonometric identities that

$$\cos\theta = \frac{e^{-j\theta} + e^{j\theta}}{2} \qquad -(2)$$

Using (2) in (1) we get

$$M(f) = \frac{1}{2}\int_{-\infty}^{\infty} [e^{-j2\pi(500)t} + e^{j2\pi(500)t}] e^{-j2\pi f t} dt$$

Rearranging the above terms we get

$$M(f) = \frac{1}{2}\int_{-\infty}^{\infty} [e^{-j2\pi(f+500)t} + e^{-j2\pi(f-500)t}]$$

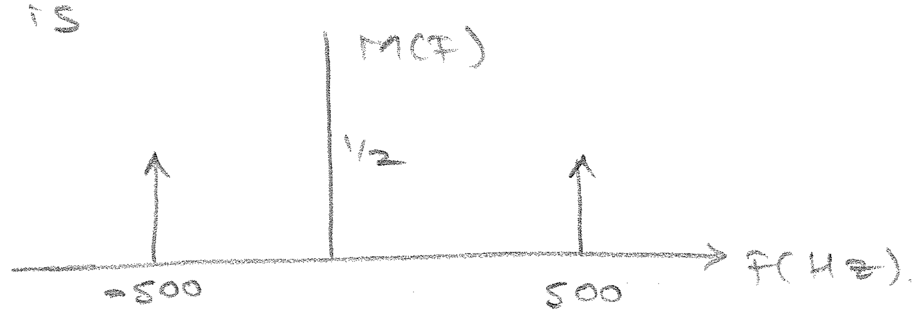
From chapter 3 we know Fourier transformation

$$e^{j2\pi f_0 t} \Longleftrightarrow \delta(f-f_0)$$

Using the same analogy we can write

$$M(f) = \frac{1}{2}[\delta(f+500) + \delta(f-500)]$$

This means that the spectrum of m(t) is an impulse that is centred at $f_c = 500$ Hz. The plot for the same is



(b) The DSB-SC (Double Side band-Suppress carrier) signal is the modulation scheme used for amplitude modulation

The spectrum of the baseband signal is shifted to the carrier frequency.

Let $y(t) = m(t) \cos 10{,}000\pi t$.

$= \cos 1000\pi t \cdot \cos 10{,}000\pi t$

→ we know the trigonometric identity,

$2 \cos a . \cos b = \cos(a+b) + \cos(a-b)$.

Using the same identity we get,

$y(t) = \frac{1}{2}[\cos 9000\pi t + \cos 11000 t]$

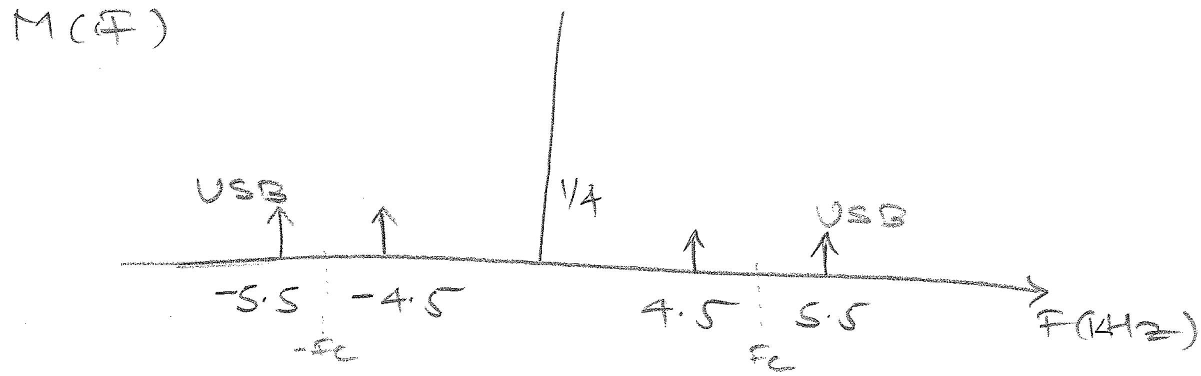
As shown earlier to plot the spectrum we need to find the Fourier transform of $y(t)$

$$F(y(t)) = \frac{1}{2}\int_{-\infty}^{\infty} [\cos 9000\pi t + \cos 11000\pi t] . e^{-j2\pi F t}$$

$$= \frac{1}{4}[\delta(F+4.5k) + \delta(F+5.5k) + \delta(F-4.5k) + \delta(F-5.5k)]$$

The last term can be thought of as shift of spectrum of m(t) from 500Hz to $(F_c - 500)$ & $(F_c + 500)$ where F_c is carrier frequency.

$[2\pi F_c = 10000\pi \Rightarrow F_c = 5000]$.



USB - Upper side band is portion of the spectrum that lies above F_c.

From the plot shown above, terms $\delta(F - 5.5K)$ and $\delta(F + 5.5K)$ are USB.

LSB - Lower side band is portion of the spectrum that lies below F_c. From the plot shown above $\delta(F - 4.5K)$ and $\delta(F + 4.5K)$ are LSB.

* As there are 2 side bands this type of modulation is called Double side band.

(ii) $m(t) = 2\cos 1000\pi t + \sin 2000\pi t$

Here we have 2 sinusiods in our base band signal. This means we have a frequency spectrum with 2 distinct frequency components.

$$F_1 = \frac{1000\pi}{2\pi} = 500\ Hz$$

$$F_2 = \frac{2000\pi}{2\pi} = 1000\ Hz.$$

We now use the following trigonometric identities.

$$\sin\theta = \cos(\theta - \pi/2) = \frac{e^{j(\theta-\pi/2)} + e^{-j(\theta-\pi/2)}}{2}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Hence we can write the baseband signal as,

$$m(t) = e^{j2\pi 500t} + e^{-j2\pi 500t} + \frac{1}{2}e^{j(2\pi 1000t - \frac{\pi}{2})} + \frac{1}{2}e^{-j(2\pi 1000t - \frac{\pi}{2})}$$

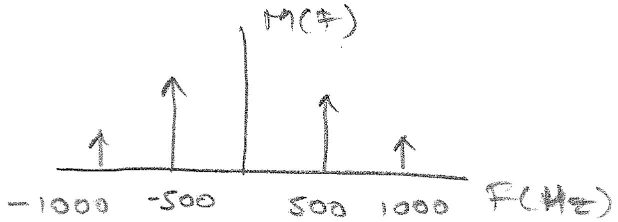
The spectrum of this signal can be obtained by taking its Fourier transform.

$$F(m(t)) = \int_{-\infty}^{\infty} e^{j2\pi 500t} + e^{-j2\pi 500t} - \frac{1}{2}\left[e^{j(2\pi 1000t - \frac{\pi}{2})} - e^{-j(2\pi 1000t - \frac{\pi}{2})}\right] e^{-j2\pi ft}\, dt$$

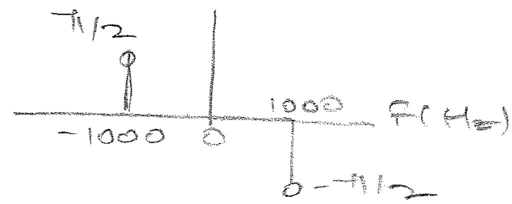
Solving the last term as calculated for(1) we get spectrum of $m(t)$ as,

$$M(f) = \delta(f-500) + \delta(f+500) + 0.5\left[\delta(f+1000)e^{-j\pi/2} \delta(f-1000)e^{j\pi/2}\right]$$

The plot of the spectrum is shown below



Magnitude



Phase

- The spectrum has phase of $\pi/2$ this is explained below, we know that

$$g(t)\cos\omega_0 t \Leftrightarrow \frac{1}{2}\left[G(\omega-\omega_0) + G(\omega+\omega_0)\right]$$

Now,

$$g(t)\cos(\omega_0 t+\theta_0) \Leftrightarrow \frac{1}{2}\left[G(\omega-\omega_0)e^{j\theta_0} + G(\omega+\omega_0)e^{-j\theta_0}\right]$$

this can be easily obtained by finding Fourier transform of $g(t)\cos(\omega_0 t+\theta_0)$ where θ_0 is the phase of the spectral component.

Now when $\theta_0 = -\pi/2$ we get

$$g(t)\cos(\omega_0 t - \pi/2) = g(t)\sin\omega_0 t.$$

Thus

$$g(t)\sin\omega_0 t \Leftrightarrow \frac{1}{2}\left[G(\omega-\omega_0)e^{-j\pi/2} + G(\omega+\omega_0)e^{j\pi/2}\right]$$

(b) $m(t) \cos 10{,}000\pi t$

Now we want to plot the spectrum of modulated signal with carrier frequency.

$F_c = 5000$ Hz.

$\therefore$ The Fourier transform of the signal is

$$F(m(t)\cos 10{,}000\pi t) = \int_{-\infty}^{\infty} (2\cos 1000\pi t \cdot \cos 10{,}000\pi t + \sin 2000\pi t \cdot (\cos 10{,}000\pi t)) e^{-j2\pi F t} dt$$

Now, using the trigonometric identities

$$2\cos a \cdot \cos b = \cos(a+b) + \cos(a-b)$$

$$2\cos a \cdot \sin b = \sin(a+b) + \sin(a-b).$$
$$= \cos(a+b-\pi/2) + \cos(a-b+\pi/2)$$

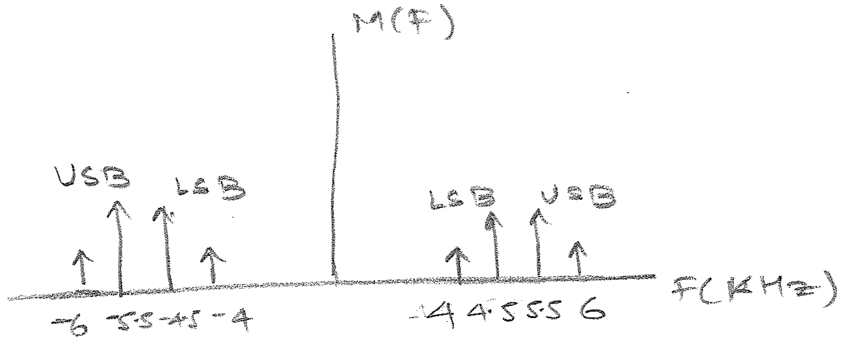
thus we have the modulated signal as

$$y(t) = \cos(11{,}000\pi t) + \cos(9000\pi t) +$$
$$\frac{1}{2}\left[\cos(12{,}000\pi t - \frac{\pi}{2}) + \cos(8000\pi t + \frac{\pi}{2})\right]$$

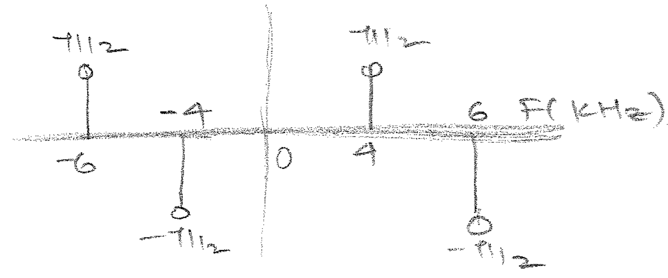
Fourier transform of the above signal can be written as

$$M(F) = 0.5\left[\delta(F-5500) + \delta(F+5500) + \delta(F-4500) + \delta(F+4500)\right] + 0.25\left[\delta(F-6000)e^{-j\pi/2} + \delta(F+6000)e^{+j\frac{\pi}{2}} + \delta(F-4000)e^{+j\pi/2} + \delta(F+4000)e^{-j\pi/2}\right]$$

Spectrum



Magnitude



Phase.

USB – As described in previous section $(F+F_c)$ terms are the upper side band

LSB – $(F-F_c)$ terms are the lower side band terms.

(iii) $m(t) = \cos 1000\pi t \cos 3000\pi t$

As used in problem (ii), we can write $m(t)$ as,

$$m(t) = \frac{1}{2}(\cos 4000\pi t + \cos 2000\pi t)$$

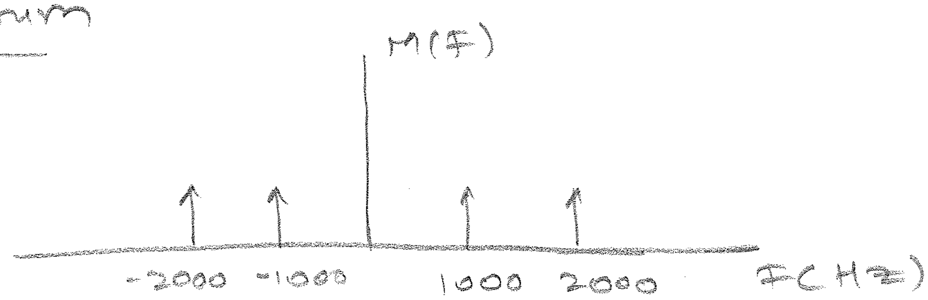
To achieve the Fourier spectrum we perform Fourier transform.

$$M(F) = \frac{1}{2}\int_{-\infty}^{\infty}(\cos 4000\pi t + \cos 2000\pi t)e^{-j2\pi F t}$$

Here we have 2 spectral components at $F_1 = 2000$ Hz & $F_2 = 1000$ Hz.

$$\therefore M(F) = \frac{1}{4}\left[\delta(F-2000) + \delta(F+2000) + \delta(F-1000) + \delta(F+1000)\right]$$

Spectrum



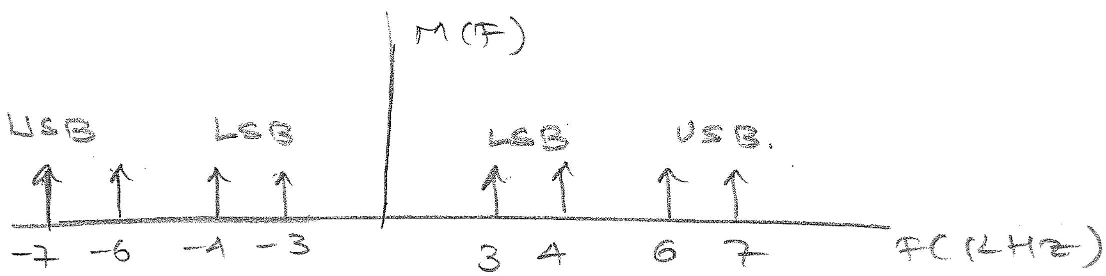
(b) $y(t) = m(t) \cos 10{,}000\pi t$.

The spectrum of $m(t)$ is now centred at carrier frequency,

$$F_c = 5000 \text{ Hz}$$

Hence the Fourier spectrum is given by.

$$M(F) = \frac{1}{8}\left[\delta(F+3000) + \delta(F+4000) + \delta(F+6000) + \delta(F+7000) + \delta(F-3000) + \delta(F-4000) + \delta(F-6000) + \delta(F-7000)\right]$$



c) USB & LSB can be found as in previous problems.

(iv) $m(t) = e^{-10|t|}$

The baseband signal is an exponent as can be seen. We can write $m(t)$ as

$$m(t) = e^{+10t} \quad \text{for } -\infty < t < 0$$
$$= e^{-10t} \quad \text{for } 0 \leq t < \infty$$

Now, taking Fourier transform of this signal we get.

$$M(f) = \int_{-\infty}^{\infty} e^{-10|t|} e^{-j2\pi f t} \, dt$$

$$= \int_{-\infty}^{0} e^{10t} \cdot e^{-j2\pi f t} \, dt + \int_{0}^{\infty} e^{-10t} \cdot e^{-j2\pi f t} \, dt$$

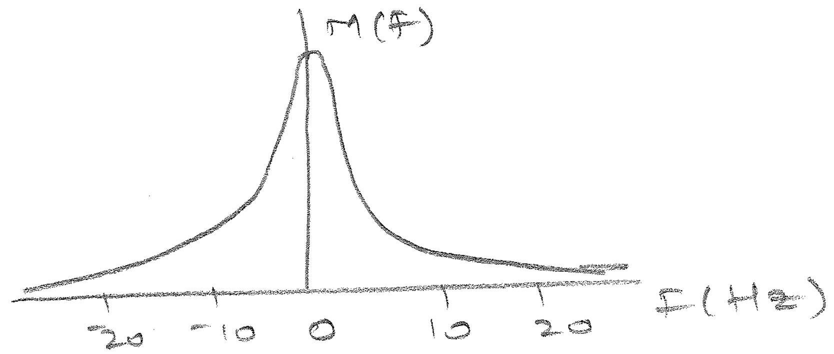
$$= \int_{-\infty}^{0} e^{(10-j2\pi f)t} \, dt + \int_{0}^{\infty} e^{-(10+j2\pi f)t} \, dt$$

$$= \frac{1}{-j2\pi f+10} + \frac{1}{j2\pi f+10}$$

$$= \frac{20}{4\pi^2 f^2 + 100}$$

The spectrum is quadratic function of f where the maximum value is obtained when $f=0$.

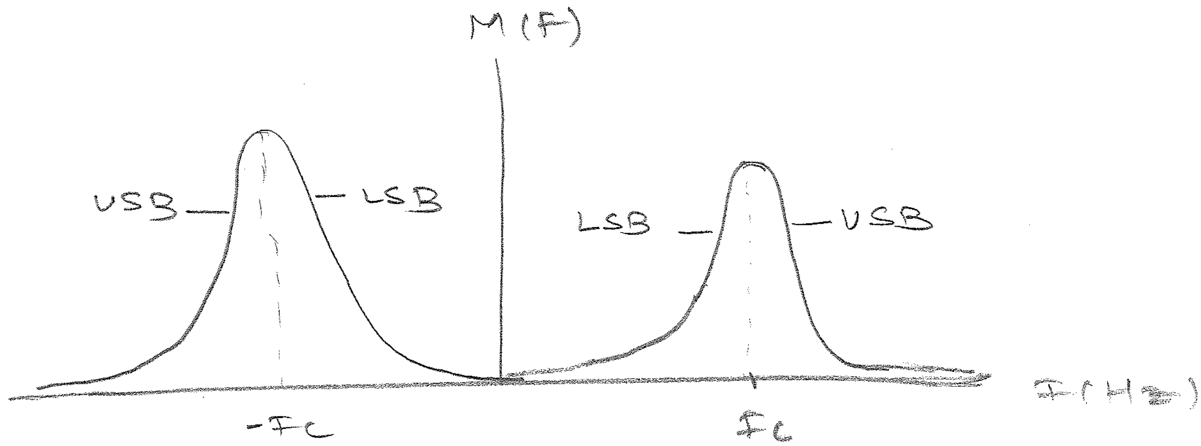
Spectrum



(b) $m(t) \cos 10{,}000\pi t$

The Fourier transform of the signal shown gives the spectrum. We can easily understand from previous examples that the spectrum of $m(t)$ is shifted and centred at $F_c = 5000$ Hz

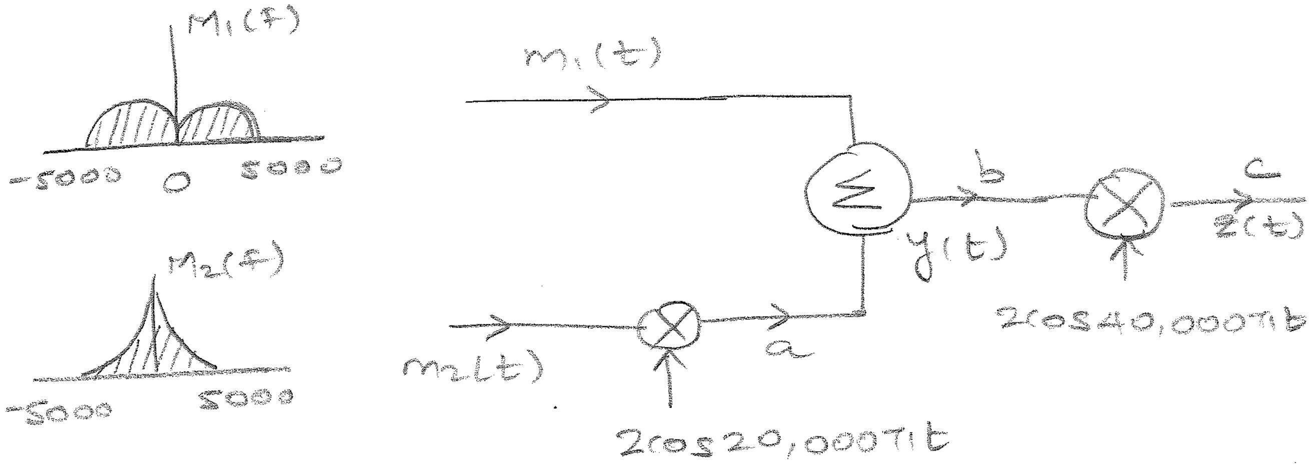
$$\therefore \quad M(F) = \frac{10}{100 + 4\pi^2(F - F_c)^2}$$



(c) USB is the spectrum for $F > F_c$

LSB is the spectrum for $F < F_c$

Problem 4.2-7

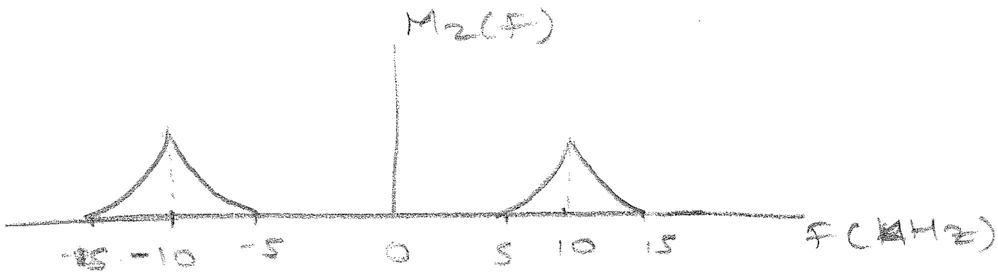


The system shown above is a modulator with 2 different signals multiplexed.

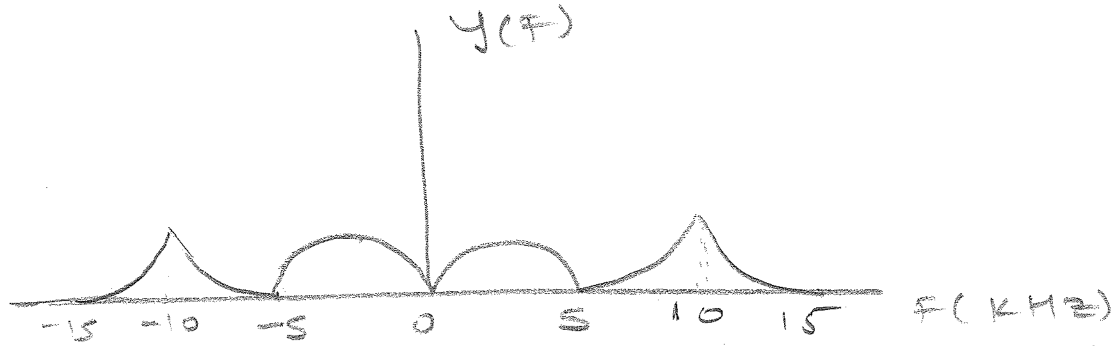
At a, the spectrum of the signal $m_2(t)$ is shifted & centered at

$f_{c_1} = 10000$ Hz.

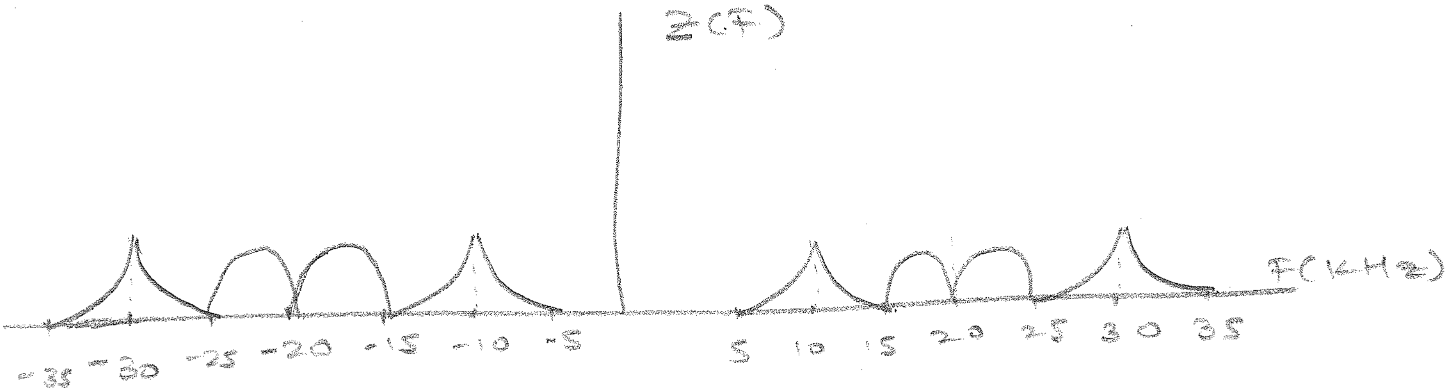
Thus at a,



At b, we have a multiplexer which sort of adds the spectrum of $m_1(t)$ and resultant signal at a. As the spectrum of both input signals are non-overlapping we can plot the spectrum as shown next.

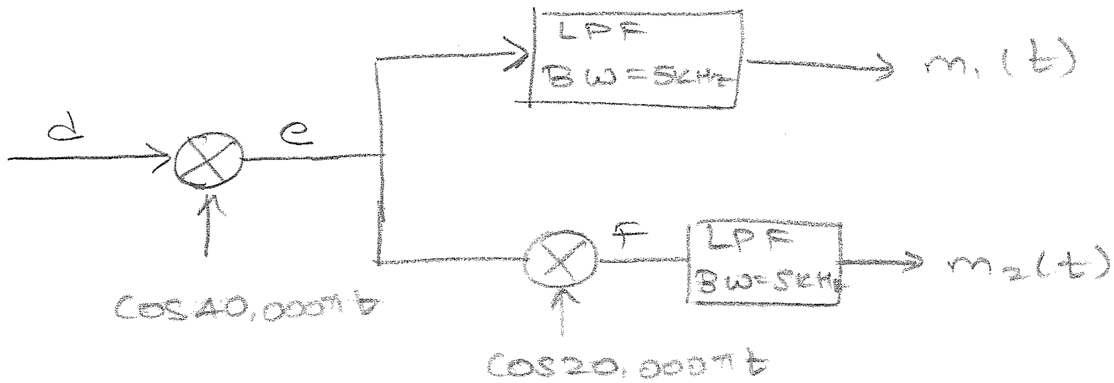


At c, y(t) is modulated by 2cos 40,000πt
This results in shift in spectrum of
y(t) shown above to F_{c2} = 20000 Hz.
The shifted spectrum is shown next.



(b) The bandwidth of a channel is
the difference between the highest
significant frequency and the lowest
significant frequency in the signal
spectrum.
From the spectrum shown above
F_L = 5KHz. F_h = 35KHz
∴ Bandwidth = $F_h - F_L$ = 30KHz

(c) The demodulator is used to demodulate or generate the baseband signal from the received (modulated) signal.



To understand the functionality of a demodulator we need to look at the trigonometric identity

$$2\cos\theta \cdot \cos\theta = 1 + \cos 2\theta$$

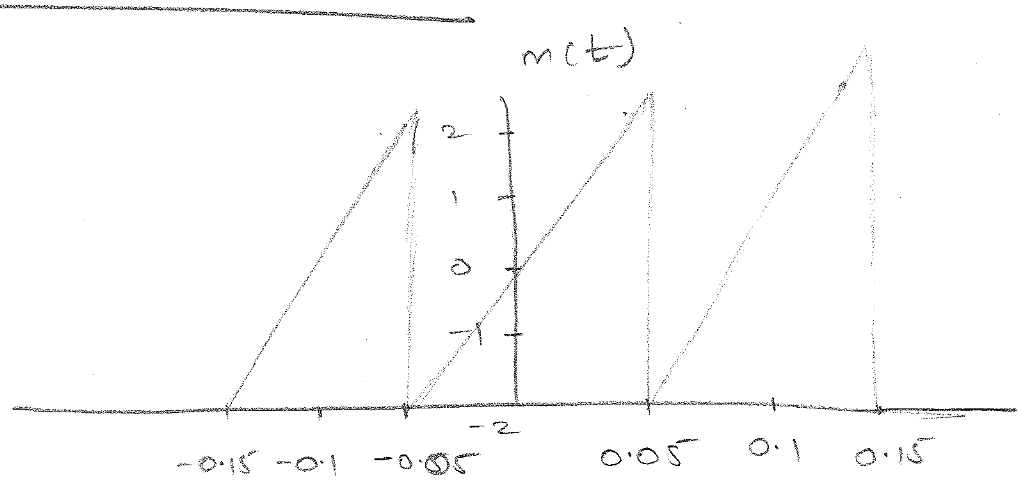
Thus, when $\cos 40{,}000\pi t$ is multiplied we get 2 components i.e. modulated $m_2(t)$ and baseband $m_1(t)$ and same signals with spectrum at 40kHz.

Now, on top branch using LPF with 5kHz bandwidth the spectral components above 5kHz are suppressed thus resulting in $m_1(t)$.

At F we get a signal $m_2(t)$ demodulated from f = 10000 Hz to its baseband spectrum with additional higher spectral components. These components are suppressed by Low Pass Filter which has a bandwidth of 5KHz.

Thus, $m_2(t)$ is recovered.

Problem 4.3-2



The amplitude modulated signal is depicted as

$\phi_{AM} = [A + m(t)] \cos \omega_c t$

where $A \cos \omega_c t$ is a carrier transmitted by the transmitter.

Modulation index μ is the fraction obtained by mp/A

$\mu = mp/A$

mp is a constant value of signal m(t).

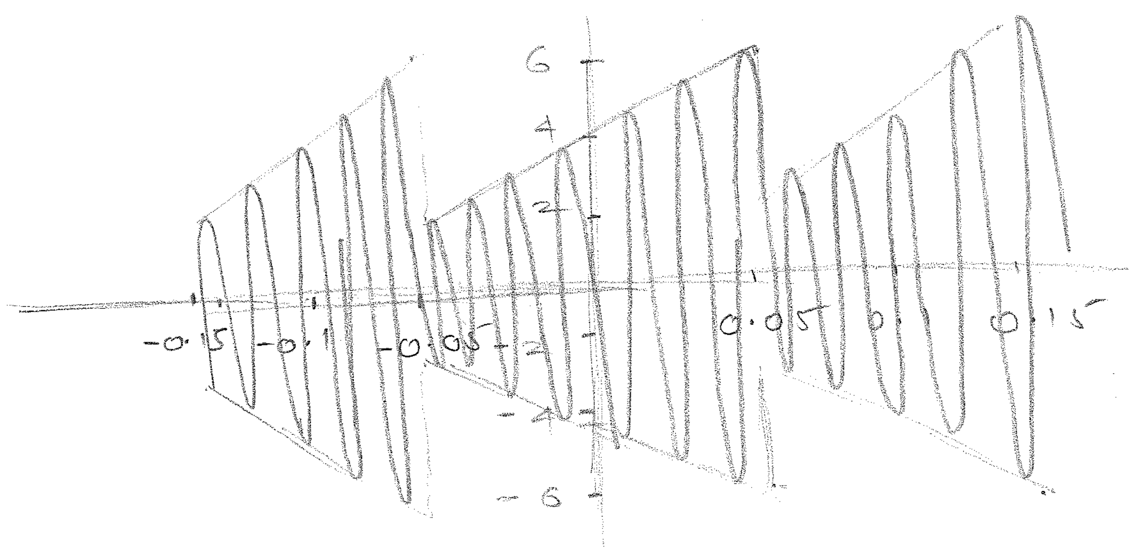
For this problem mp = 2. i.e.

mp = mp(t) = $|m_n(t)|$ = 2. (peak value)

a) modulation index = μ = 0.5

$\therefore A = \frac{mp}{\mu} = \frac{2}{0.5} = 4$

The value of A = 4 means that the amplitude modulated signal has the DC level shifted by 4 units.

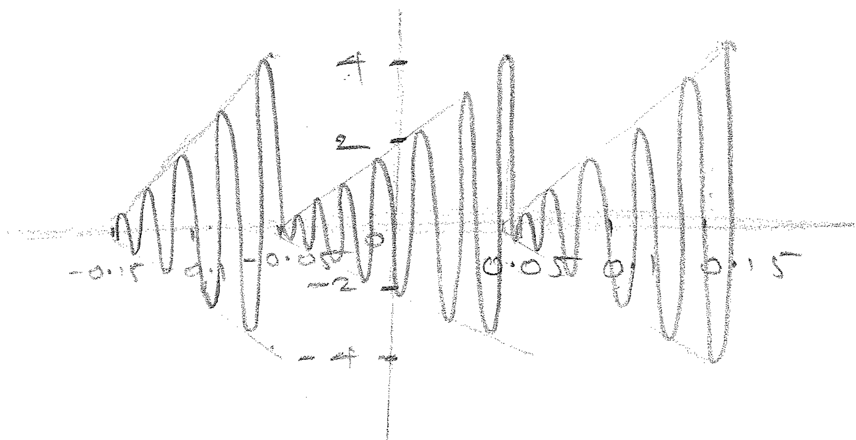


We can see that the envelope of the signal has shifted from 2 to 6 units.

(b) Modulation index $\mu = 1$

$\therefore A = \frac{mp}{\mu} = 2$

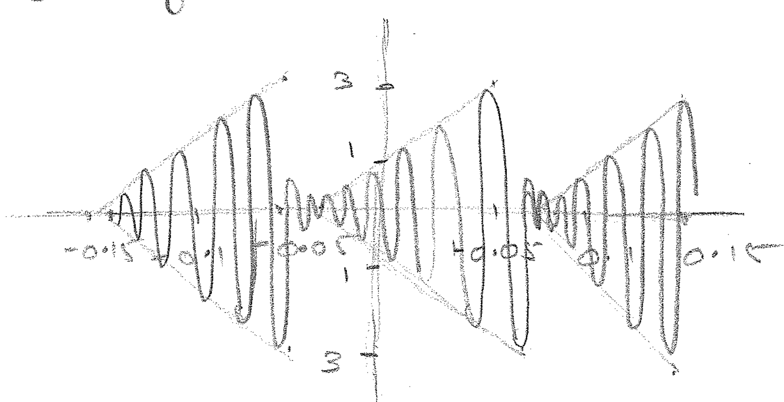
Thus for this case DC level is shifted by 2 units



(c) Modulation index $\mu = 2$

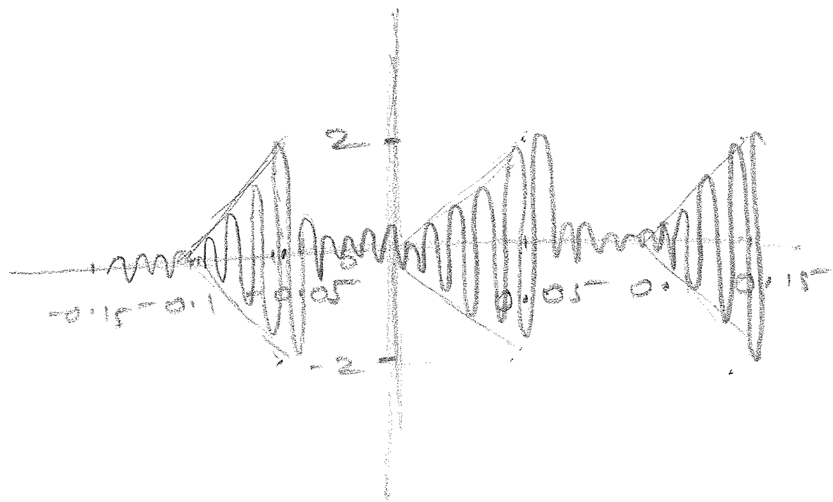
$\therefore A = \frac{mp}{\mu} = 1$

Thus for this case DC level is shifted by 1 unit.



Thus for $A = 1$ we see that some part of envelope is negative and thus information is lost and m(t) cannot be recovered from the envelope.

(d) $\mu = \infty$ $\therefore$ $A = \frac{m_p}{\mu} = 0$ Hence there is no carrier signal. Its is suppressed carrier signal.



We can see from this problem that as the value of modulation index increases we start loosing information as more & more portion of envelope starts becoming negative.

Problem 4.3-5

Message signal $m(t) = e^{-3t} u(t-2)$

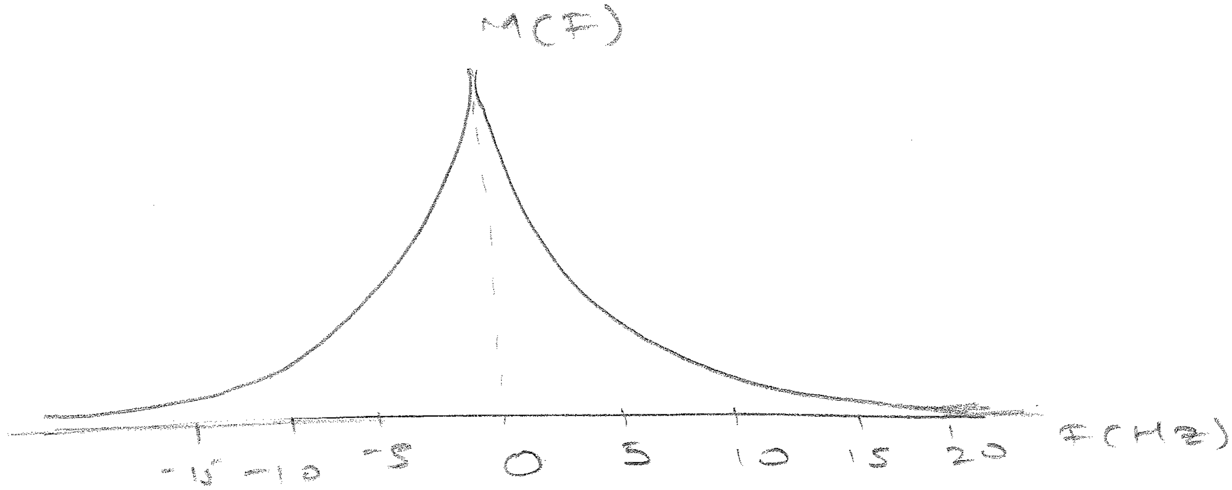
We can understand from the function $m(t)$ that it has a step function which starts at $t=2$. Thus the limits for Fourier transform start from $t=2$ as the function is zero for $t<2$

$$M(f) = \int_2^{\infty} e^{-3t} e^{-j2\pi f t} dt$$

$$= \frac{-1}{j(2\pi f+3)} \left[-e^{-(4\pi f+6)} \right]$$

$$= \frac{e^{-(4\pi f+6)}}{j(2\pi f+3)}$$

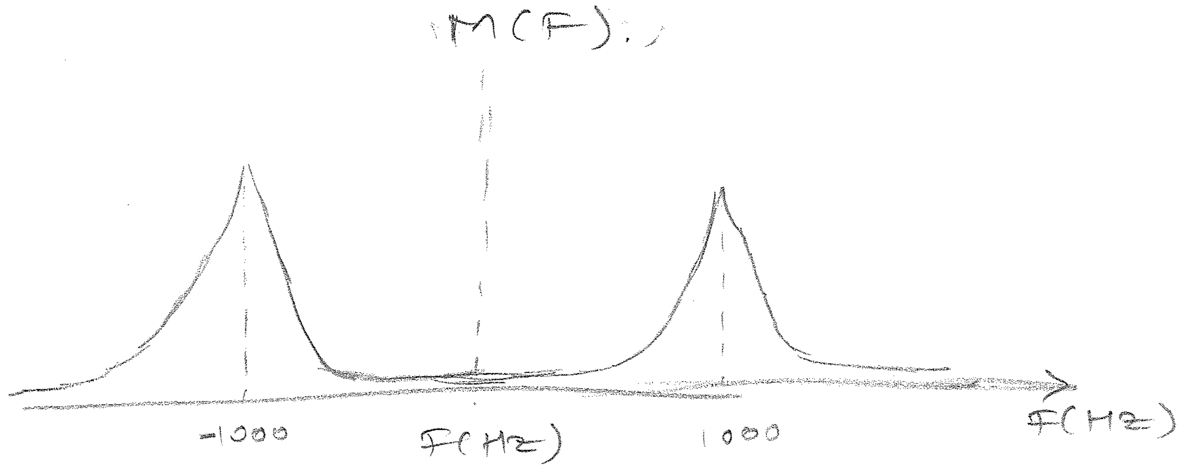
Below we plot the magnitude plot of the spectrum. The spectrum has maximum value at $f=0$ and then starts reducing exponentially.



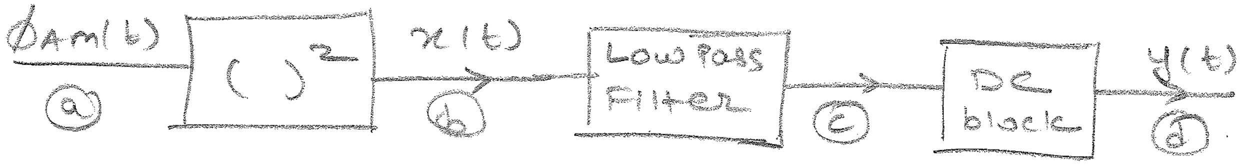
(b) The carrier signal used for modulation is cos 2000πt. Thus the carrier frequency can be calculated as

$F_c = 1000$ Hz.

The spectrum of the modulated signal is shifted and centered at $F_c = 1000$ Hz.



Problem 4.3-8



As described in the problem the system shown above is a demodulator circuit. The input to the system is the Amplitude modulated signal.

Thus, at (a) we have

$$\phi_{AM} = [A + m(t)] \cos(\omega_c t)$$

Next block is a crystal which has a square function. This block generates baseband and $2f_c$ components as shown below.

At (b)
$$x(t) = [A + m(t)]^2 \cos^2(\omega_c t)$$
$$= \frac{1}{2}[A^2 + 2Am(t) + m^2(t)][1 + \cos 2\omega_c t]$$

Now, the signal passes through a low pass filter. Low pass filter suppress the high frequency ($2\omega_c$) components and allow the baseband signal to pass.

Hence at ©

$$z(t) = \frac{A^2 + 2m(t)A + m^2(t)}{2}$$

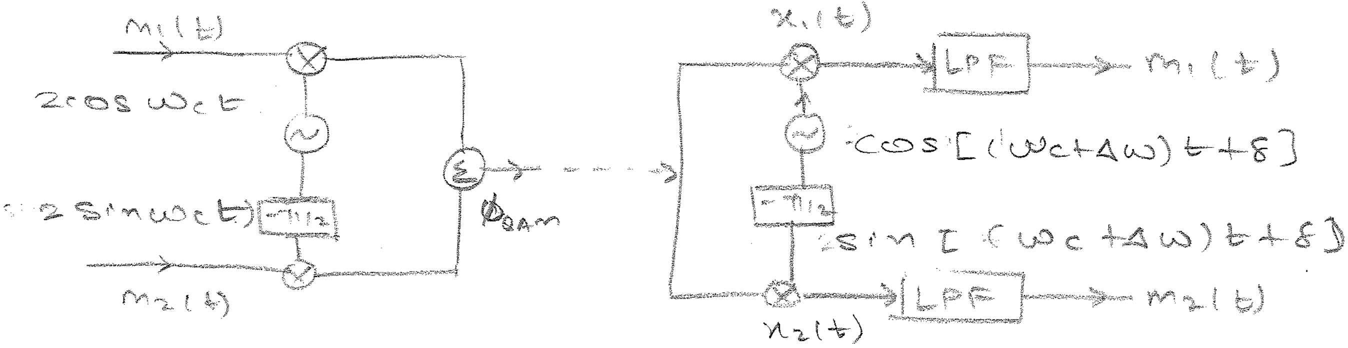
$$= \frac{A^2}{2}\left[1 + \frac{2m(t)}{A} + \left(\frac{m(t)}{A}\right)^2\right]$$

$\frac{m(t)}{A} << 1$. This condition gets violated only when $m(t)$ is near the peak. Thus we get

$$z(t) \approx \frac{A^2}{2} + Am(t)$$

Next we have a DC block which suppresses dc term $\frac{A^2}{2}$ thus the output is $m(t)$. The distortion component is $\frac{m^2(t)}{2}$.

Problem 4.4-1



As DSB signals occupy twice the bandwidth for baseband, QAM (Quadrature Amplitude modulation) technique is used where modulated signals occupy same band but their phase are shifted. They are detected by receiver using synchronous detection.

In this problem locally generated carrier at receiver has ~~Frequency~~ frequency error "$\Delta\omega$" and phase error "δ" as shown above.

$\Rightarrow$ $\phi_{QAM} = 2m_1(t)\cos\omega_c t + 2m_2(t)\sin\omega_c t.$

$\Rightarrow$ $x_1(t) = 2m_1(t)\cos\omega_c t\cos((\omega_c+\Delta\omega)t+\delta) + 2m_2(t)\cos\omega_c t.\sin((\omega_c+\Delta\omega)t+\delta)$

Here from $x_1(t)$ we can see that receiver has frequency error $\Delta\omega$ and phase error δ. Solving for $x_1(t)$ using trigonometric identities

$\Rightarrow$ $x_1(t) = m_1(t)[\cos(\Delta\omega t+\delta)+\cos((2\omega_c+\Delta\omega)t+\delta)] + m_2(t)[\sin(-\Delta\omega t-\delta)+\sin((2\omega_c+\Delta\omega)+\delta)]$

Now, we know that when we pass a signal spectrum through LPF, higher frequency components will be suppressed. The bandwidth of LPF is $B = \frac{\omega_c}{2\pi}$

Thus after signal passes through LPF we get

$m_1'(t) = m_1(t)[\cos(\Delta\omega t + \delta)] - m_2(t)[\sin(\Delta\omega t + \delta)]$

Thus, the received signal's spectrum will be deviated by a fraction ΔF and phase will have an error δ. Thus it is very important both at transmitter and receiver to be synchronized.

Similar to $m_1'(t)$ we can find $m_2'(t)$

$m_2'(t) = m_1(t)\sin[\Delta\omega t + \delta] + m_2(t)\cos[\Delta\omega t + \delta]$

Problem 4.42

a) $m(t) = \cos 100\pi t + 2\cos 300\pi t$

The base band signal $m(t)$ has 2 frequency components.

$2\pi F_1 = 100\pi$ $\qquad$ $2\pi F_2 = 300\pi$

$\therefore F_1 = 50\text{Hz}$ $\qquad$ $\therefore F_2 = 150\text{Hz}$

The Fourier transform of $m(t)$ gives its spectrum

$$F(m(t)) = \int_{-\infty}^{\infty} \cos 2\pi 50 t\, e^{-j2\pi F t} + 2\cos 2\pi 150 t\, e^{-j2\pi F t}\, dt$$

As proved earlier & from identities in Chapter 3

$M(F) = 0.5[\delta(F+50) + \delta(F-50)] + [\delta(F+150) + \delta(F-150)]$

Thus the spectrum has impulse at $F_1 = 50\text{Hz}$ & $F_2 = 150\text{Hz}$

(ii) Modulated signal is

$y(t) = 2m(t)\cos 1000\pi t$.

Thus, the carrier is at $F_c = 500$Hz. Hence the spectrum will now be centred at 500Hz

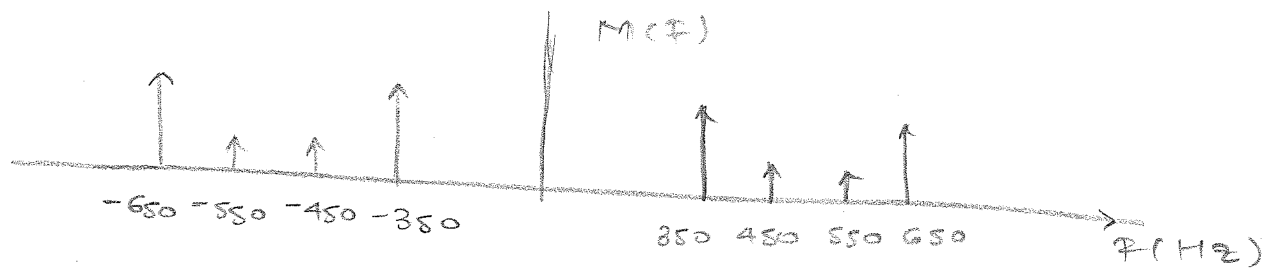
$y(t) = 2\cos 100\pi t \cos 1000\pi t + 4\cos 300\pi t \cos 1000\pi t$

Using trigonometric identities we get

$y(t) = \cos 2\pi(550)t + \cos 2\pi(450)t + 2\cos 2\pi(650)t + 2\cos 2\pi(350)t$.

Applying Fourier transform

$$M(F) = 0.5[\delta(F+550) + \delta(F+450) + \delta(F-450) + \delta(F-550)] + [\delta(F+650) + \delta(F+350) + \delta(F-350) + \delta(F-650)]$$



(iii) Suppressing the LSB term we get

$\phi_{USB} = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$.

Now Hilbert transform delays the phase of each spectral component by $\pi/2$.

$\therefore m_h(t) = \cos(\omega_m t - \pi/2) = \sin\omega_m t$.

$= \cos 100\pi t \cos\omega_c t - \sin 100\pi t \sin\omega_c t + 2[\cos 300\pi t \cdot \cos\omega_c t - \sin 300\pi t \sin\omega_c t]$

Using the trigonometric identity

$\cos(u+v) = \cos u \cos v - \sin u \sin v$.

$= \cos(\omega_c + 100\pi)t + 2\cos(\omega_c + 300\pi t)$

$= \cos(1100\pi t) + 2\cos(1300\pi t)$

Thus the Fourier transform gives.

$$\phi_{USB} = 0.5[\delta[f-550] + \delta[f+550]] + \delta[f-650] + \delta[f+650].$$

(iv) Now suppressing ϕ_{USB} we get

$$\phi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

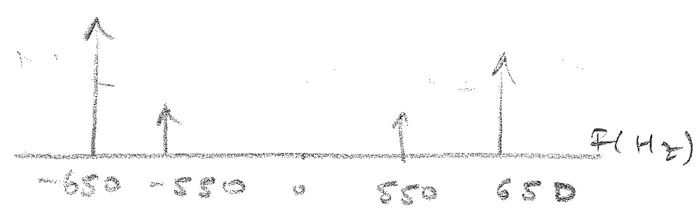
Similar to ϕ_{USB} solving ϕ_{LSB} we get

$$\phi_{LSB}(t) = \cos(\omega_c - 100\pi)t + 2\cos(\omega_c - 300\pi)t$$

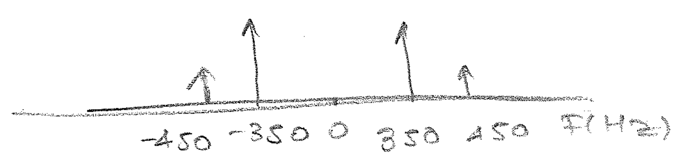
Thus the spectrum of ϕ_{LSB} is given by

$$\phi_{LSB} = 0.5[\delta(f-450) + \delta(f+450)] + [\delta(f-350) + \delta(f+350)]$$

ϕ_{USB}



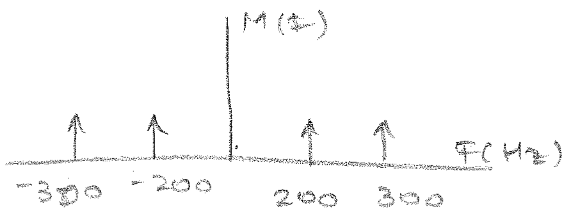
ϕ_{LSB}.



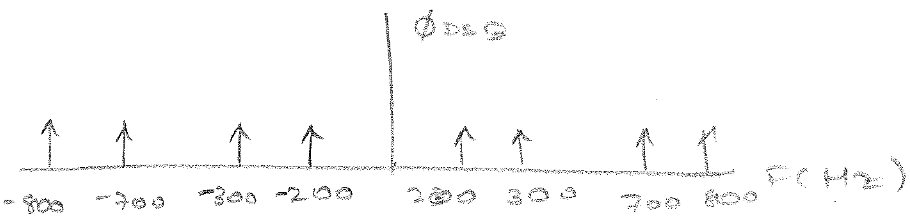
(b) $m(t) = \sin 100\pi t \sin 500\pi t$.

Part b can be solved in terms as a.

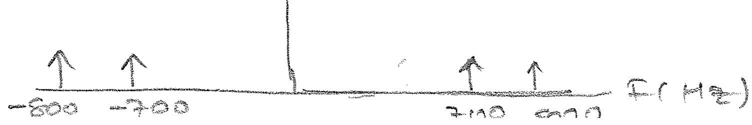
1). $m(t) = \sin 100\pi t \sin 500\pi t$

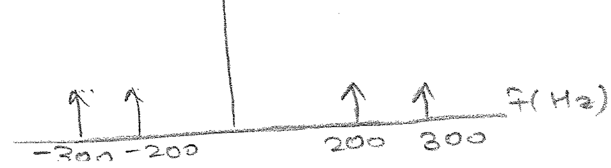


ii). Modulated signal



(iii - v)



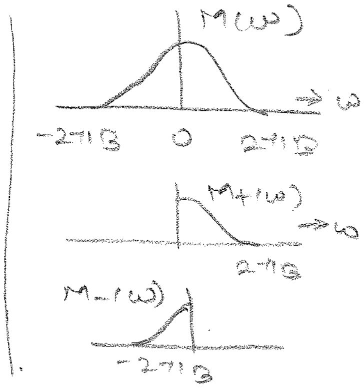


Problem 4.4-5

Single side band (SSB) signal can be expressed as

$$m_+(t) = \frac{1}{2}(m(t) + j m_h(t))$$

$$m_-(t) = \frac{1}{2}(m(t) - j m_h(t))$$



From the figure drawn we understand that $M_+(\omega)$ & $M_-(\omega)$ are conjugates. Thus $m_+(t)$ and $m_-(t)$ cannot be real. Hence, we have a complex term $m_h(t)$

Moreover $M_+(F) = M(F)u(F)$

i.e. $M_+(\omega)$ is spectrum of $M(\omega)$ multiplied by a unit step.

$$M_+(F) = \frac{1}{2}M(F)[1 + sgn(F)]$$

$$= \frac{1}{2}M(F) + \frac{1}{2}sgn(F).M(F)$$

$$\therefore M_h(F) = -jM(F)\,sgn(F)$$

Now say input of Hilbert transformer is $m_h(t)$

$$Y(F) = M_h(F)\,H(F) \qquad [H(F) = -j\,sgn(F)]$$

$$= -jM(F)sgn(F).-j\,sgn(F)$$

$$= -M(F)$$

Thus the Hilbert transform of $m_h(t) = -m(t)$

(ii) Signal energy is expressed as

$$E_g = \int_{\infty}^{\infty} g^2(t)\, dt.$$

$\therefore$ Energy of $m(t)$

$$E_m = \int_{-\infty}^{\infty} m^2(t)\, dt \quad \text{in time domain}$$

$$= \int_{-\infty}^{\infty} |M(f)|^2\, df \quad \text{in frequency domain.}$$

Now energy of $m_h(t)$

$$E_{m_h} = \int_{-\infty}^{\infty} m_h(t)\, dt \quad \text{in time domain}$$

$$= \int_{-\infty}^{\infty} |M_h(f)|^2\, df$$

$$= \int_{-\infty}^{\infty} |M(f)|^2 |\mathrm{sgn}(f)|^2\, df$$

$$= \int_{-\infty}^{\infty} |M(f)|^2\, df = \underline{E_m}$$

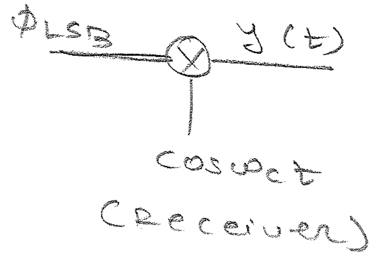
Thus the energy of $m(t)$ and $m_h(t)$ are identical.

Problem 4.4-6

The incoming SSB signal at the receiver is

$\phi_{LSB}(t) = 2m(t)\cos[(\omega_c + \Delta\omega)t + \delta] + 2m_h(t)\sin[(\omega_c + \Delta\omega)t + \delta]$

The local oscillator at the receiver runs at $\cos\omega_c t$.



Hence the output signal $y(t)$ is shown below.

$$y(t) = \phi_{LSB}(t)\cos\omega_c t$$

$$= 2[m(t)\cos[(\omega_c + \Delta\omega)t + \delta] + m_h(t)\sin[(\omega_c + \Delta\omega)t + \delta]] \cdot \cos\omega_c t$$

$$= [m(t)[\cos(\Delta\omega t + \delta) + \cos((2\omega_c + \Delta\omega)t + \delta) + m_h(t)[\sin(\Delta\omega t + \delta) + \sin((2\omega_c + \Delta\omega)t + \delta)$$

when the signal passes through LPF

$$y(t) = m(t)\cos(\Delta\omega t + \delta) + m_h(t)\sin(\Delta\omega t + \delta)$$

(a) when $\delta = 0$

$$y(t) = m(t)\cos(\Delta\omega t) + m_h(t)\sin(\Delta\omega t)$$

Thus the demodulated signal when there is no phase error has spectrum shifted by $\Delta\omega$.

This error can be easily detected by human ear as human ear can respond to minute pressure variations in the air if they are in audible ~~frequency~~ range i.e. 20Hz - 20kHz. The sound waves are pressure waves with frequency between 20 - 20kHz. Any subtle variation in this can be detected by human ear.

(b) when $\Delta\omega = 0$

$y(t) = m(t)\cos\delta + m_h(t)\sin\delta$

The Fourier transform of this signal has an erroneous phase component δ. Here the amplitude spectrum remains constant as that as $m(t)$. But the phase component varies. as we have $M(\omega)e^{j\delta}$ and $M(\omega)e^{-j\delta}$ terms. Human ear is insensitive to this as hearing is based on amplitude of frequencies. Phase distortion is delay in frequencies. In applications where signal is to be interpreted by digital hardware, and image processing, where signal is used to reconstruct image phase distortion is intolerable.