Chapter 5 Solution

Q5.1.1

Given $\omega_c = 10^8$ $K_f = 10^5$ $K_p = 25$.

ω_c = It is the carrier frequency of the signal.

K_p = In phase modulation the phase of the signal varies linearly with amplitude of message signal. The linearity constant is 'K_p'.

K_f = In Frequency modulation the instantenous Freq is proportional to amplitude of message signal and the proportionality constant is K_f.

i) Sketch $\phi_{FM}(t)$.

As mentioned above the instantenous Frequency is proportional to amplitude of message signal.

$$\omega_i = \omega_c + K_f m(t).$$

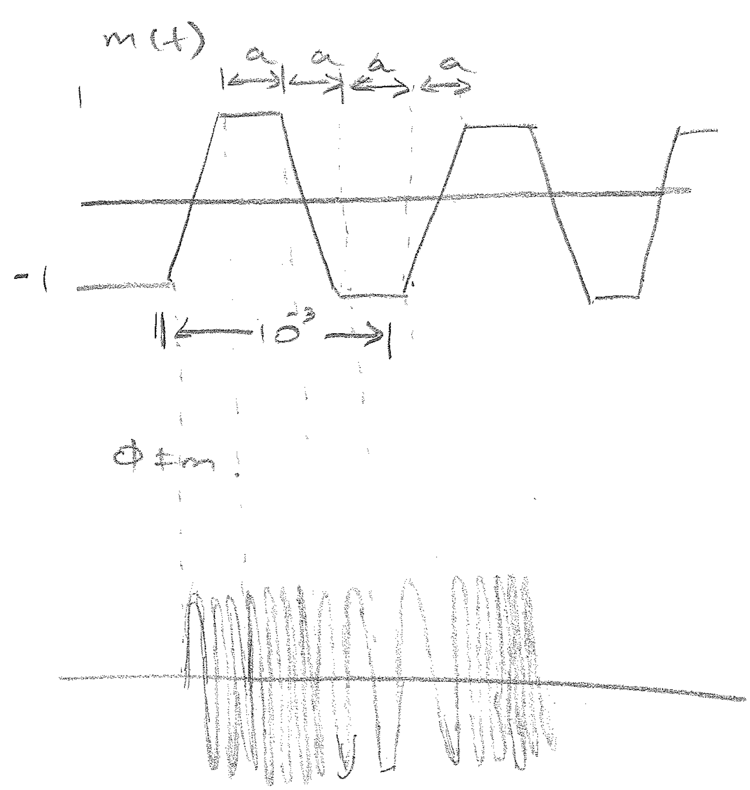
Here $K_f m(t)$ can also be defined as the shift in Freq $\Delta\omega$.

$$\therefore \; F_i = F_c + \frac{K_f}{2\pi} m(t) \qquad [\because \omega = 2\pi F)$$

As the Frequency of the modulated signal varies according to the amplitude of message signal we try to find the minimum & maximum amplitude and thus the low & high Freq.

$m_{p\,max} = 1$

$m_{p\,min} = -1$



Maximum Frequency corresponds to maximum amplitude & vice versa.

$\therefore f_{i\,max} = f_c + \frac{k_f}{2\pi} m_{p\,max}$

$= \frac{10^8}{2\pi} + \frac{10^5}{2\pi}(1)$

$= \underline{15.93 \text{ MHz}}$

$\therefore f_{i\,min} = f_c + \frac{k_f}{2\pi} m_{p\,min}$

$= \frac{10^8}{2\pi} + \frac{10^5}{2\pi}(-1)$

$= \underline{15.90 \text{ MHz}}$

We can see from the graph that the frequency increases when the amplitude of m(t) increases and decreases when amplitude decreases.

(ii) <u>Phase Modulation</u>

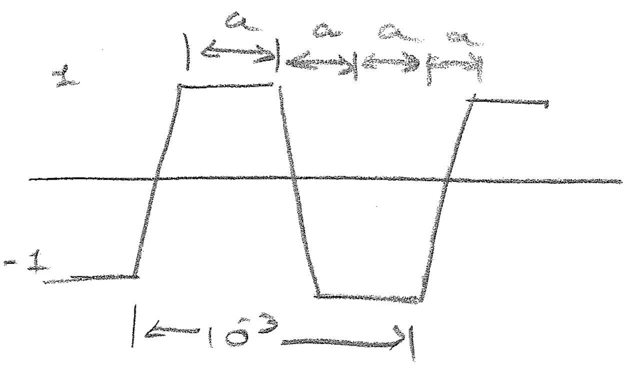
For the phase modulation the phase is proportional to the amplitude of message signal

$\therefore \theta(t) = \omega_c t + k_p m(t)$

The instantaneous frequency is equal to the slope of $\theta(t)$ at t.

$$\therefore \frac{d\theta(t)}{dt} = \omega_i = \omega_c + K_p \dot{m}(t)$$

First we need to find the derivative of message signal.



From m(t) we can easily infer that

$a = 25 \times 10^{-5}$

Now, $\frac{dm(t)}{dt} = \frac{m_{p(max)} - m_{p(min)}}{a}$

$$= \frac{2}{25 \times 10^{-5}}$$

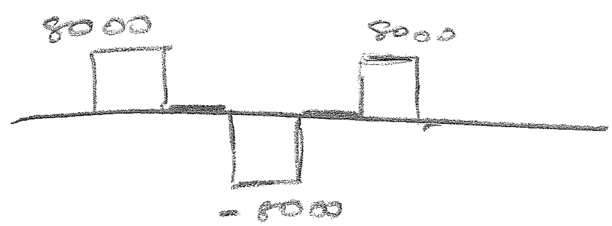
$$= 8000 \quad \text{(For rising)}$$

$\frac{dm(t)}{dt} = \frac{m_{p(max)} - m_{p(max)}}{a} = 0$ for constant amplitude

$\frac{dm(t)}{dt} = \frac{m_{p(min)} - m_{p(max)}}{a} = -8000$ for falling.

$\therefore \dot{m}(t)_{max} = 8000$

$\dot{m}(t)_{min} = -8000$.



$$\therefore \quad f_{i_{max}} = f_c + \frac{k_p}{2\pi} \dot{m}(t)_{max}$$

$$= \frac{10^8}{2\pi} + \frac{25}{2\pi} \times 8000$$

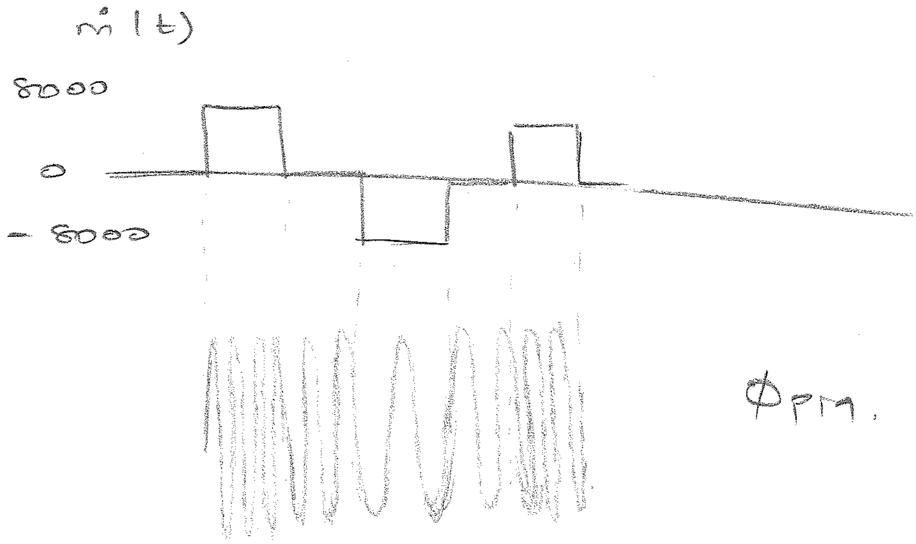
$$= 100.2\ MHz$$

$$f_{i_{min}} = f_c + \frac{k_p}{2\pi} \dot{m}(t)_{min}$$

$$= \frac{10^8}{2\pi} + \frac{25 \times (-8000)}{2\pi}$$

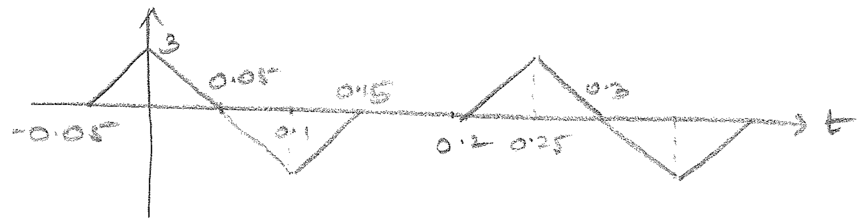
$$= 99.8\ MHz.$$

$$f_{i\,(\dot{m}(t)=0)} = f_c$$



Thus at mp max we have highest Frequency at mp=0 we have instantaneous Frequency = fc (carrier Frequency) ad For mp min we have lowest Frequency.

Q 5.1.3.



The carrier frequency $\omega_c = 2\pi \times 10^3$

$\therefore f_c = 10^3$ Hz.

a) FM signal $k_f = 20\pi$.

As discussed in the previous problem for FM the instantaneous frequency is given by

$$\omega_i = \omega_c + k_f m(t)$$

$$\therefore f_i = f_c + \frac{k_f}{2\pi} m(t).$$

$m_{p\,max} = 3$

$m_{p\,min} = -3$

$$\therefore f_{i\,max} = f_c + \frac{k_f}{2\pi} m_{p\,max}$$

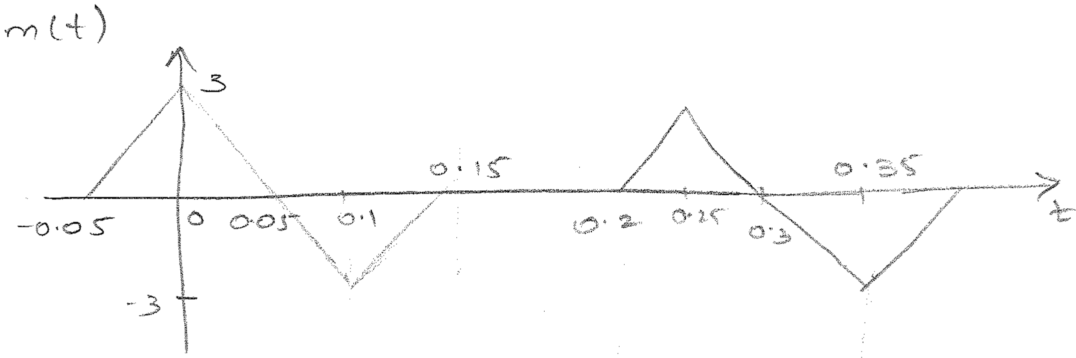
$$= 10^3 + \frac{\cancel{20\pi}^{10}}{\cancel{2\pi}} (+3)$$

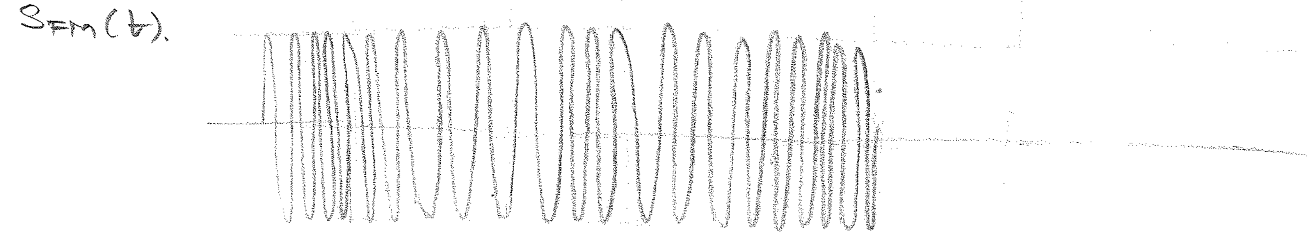
$$= 1030 \text{ Hz}$$

$$f_{i\,min} = f_c + \frac{k_f}{2\pi} m_{p\,min}$$

$$= 10^3 + \frac{20\pi}{2\pi} (-3)$$

$$= 970 \text{ Hz}.$$





(b). PM signal $K_p = \pi/2$

Solving for phase modulation as per previous problem.

$\omega_i = \omega_c + \dot{m}(t)$

$\dot{m}(t)$ is calculated as shown below.

$$\frac{dm(t)}{dt} = \frac{(3-0)}{(0+0.05)} = 60 \quad \text{(rising)}$$

$$= \frac{(-3-3)}{0.1} = -60 \quad \text{(falling)}$$

$$= 0 \quad \text{(between 0.15 and 0.2)}$$

$\dot{m}(t)_{max} = 60$

$\dot{m}(t)_{min} = -60$.

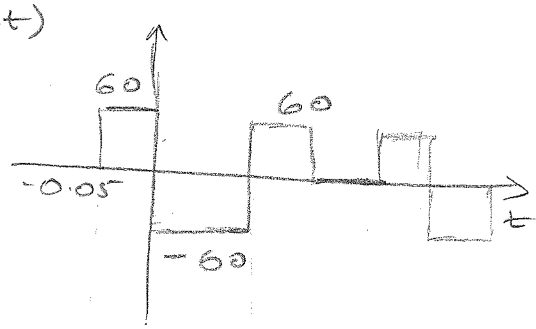
$$f_i = f_c + \frac{K_p}{2\pi}\dot{m}(t)$$

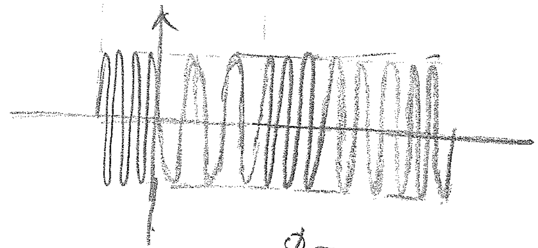
$f_{imax} = 10^3 + \frac{\pi}{2\times 2\pi} \times 60$

$= 1015\ Hz$

$f_{imin} = 10^3 + \frac{\pi}{2\times 2\pi} \times (-60)$

$= 985\ Hz.$



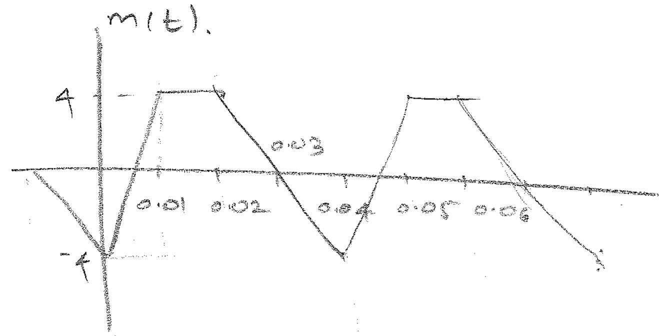


Q5.1.5 Carrier Frequency $\omega_c = 4\pi \times 10^3$

$F_c = 2\times 10^3\ Hz.$

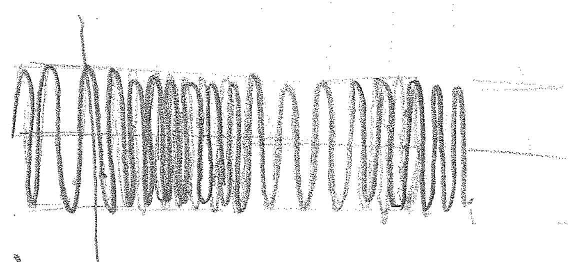
a). FM with $k_f = 500\pi$.

As solved in previous problems



$\omega_i = \omega_c + k_f m(t)$

$f_i = f_c + \frac{k_f}{2\pi} m(t)$



$m_{pmax} = 4 \quad m_{pmin} = -4.$

$\therefore f_{max} = 2\times 10^3 + \frac{500\pi}{2\pi} \times (4)$

$= 3\ kHz.$

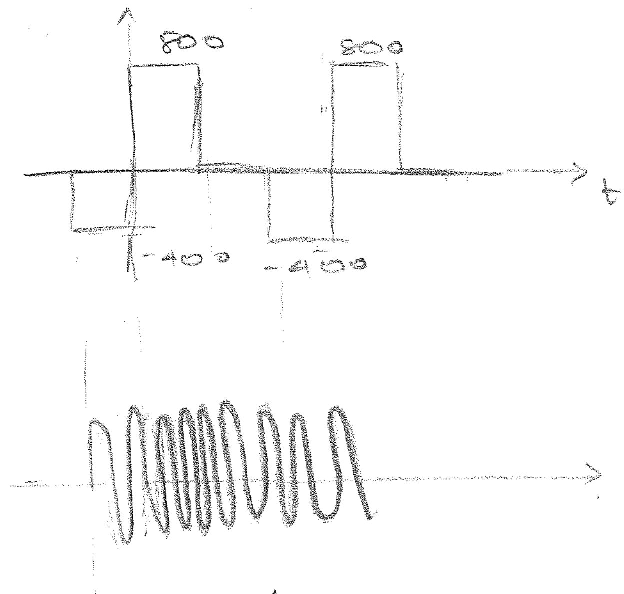
$f_{min} = 2\times 10^3 + \frac{500\pi}{2\pi} \times (-4)$

$= 1\ kHz$

(b) PM for $k_p = 0.2\pi$.

$$\dot{m}(t)_{max} = \frac{4-(-4)}{0.01} = 800$$

$$\dot{m}(t)_{min} = \frac{-4-4}{0.02} = -400$$



ϕ_{PM}.

$$\omega_i = \omega_c + k_p \dot{m}(t)$$

$$\therefore f_{i\,max} = 2\times10^3 + \frac{0.2\pi}{2\pi} \times 800$$

$$= 2\times10^3 + 100$$

$$= 2.1\ kHz$$

$$f_{i\,min} = 2\times10^3 + \frac{0.2\pi}{2\pi} \times(-400)$$

$$= 2\times10^3 - 50$$

$$= 1.95\ kHz.$$

$$f_{i_0} = 2\ kHz.$$

Q5.2.4

For the angle modulated signal

$$\phi_{EM}(t) = 10\cos(\omega_c t + 0.1 \sin 2000\pi t)$$

the carrier frequency $\omega_c = 2\pi \times 10^6$

$f_c = 10^6$ Hz.

The phase of above signal can be written in the form

$$\theta(t) = \omega_c t + 0.1 \sin 2000\pi t.$$

a) Power of a modulated wave,

Although the instaneous frequency and phase of an angle modulated wave can vary with time, the amplitude A always remains constant.

$\therefore$ Power $= \frac{A^2}{2}$ $\qquad A = 10.$

$= \frac{100}{2}$

$= 50$

b). ΔF (Frequency deviation).

The frequency deviation of an angle modulated signal is the variation in frequency proportional to the signal amplitude.

$\Delta F \propto m(t)$

$\Delta F = K_F m(t)$ where K_F is the proportionality constant

Now, $\theta(t) = \omega_c t + 0.1 \sin 2000\pi t$.

$\therefore \frac{d\theta(t)}{dt} = \omega_i = \omega_c + 200\pi \cos 2000\pi t$

(Instantenous Frequency is equal to the slope of the angle at each time t).

$\therefore \Delta\omega = 200\pi = K_F m(t)$

$\therefore \Delta F = \frac{200\pi}{2\pi} = \underline{100\ Hz}$.

c) $\underline{\Delta\theta}$

The phase deviation of given signal can be obtained from

$\theta(t) = \omega_c t + 0.1 \sin 2000\pi t$.

where $\underline{\Delta\theta = 0.1} = K_p m(t)$.

K_p is the proportionality constant for phase modulation.

(d). Bandwidth of $\phi_{FM}(t)$.

The estimated Bandwidth of FM signal is given by

$$B_{FM} = 2(\Delta F + B).$$

where B is the highest signal frequency in the signal or its derivative and ΔF is the deviation $\frac{k_f}{2\pi} m(t)$.

From the signal,

$$\theta(t) = \omega_c t + 0.1 \sin 2000\pi t.$$

$$B = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}.$$

$$\therefore B_{FM} = 2(100 + 1000)$$

$$= \underline{2.2 \text{ kHz}}$$

Q 5.2.8 $\quad m(t) = e^{-t^2/100} \quad f_c = 10^4 \text{ Hz}$

$k_f = 6000\pi \quad k_p = 8000\pi.$

a) FM

As solved in previous problems we first find maximum & minimum value of $m(t)$

$$m_{max} = e^{-0/100} = 1$$

$$m_{min} = e^{-\infty/100} = 0.$$

$\therefore \Delta F = \frac{K_F}{2\pi} \times m_{max} - \frac{K_F}{2\pi} \times m_{min}$

$= \frac{6000\pi}{2\pi}(1-0)$

$= \underline{3000 \text{ Hz}}.$

<u>For PM</u> we need to find $\dot{m}(t)$

$\therefore \dot{m}(t) = \frac{d}{dt} e^{-t^2/100} = -\frac{t}{50} e^{-t^2/100}$

It is not directly possible to find maximum value of this function. $\therefore$ we take derivative to this and equate to zero. (The tangent of maximum value of a function is horizontal $\therefore$ zero slope).

$\therefore \ddot{m}(t) = \frac{-1}{50} e^{-t^2/100} + \frac{t^2}{2500} e^{-t^2/100} = 0$

$= -\frac{e^{-t^2/100}}{50}\left[1 - \frac{t^2}{50}\right] = 0$

$\Rightarrow t = \sqrt{50}$

$\therefore \ddot{m}(\sqrt{50}) = 0$

$\therefore M_{pmax} = \dot{m}(\sqrt{50}) = 0.0858.$

$m_{pmin} = 0$

$\Delta F = \frac{k_p \dot{m}_p}{2\pi} = \frac{6000\pi \times 0.0858}{2\pi} = \underline{343 \text{ Hz}}.$

b) To find the bandwidth we find $M(f)$.

For $m(t) = e^{-t^2/100}$ using the transform pair

$$e^{-t^2/2\sigma^2} \Longleftrightarrow \sigma\sqrt{2\pi}\, e^{-\sigma^2\omega^2/2}$$

$$\therefore M(\omega) = 10\sqrt{\pi}\, e^{-25\omega^2}$$

To find 3dB bandwidth, we find $V_{max} = 17.7$

$$3\text{dB power} = \frac{V_{max}^2}{2}$$

$$3\text{dB Voltage } V_{3dB} = \sqrt{\frac{V_{max}^2}{2}} = 12.5$$

$$\therefore V_{3dB} = 10\sqrt{\pi}\, e^{-25\omega^2}$$

$$\therefore \omega_{3dB} = 0.118$$

$$\therefore F_{3dB} = 0.0188 \text{ Hz}.$$

$$\therefore B_{FM} = 2(\Delta f + B) \simeq 2\Delta f = \underline{6000 \text{ Hz}}$$

For PM

$$\mathcal{F}\{\dot{m}(t)\} = j2\pi f M(f)$$

$$= j2\pi f\,(10\sqrt{\pi}\, e^{-100\pi^2 f^2})$$

$$= j\,20\pi^{3/2} f e^{-100\pi^2 f^2}$$

From above calculations we can assume f_{3dB} for PM will be small compared to Δf

$$\therefore B_{PM} = 2(\Delta f + B) \simeq 2\Delta f$$

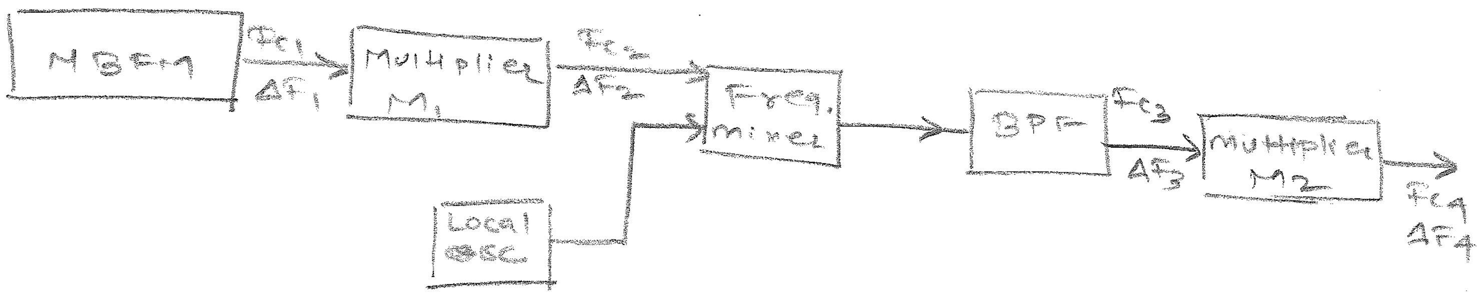
$$= \underline{686 \text{ Hz}}$$

Q 5.3.2 Amstrong Indirect FM modulator.

$F_{c4} = 98.1\,MHz$ $\quad f_{c1} = 100\,KHz$ $\quad 10\,MHz \leq f_{LO} \leq 11\,MHz$.

$\Delta F_4 = 75\,KHz$ $\quad \Delta F_1 = 10\,Hz$.

The modulator block diagram is shown below.



In the above block diagram we need to find M_1, M_2 and f_{LO}.

Now,

$f_{c1} = 100\,KHz$

$\Delta F_1 = 10\,Hz$.

$\therefore$ Total factor of frequency multiplication

$$M_1 \cdot M_2 = \frac{\Delta F_4}{\Delta F_1} = \frac{75 \times 10^3}{10} = 7500.$$

Let us assume

$M_1 = x$

$M_2 = y$.

$\therefore x \cdot y = 7500.$

From the figure,

$f_{c2} = x f_{c1}$ and $f_{c4} = y f_{c3}$.

To find f_{LO}, there are 3 possibilities.

$$f_{c3} = f_{c2} + f_{LO}$$

$$f_{c3} = f_{c2} - f_{LO} \quad (f_{c2} > f_{LO})$$

$$f_{c3} = f_{LO} - f_{c2} \quad (f_{LO} > f_{c2}).$$

The requirement is $10\,\text{MHz} < f_{LO} < 11.94\,\text{MHz}$.

a) we shall try for $f_{c3} = f_{c2} + f_{LO}$.

Now, $f_{c4} = y f_3$

$$\therefore 98.1\,\text{MHz} = y(f_{c2} + f_{LO})$$

$$= y(x \cdot f_{c1} + f_{LO})$$

$$= 7500 \times 100 \times 10^3 + y f_{LO}.$$

$\therefore$ $y.f_{LO}$ becomes negative which is

<u>incorrect</u>

Now let us try for $f_3 = f_{c2} - f_{LO}$.

$$f_{c4} = y f_3.$$

$$98.1 \times 10^6 = y(f_{c2} - f_{LO})$$

$$= y x f_{c1} - y f_{LO}.$$

$\therefore\ 98.1 \times 10^6 = 7500 \times 100 \times 10^3 - y f_{LO}$

$\therefore\ y f_{LO} = 651.9 \times 10^6$

$$\therefore\ f_{LO} = \frac{651.9 \times 10^6}{y}$$

Lets assume $y = 50$ $\therefore f_{LO} = 13.03\,MHz$.

This is not correct solution as $10MHz < f_{LO} < 11MHz$.

Lets take $\underline{y = 60}$ $\therefore f_{LO} = 10.865\,MHz$.

This solution is true hence we can calculate y.

$$x = \frac{7500}{y} = \underline{125}$$

$\therefore\ M_1 = 125$

$M_2 = 60.$

The third case too is incorrect as $y.F_{LO}$ becomes negative.

(b). The tunable range of carrier frequency is range of f_{c4}. Now, f_{c4} is given by

$$f_{c4} = 60.\ f_{c3}.$$

$f_{c4} = 60 \cdot f_{c3}$

$= 60(f_{c2} - f_{LO})$

$= 60(125 * f_{c1} - f_{LO})$ $\quad [\because f_{c2} = n \cdot f_{c1}]$

$= 60(125 * 100 * 10^3 - f_{LO})$

$= 60(12.5 * 10^6 - f_{LO})$.

If $f_{LO} = 10MHz$.

$f_{c4} = 60(12.5 - 10) * 10^6$

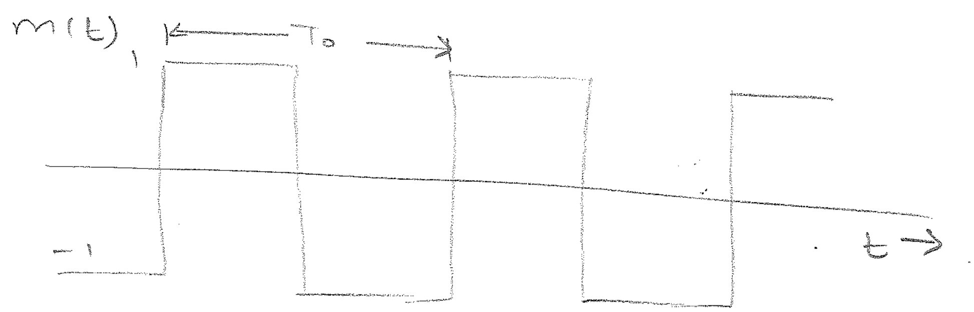
$= 150MHz$.

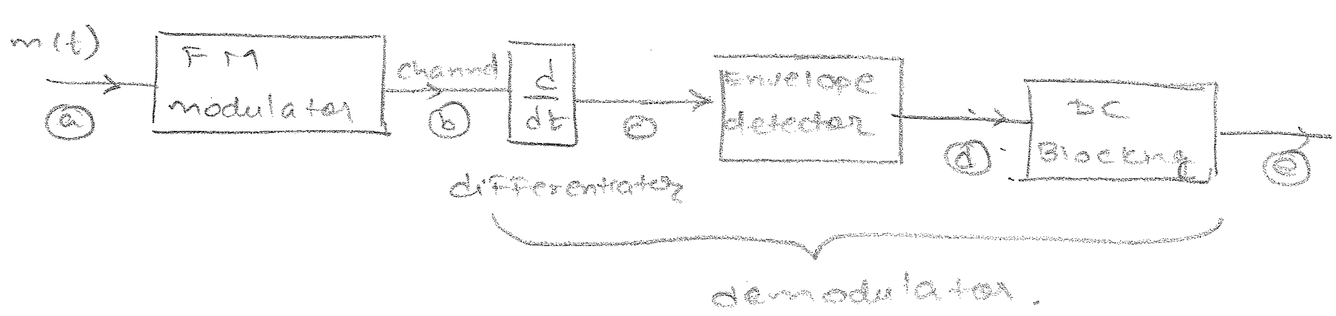
$F_{LO} = 11MHz$.

$f_{c4} = 60(12.5 - 11) * 10^6$

$= 90MHz$.

<u>Q5.4.2</u> The carrier used for modulation has $f_c = 10kHz$. with frequency deviation $\Delta f = 1kHz$.





a). We know that for a FM wave the instantenous frequency is given by

$\omega_i = \omega_c + K_F m(t)$.

Frequency deviation $\Delta F = K_F m(t)$.

$\therefore \Delta\omega = 2\pi K_F m(t)$.

$= 2\pi * 1kHz$.

$= 2000\pi$.

The minimum value of ΔF is when $m(t) = -1$
similarly the maximum value of ΔF is when $m(t) = 1$.

$\therefore \omega_i = 10k * 2\pi \pm 2\pi * 1k$

$= 20000\pi \pm 2000\pi$

How $\omega_i = \frac{d\theta}{dt}$.

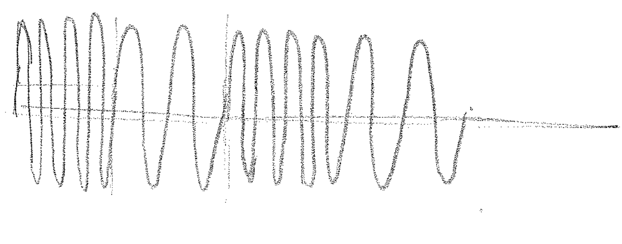
$\therefore \theta(t) = \int \omega_i dt$

$= 20000\pi t \pm 2000\pi t$.

$\therefore \phi_{FM} = A\cos(20000\pi t \pm 2000\pi t)$

where A is given carrier amplitude.

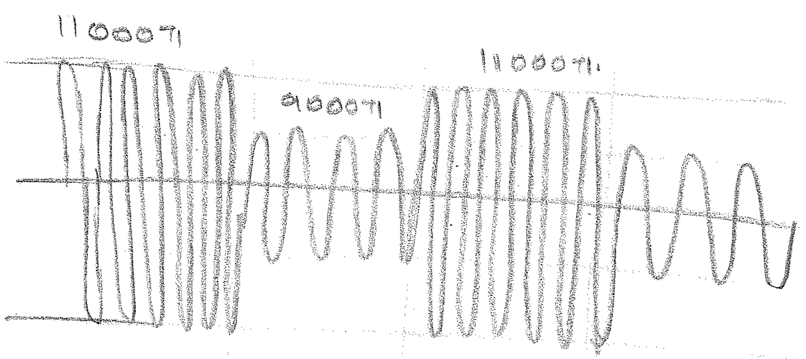
Hence the plot at (a)



A+c Once the signal is received at the receiver end it is differentiated. A differentiator is used as the information in FM signal resides in instantenous frequency ω_i, and frequency selective network with transfer function of form $|H(\omega)| = a\omega + b$ would yield ω_i.

$$\therefore \phi_{FM}(t) = \frac{d}{dt}\left[A\cos(20000\pi t \pm 2000\pi t)\right]$$

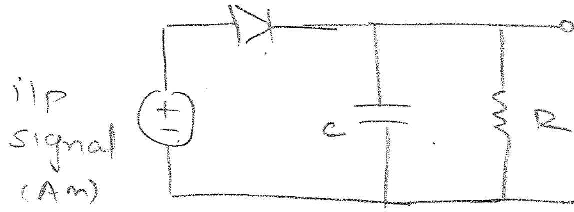
$$= -(20000\pi \pm 2000\pi)\, A\sin(20000\pi t \pm 2000\pi t).$$



We can deduce from this that the signal at (C) is both amplitude modulated as well as frequency modulated.

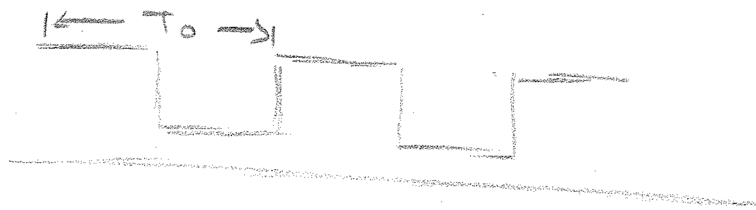
A+d The signal at (C) has
$\Delta\omega = k_f m_p < \omega_c$ and $\omega_c + k_f m(t) > 0$ for all t
hence $m(t)$ can be obtained by envelope detector.

An envelope detector has a Diode that conducts in positive cycle and the capacitor charges to the peak value.



Envelope detector

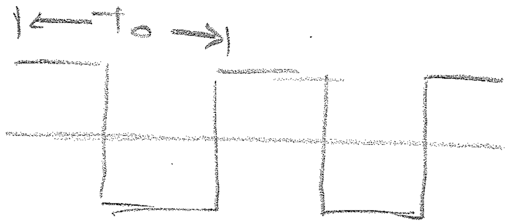
In negative cycle diode is off and capacitor slowly discharges. It again charges in positive cycle. Hence the output of Envelope detector is shown below.



at d

The output has a periodic square wave proportional to $(20000\pi \pm 2000\pi)A$ with a dc offset.

At e After DC block the result is periodic square wave proportional to $m(t)$.



At e

Q 5.4.3

$$S(t) = 2\cos[10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)].$$

a) Band width of FM signal.

$$B_{FM} = 2(\Delta F + B).$$

B - Band width required for maximum signal frequency component in the signal or its derivative.

$$\therefore B = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}.$$

Now $\theta(t) = 10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)$

Instantenous Frequency.

$$\omega_i = \frac{d\theta}{dt} = 10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)$$

The carrier deviation is given by.

$$\Delta\omega = 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)$$

When these 2 sinusoids add in phase we get the maximum deviation.

$\therefore \Delta\omega_{max} = 4300\pi$

$\therefore \Delta F = \dfrac{4300\pi}{2\pi}$

$= 2150 \text{ Hz}$

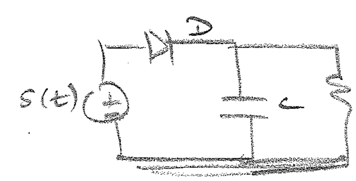
$\therefore B_{FM} = 2(\Delta F + B)$

$= 2(2150 + 1000)$

$= \underline{6.3 \text{ kHz}}$

(b) As $s(t)$ has a constant amplitude passing it through envelope detector would produce a constant at the output (The diode is always forward biased and charge on capacitor does not vary).

(c) On differentiating $s(t)$ we get



$$\dot{s}(t) = -2[10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) - 300\pi\sin(100t)]$$
$$* \sin[10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)]$$

Thus the output of the envelop detector will be.

$$2[10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) - 300\pi\sin(100t)]$$

(d). It is obvious that the signal is first differentrated so that we have an AM signal in addition to FM signal.

Now, we know that

$$k_f\, m(t) = 4000\pi \cos(2000\pi t + 0.3\pi) + 300\pi \sin(100t)$$

$$\therefore m(t) = [4000\pi \cos(2000\pi t + 0.3\pi) + 300\pi \sin(100t)]/k_f$$

$$k_f = 200\pi.$$

$$\therefore m(t) = 20\cos(2000\pi t + 0.3\pi) + 1.5\sin(100t).$$

Q 5.5.1

Pre emphasis – Boosting of weaker frequency components before modulation.

Deemphasis – Attenuating the frequency components (higher frequency components) boosted in pre emphasis.

$$H_p(\omega) \cdot H_d(\omega) = 1.$$

Here we have

$$H_p(f) = j2\pi f.$$

This is a differentiator.

Now, $H_d(f) = 1/H_p(f)$

$= 1/j2\pi f$

This is an integrator.

We know that in phase modulation the signal is differentiated and the modulation is proportional to $\dot{m}(t)$.

Moreover, while demodulating we need to integrate the differentiated message signal to get $m(t)$.

Q 5.6.1 AM signal $F_c = 1530\,kHz$

$F_{IF} = 455\,kHz$.

In an AM receiver the frequency mixer translates carrier ω_c to fixed IF of 455 kHz where IF is Intermediate Frequency

$\therefore$ The frequency of the local oscillator is $F_{LO} = F_c \pm F_{IF}$.

The reason behind IF is that all the stations in AM receiver are translated to a fixed carrier frequency of 455KHz. This helps in achieving adequate selectivity.

Now, for this case

$\therefore F_{LO} = 1530K + 455K$

$= \underline{1985KHz}$

or

$F_{LO} = 1530K - 455K$

$= 1075KHz.$

Let us assume $F_{LO} = 1075KHz$. Then a station at $f_c = 1530KHz$ will be tuned clearly. For $F_{LO} = 1075KHz$ the image station is at $f_c' = 1530K - 2 \times F_{IF}$

$= 620KHz.$

So 620KHz is the image frequency for station at 1530KHz. And owing to a poor RF stage bandpass filter f_c can be heard at f_c'.