Chapter 7 Solutions

Q 7.2.1 $\quad p_1(t) = \Delta(t/T_b)$

(a) Find PSD for polar, on-off, and bipolar signal.

To find PSD for any line code we need to find the fourier transform

From table 3.1, 19th equality

$$\Delta(t/T_b) \Longleftrightarrow \frac{T_b}{2}\,\text{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

$$\therefore P(f) = \frac{T_b}{2}\,\text{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

PSD for line codes is given by the general expression as given in eq. 7.11c

$$S_y(\omega) = \frac{|P(f)|^2}{T_b}\left(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n 2\pi f T_b\right)$$

i) For polar signal

$R_0 = 1$ eq. 7.12a

$R_n = 0, (n \geq 1)$ eq. 7.12c

$$\therefore S_y(f) = \frac{|P(f)|^2}{T_b}$$

$$= \frac{\left(\frac{T_b}{2}\right)^2 \text{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{T_b}$$

$$\boxed{S_y(f) = \frac{T_b}{4}\,\text{sinc}^4\left(\frac{\pi f T_b}{2}\right)}$$

For on-off signaling

$$R_0 = \lim_{N \to \infty} \frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2}$$

$$R_n = \lim_{N \to \infty} \frac{1}{N}\left[\frac{N}{4}(1) + \frac{3N}{4}(0)\right] = \frac{1}{4} \quad (n \geq 1)$$

$\therefore$ From 7.19b eq

$$S_y(\omega) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right]$$

$$\boxed{S_y(\omega) = \frac{T_b \operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{16}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_b}\right)\right]}$$

For bipolar signal

From eq^n 7.21(b)

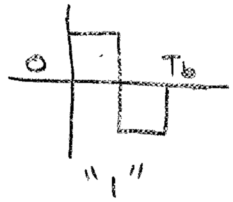
$$S_y(f) = \frac{|P(f)|^2}{T_b}\sin^2(\pi f T_b)$$

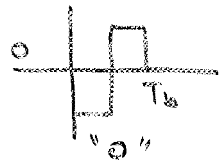
$$= \frac{T_b^{\cancel{2}}}{4}\,\frac{\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)}{\cancel{16}}\sin^2(\pi f T_b)$$

$$\boxed{S_y(f) = \frac{T_b}{4}\operatorname{sinc}^4\left(\frac{\pi f T_b}{2}\right)\sin^2(\pi f T_b)}$$

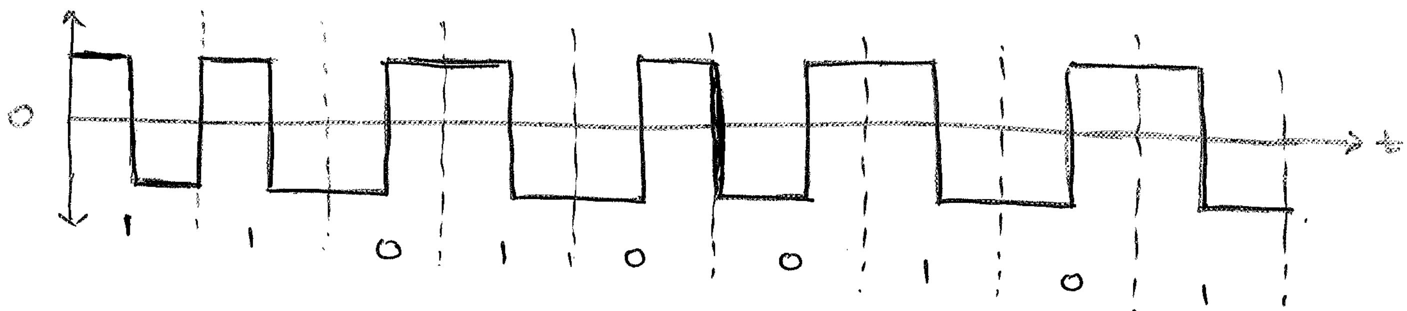
Q 7.2.4

(a) $p(t) =$

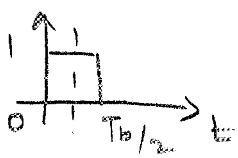




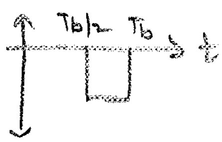
$y(t) = 1\,1\,0\,1\,0\,0\,1\,0\,1 \ldots$



(b)



$\longrightarrow \quad \Pi\left(\frac{t+T_b/4}{T_b/2}\right)$



$\longrightarrow \quad -\Pi\left(\frac{t-T_b/4}{T_b/2}\right)$

$$\therefore \; p(t) = \Pi\left(\frac{t+T_b/4}{T_b/2}\right) - \Pi\left(\frac{t-T_b/4}{T_b/2}\right)$$

<u>Fourier</u> <u>transform</u> <u>of</u> <u>p(t)</u>

$$\Pi(t/\tau) \Leftrightarrow \tau \operatorname{sinc}(\pi f \tau)$$

$$\therefore \; \Pi\left(t/\tfrac{T_b}{2}\right) \Leftrightarrow \frac{T_b}{2} \operatorname{sinc}\left(\frac{\pi f T_b}{2}\right)$$

$$\therefore \; \Pi\left(\frac{t+T_b/4}{T_b/2}\right) \Leftrightarrow \frac{T_b}{2} \operatorname{sinc}\left(\frac{\pi f T_b}{2}\right) e^{j2\pi f T_b/4}$$

$$\therefore\ P(f) = \frac{T_b}{2}\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)e^{j\frac{\pi f T_b}{2}} - \frac{T_b}{2}\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)e^{-j\frac{\pi f T_b}{2}}$$

$$= \frac{T_b}{2}\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)\left[e^{j\frac{\pi f T_b}{2}} - e^{-j\pi f T_b/2}\right]$$

$$= \frac{T_b}{2}\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)\left[\cancel{\cos\frac{\pi f T_b}{2}} + j\sin\frac{\pi f T_b}{2} - \cancel{\cos\frac{\pi f T_b}{2}} + j\sin\frac{\pi f T_b}{2}\right]$$

$$= j\,T_b\,\text{sinc}\left(\frac{\pi f T_b}{2}\right)\sin\left(\frac{\pi f T_b}{2}\right)$$

Now, 1 & 0 are generated with 50%. probability. Also the signaling is polar (1 & −1)

$$\therefore\ R_0 = \lim_{N\to\infty}\frac{1}{N}\sum_k a_k^2$$

$a_k^2 = 1$ as 1 & −1 are equally likely.

$$= \lim_{N\to\infty}\frac{1}{N}\left(\frac{N}{2}(1)^2 + \frac{N}{2}(-1)^2\right)$$

$$= \lim_{N\to\infty}\frac{1}{N}\times N$$

$$= 1$$

Now, $R_n = 0 \qquad n \geq 1$

$$(\because\ R_1 = \lim_{N\to\infty}\frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$

$\because\ a_{k=1}$ and a_{k+1} are either 1 or −1,

$$\therefore \quad S_x(f) = \frac{1}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos n2\pi f T_b\right]$$

$$= \frac{1}{T_b} R_0$$

$$\therefore \quad S_y(f) = |P(f)|^2 S_x(f)$$

$$= \frac{|P(f)|^2 R_0}{T_b}$$

$$= \frac{T_b^2 \operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}{T_b}$$

$$\boxed{S_y(f) = T_b \operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

Q.7.3.1 Bit rate $R_b = 5 * 10^6$ bit/s

Roll off factor $r = 0.25$

$\therefore$ Minimum band width $B_T = \frac{(1+r) R_b}{2}$

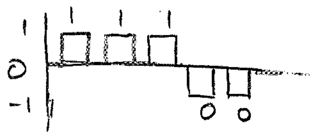
$$= \frac{(1+0.25)\, 5*10^6}{2}$$

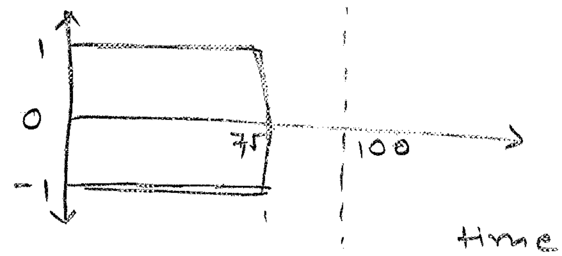
$$\boxed{B_T = 3.125 * 10^6 \text{ Hz}}$$

Q 7.6.1 $\quad p(t) = \Pi\left(\frac{t}{3T_b/4}\right)$

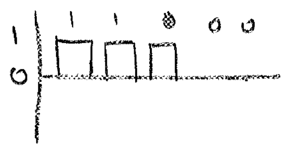
$3T_b/4$ is $3/4$ of T_b. $\quad$ Eye Diagram

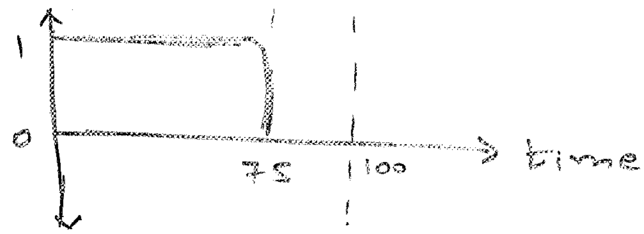
(a) Polar



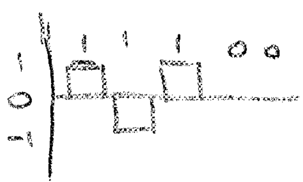


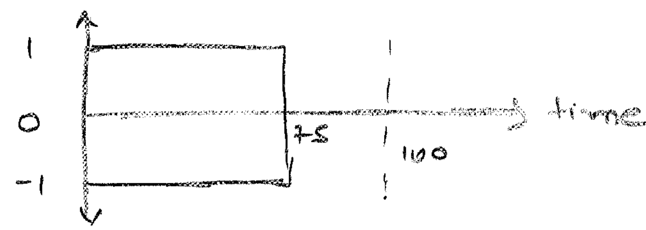
(b) On-off





(c) bipolar





(d) duobinary $\quad$ 1 1 1 0 0

1 2 2 0 -2