Chapter 6 Solutions

Q6.1.2

a) $\text{sinc}(2100\pi t)$

From table 3.1 we find

$$2B\,\text{sinc}(2\pi Bt) \Longleftrightarrow \Pi(f/2B)$$

$$\therefore \text{sinc}(2100\pi t) \Longleftrightarrow \frac{1}{2100}\Pi(f/2100)$$

$\therefore 2B = 2100$ rad/s

Nyquist rate $= 2*B = 2100$ samples/s

Nyquist interval $= 1/2B = 1/2100$ sec.

b) $5\,\text{sinc}^2(200\pi t)$

From table 3.1

$$B\,\text{sinc}^2(\pi Bt) \Longleftrightarrow \Delta(f/2B).$$

$$\therefore 5\,\text{sinc}^2(200\pi t) \Longleftrightarrow 0.025\,\Delta(f/400)$$

$\therefore 2B = 400.$

$\therefore$ Nyquist rate $= 2B = 400$ samples/s

Nyquist interval $= 1/2B = 1/400$ sec.

(c) $\text{sinc}(2100\pi t) + \text{sinc}^2(200\pi t)$

Taking results from part (a) & (b)

$$\text{sinc}(2100\pi t) + \text{sinc}^2(200\pi t) \Leftrightarrow \frac{1}{2100}\Pi(f/2100) +$$

$$0.005\,\Delta(f/400)$$

For first signal bandwidth $B_1 = 1050$ Hz.

For second signal bandwidth $B_2 = 200$ Hz.

When 2 signals are added higher bandwidth signal is used to calculate Nyquist rate.

$\therefore$ Nyquist rate $= 2B_1 = 2100$ sample/sec

Nyquist interval $= 1/2B_1 = 1/2100$ sec.

(d) $\text{sinc}(200\pi t)\,\text{sinc}(2100\pi t)$

The width property of convolution says the following

$x_1(t) \Leftrightarrow y(T_1)$ $\qquad$ $x_2(t) \Leftrightarrow y(T_2)$

then

$$x_1(t).x_2(t) \Leftrightarrow y(T_1+T_2).$$

Similarly,

$$\text{sinc}(200\pi t) \Leftrightarrow 0.005\,\Pi(f/200)$$

$$\text{sinc}(2100\pi t) \Leftrightarrow \frac{1}{2100}\Pi(f/2100)$$

$\therefore$ $B_1 = 100$ Hz

$B_2 = 1050$ Hz

$\therefore$ Band width of resultant signal $= B_1 + B_2$

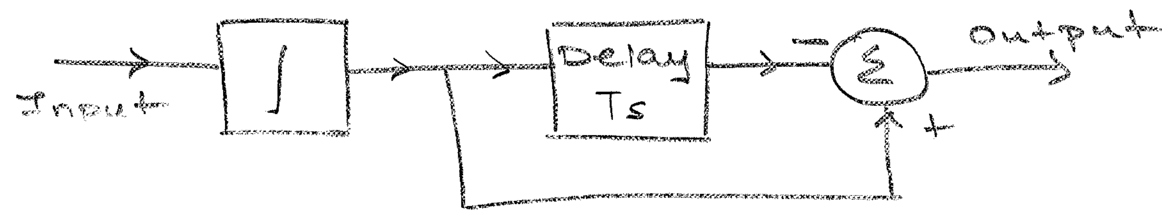
$= 1150$ Hz.

$\therefore$ Nyquist sampling rate $= 2(B_1 + B_2)$

$= 2300$ sample/sec

Nyquist sampling interval $= 1/2300$ sec

Q 6.1-6



(a) unit impulse $\rightarrow \delta(t)$

$$\therefore h(t) = \int_0^t [\delta(\tau) - \delta(\tau - T_s)]\, d\tau$$

$$= \int_0^t \delta(\tau)\, d\tau - \int_0^t \delta(\tau - T_s)\, d\tau$$

$$= u(t) - u(t - T_s)$$

$$\therefore h(t) = \Pi\left(\frac{t - T_s/2}{T_s}\right)$$

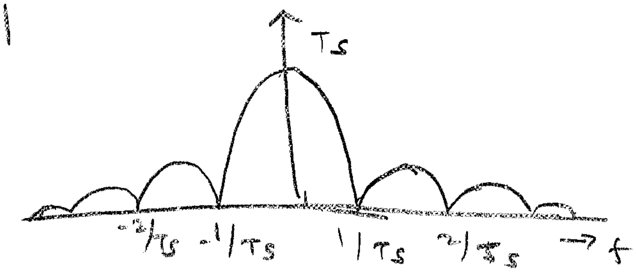
b). Transfer function $H(f)$ from 17th formula of table 3.1.

$$\Pi(t/\tau) \Longleftrightarrow \tau \,\text{sinc}(\pi f \tau)$$

$$\Pi\left(\frac{t - T_s/2}{T_s}\right) \Longleftrightarrow T_s \,\text{sinc}(\pi f T_s)\, e^{-j\pi f T_s}$$

∴ The magnitude of $H(f)$ is

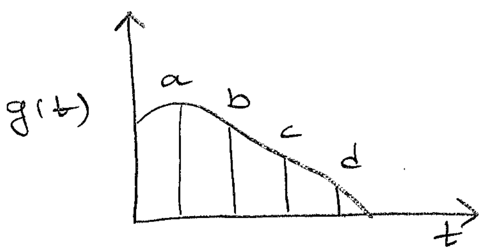
$$|H(f)| = T_s\, |\text{sinc}(\pi f T_s)|$$



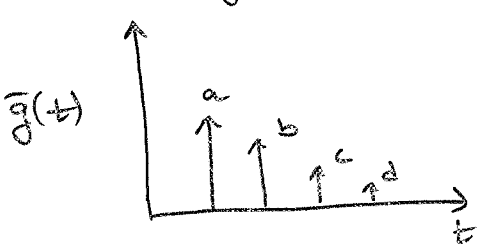
(c) $\bar{g}(t)$

$\bar{g}(t)$ is sampled signal of $g(t)$

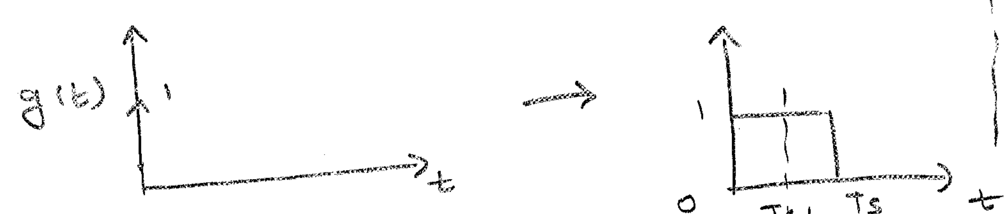
For eg $g(t)$



Sampled version of $g(t)$
$\bar{g}(t)$



Now from part (a)



So finally for $\bar{g}(t)$ we get a step like response

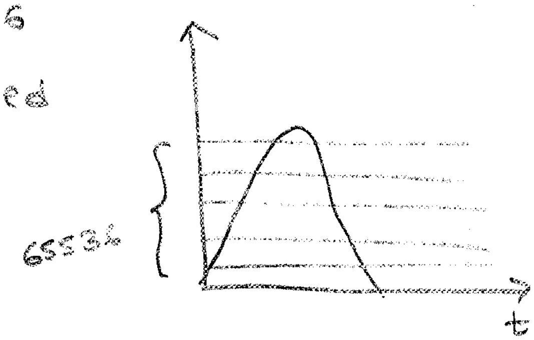
Similarly

same is the case for c & d

Q6.2.1 Bandwidth $B = 15\,kHz$

a) Quantization levels = 65536

∴ Number of samples required to encode a sample is



$$N = \log_2 L = \log_2 65536$$
$$= 16$$

(or in other sense $2^N = L$

∴ $L = 2^{16} = 65536$)

(b) Average power = $\overline{m^2(t)} = 0.1\,W$

Peak voltage = $m_p = 1\,V$

$$SQNR = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} = 3 \times (65536)^2 \times \frac{0.1}{1^2}$$

$$= 1.288 \times 10^9$$

SQNR is measured in dB

∴ $SQNR = 10 \log 1.288 \times 10^9$

$= \underline{91.1\,dB}$

(c) Number of binary digits per second (bits/s) to encode whole signal is calculated by

Nyquist rate × No. of bits for levels

∴ $15000 \times 2 \times 16$

$= \underline{480{,}000 \text{ bits/s}}$

(d) Sample rate = 44100 samples/s

$L = 65536$

$\therefore \; n = 16$

$\therefore$ No of bits/s = $44100 * 16$

$= 705600$ bits/s

Minimum bandwidth = $\frac{\text{Nyquist rate}}{2}$

$= \underline{352,800}$ Hz

<u>Q6.2.4</u> $m_p = 1 \quad \widetilde{m^2(t)} = 20mW$

$\therefore \; 43dB = 10 \log 3L^2 \frac{\widetilde{m^2(t)}}{m^2p}$

$\therefore \; 4.3 = \log 3L^2 \frac{\widetilde{m^2(t)}}{m^2p}$

$\therefore \; 19952.6 = 3L^2 \frac{\widetilde{m^2(t)}}{mp^2}$

$\therefore \; L = 576.7$

For 576.7 levels we need minimum no

10 bits i.e, $2^{10} = 1024 \quad (\because 2^9 = 512)$

Now for 10 bits

$L = 1024$

$\therefore$ SNR $= 10 \log \left(3 * (1024)^2 * \frac{20m}{1^2}\right)$

$= \underline{47.98 \text{ dB}}$

Q6.2.7 According to u- compandor law

if $\mu^2 \gg \frac{m_p^2}{\widetilde{m^2(t)}}$ $(\because 100^2 \gg 50)$

$\therefore\ 43 = 10 \log \frac{3L^2}{[\ln(1+\mu)]^2}$

$\therefore\ 10^{4.3} = 0.14 L^2$

$\therefore\ L = 377.5$

Hence minimum no. of bits required are 9 $(2^9 = 512)$

Now with 9 bits $L = 512$

$\therefore\ SNR = 10 \log \frac{3 \times (512)^2}{[\ln(1+100)]^2}$

$= \underline{45.67\ dB}$