Cell Radius Inaccuracy: A New Measure of Coverage Reliability

PETE BERNARDIN, MEMBER, IEEE, MENG YEE, MEMBER, IEEE, AND THOMAS ELLIS

Preprint of Publication in the November 1998 Issue (Vol. 47, No.4, pp. 1215-1226) of the IEEE Transactions on Vehicular Technology

Abstract [*]**-- A robust method for determining the boundaries of cells and the associated reliability of the RF coverage within these boundaries is presented. The procedure accurately determines the effective cell radius using a linear regression of RF signal strength samples. The accuracy of this estimate is quantified both as a radius uncertainty (e.g., ± 100 meters) and as a coverage (i.e., area/edge) reliability error through 1) simulation, 2) analysis of real data, and 3) theoretical analysis. It is shown that if the estimate of the cell radius meets the desired accuracy, then the corresponding estimates of coverage reliability (both area and edge) are more than sufficiently accurate. Through a sensitivity analysis, it is discovered that estimating the cell radius is a much more critical step in determining the quality of RF coverage than the more common practice of simply estimating the area reliability. In addition, a formula for estimating area reliability is given and shown to be more accurate than can be obtained by current approaches. The verification method presented here is particularly useful in wireless planning since it effectively determines the geographic extent of reliable RF coverage. It is recommended that radio survey analyses select cell radius estimation as the preferred method of coverage verification.**

I. INTRODUCTION

THE TWO MOST COMMONLY used measures of the reliability of RF coverage are 1) cell edge reliability and 2) cell area reliability. Cell edge reliability refers to the probability that the RF signal strength measured on a circular contour at the cell edge will meet or exceed a desired quality threshold (e.g., -90 dBm). Whereas, cell area reliability is the probability that RF signal will meet or exceed the quality threshold after integrating the contour probability over the entire area of the cell (i.e., across all of the contours of the cell, including the cell edge). D.O. Reudink showed that, for a given propagation environment, cell edge reliability and cell area reliability are deterministically related [2] (see also section IV and the Appendix of this paper). Because of this relationship, estimating the distance to the cell edge can be shown to be theoretically equivalent to determining the reliability of the signal strength within the cell (e.g., see equation (5) and equation (a7) in the Appendix). In this study we describe a new measure of RF reliability that has previously not been reported in other wireless investigations. We call this coverage criterion "cell radius inaccuracy," ΔR. We have found this criterion to be very useful in answering the following two (related) questions:

1) How many signal strength measurements are needed to accurately estimate the spatial extent of reliable coverage?
2) How do we best estimate the coverage reliability of isolated cells with a finite number of signal strength measurements?

In answering the first question, the equivalent circular contour (i.e., the effective radius, R) of the cell is estimated, as shown in Figure 1. The relationship between the inaccuracy (ΔR) of this radius estimate and the amount of lognormal fading, σ, in each cell is empirically derived as a function of the number of independent signal strength measurements, N (see equation (14)).

Regarding the second question, perhaps the most important finding of this study is that it is the accuracy of the cell radius estimate (i.e., ΔR), not the accuracy of the area reliability estimate that is the limiting factor in determining the quality of RF coverage. The relationship between cell radius inaccuracy and area reliability is also discussed.

Typically, cell radius estimation and area reliability analyses are not considered together in propagation optimization. It is found that these two problems cannot be considered independently, and the consequence of doing so can lead to inaccurate estimates of RF coverage.

Many vendors of RF predictive tools already use regression to determine the best linear approximation to the median path loss for the purpose of tuning RF prediction models. In this paper we investigate the advantages of also measuring the lognormal fading within each cell to more precisely determine the radius of reliable cellular coverage. These measurements are used to compute a fade margin for each cell, which is then incorporated in the estimation of the cell's radius. Thus, the cell radius is defined explicitly in terms of the desired quality of coverage.

It is recommended that, in addition to area reliability, future wireless verifications also consider cell radius inaccuracy.

II. OVERALL APPROACH

Figure 1(a) shows a high level view of the coverage estimation process for an omni cell. The signal strength is shown as a path loss surface which decreases with the logarithm of the distance from the cellular base station. After signal strength measurements have been taken uniformly over the area of the

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[*] Manuscript received August 12, 1996; revised February 17, 1997 and February 27, 1998.

P. Bernardin and M. Yee are with Northern Telecom Wireless Engineering Services, Richardson, TX, 75083-3805.

T. Ellis is with Wireless Software Design & Consulting, 13447 Forestway Drive, Dallas, Tx, 75240.

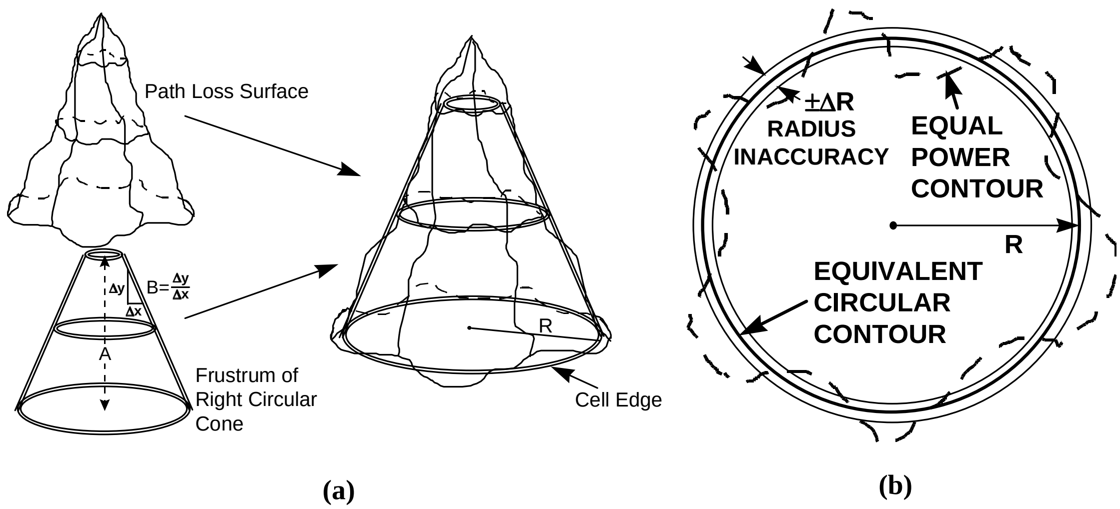

Figure 1. (a) Fitting the path loss surface via linear regression. Signal strength as a decreasing function of the logarithm of the distance from the base station for 360 degrees of azimuth. The area of the path loss surface is uniformly sampled and approximated with a cone via linear regression. The cell edge is approximated as the radius of the base of the best-fitting cone. **(b) Enlargement of the base of the cone in (a).** The measurement approach computes the best circular approximation to the equal power contour. The effective radius, *R*, of the cell is measured and the accuracy quantified in terms of a radius inaccuracy ring, $\pm\Delta R$. The average signal strength on the circular contour is equal to the signal strength of the equal power contour.

surface, the path loss is fit with a cone via linear regression. The cell edge is approximated as the radius of the base of the best-fitting cone.

The proposed method estimates the best circular boundary that matches the cell edge at the desired area reliability, as illustrated in Figure 1(b). It should be emphasized that this

the signal power is above -90 dBm). It is the radius of this fitted circle that is estimated. Thus, this radius can be considered the "effective radius" of the cell and is well defined for any cell, circular or otherwise.

The accuracy of the cell radius estimate is quantified in terms of a radius inaccuracy ring, $\pm\Delta R$, also shown in Figure 1(b), where the dimension of ΔR is expressed in units of distance. The width of this ring depends mostly on the number of signal strength samples in the regression, and also upon the amount of lognormal fading in the cell.

Two methods for determining area reliability from drive test data are compared. The first method is the standard approach of estimating the proportion of signal strengths that are above a desired reliability threshold [3].

The second technique is the preferred method, which is used throughout this study. This method involves determining the propagation parameters of individual cells and using this information in conjunction with Reudink's analysis (see equation (a7) in the Appendix) to estimate the area reliability. The propagation path is approximated with a two-parameter model similar to Hata [1]. A fade margin based on the actual signal variation within each cell is calculated to ensure the desired cell edge reliability. It is shown that this technique provides area reliability estimates that are much more accurate than those obtained from the first method.

approach does not in any way require that the true cell edge be circular. Rather, even the most irregular cell edge can be fitted with a circle such that the average power along the circumference is equal to the power of the true cell edge. This circle encloses the area over which the RF signal meets or exceeds the desired area reliability (e.g., over 90% of the area,

The proposed approach can be used to quickly determine the validity of drive test data. Given enough measurements, simulations show that this technique can be made almost arbitrarily exact. It is recommended that this method be included as part of the pre-build verification procedure for any wireless technology (TDMA, AMPS, CDMA, etc.).

III. APPROACH FOR ESTIMATING THE CELL RADIUS

We seek to characterize the propagation effects of the environment (terrain and clutter), not of the antenna. Thus, the signal strength measurements are first inverse filtered to remove the anisotropic weighting introduced by the horizontal antenna pattern.
The proposed approach for estimating the cell radius is graphically summarized in Figure 2. The measurement method is based on a two-parameter propagation model similar to the prediction formulas of Hata

$$P_r = P_t - P_L = P_t - A - B\log_{10} r \qquad (1)$$

where P_r is the received power (dBm), P_t is the transmitted power (EIRP) of the base station plus the receiver gain (e.g., P_t =50 dBm EIRP+G_r), P_L is the path loss (dB), r is the range (km) from the base station, and A and B are the unknown constants to be estimated from the RF data via linear regression [1]. A fade margin based on the actual signal

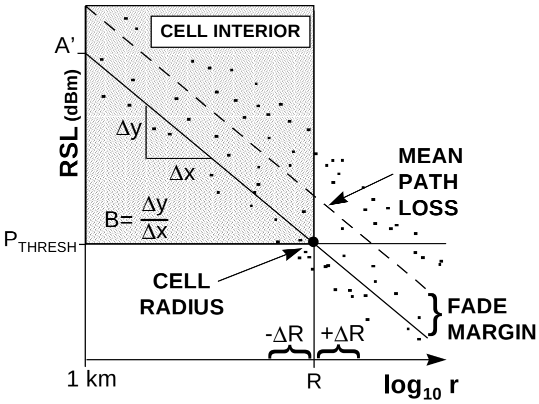

Figure 2. The graphical approach to estimating the cell radius to within ±*ΔR*. The received signal strength level (RSL) is plotted versus the range from the base station to each measurement. The mean path loss is computed via linear regression and offset by the fade margin. The cell radius is defined in terms of the desired coverage reliability as the point where the faded line crosses the reliability threshold, P_{THRESH}.

variation within each cell is calculated to ensure the desired cell edge reliability.

Because of the similarity to Hata's model, it is important to clarify that the method does not incorporate Hata's coefficients. Instead, the salient propagation parameters are estimated from the data since the major goal in this study is RF verification, not RF prediction.

The interior of each cell is divided into approximately 5000 bins which are uniformly sampled both in range and azimuth (i.e., uniform area sampling). The signal strength measurements in each bin are averaged to produce a single (average power) value per bin [4]. The range is then computed from the base station to the center of all of the bins that contain measurements. Thus, each bin represents an average power measurement at a certain range from the base station. The range axis is then mapped to a logarithmic (common log) scale, the transmit power is combined with the parameter, A, and the two parameters of the following equivalent model are estimated via linear regression

$$P_r = A' - Br_L \tag{2}$$

where $r_L = \log_{10} r$ and $A' = P_t - A$.

Once the constants A' and B have been estimated, the mean trend of the propagation data is subtracted from the signal strength measurements and the standard deviation, σ, of the remaining zero-mean process is estimated. The value of σ represents the composite variation due to two primary factors: lognormal fading and measurement error. Both of these factors tend to introduce uncorrelated variations around the the mean since the regression is computed for range measurements across all azimuth angles which greatly reduces most spatial correlation effects.

A fade margin, FM_σ, that ensures the desired cell edge reliability, $F(z)$, can then be approximated (see equation (a4) in Appendix)

$$FM_\sigma = z\,\sigma \tag{3}$$

where $$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{2}}\, dt \tag{4}$$

For example, cell edge reliabilities of 75% and 90% correspond to fade margins of about 0.675σ and 1.282σ, respectively.

It is now straightforward to derive the distance to the cell edge, R , at any desired signal strength threshold, P_{THRESH}, and service reliability, $F(z)$. From equations (1), (2), (3), and (4)

$$R = 10^{-(P_{THRESH} + FM_\sigma - A')/B} \tag{5}$$

Any additional static (nonfading) margin, such as building penetration losses, can also be easily incorporated into the P_{THRESH} term. Thus, A',B and, σ are all that is needed to determine the range from the base station to the cell edge.

Example:
Compute the range to the cell edge assuming the Hata (Cost-231) urban model constants for 1900 MHz and a base station antenna height of 30 meters:

A= 140
B= 35.2

Also assume

σ = 8 dB
P_{THRESH}= -95 dBm
P_t= 50 dBm (EIRP)
$F(z)$ = 75% (i.e., $FM_\sigma = 0.675\sigma$)

From equation (5), for 75% cell edge reliability the estimated radius is

$$R = 10^{-(-95+5.4-50+140)/35.2} = 0.974 km$$

Similarly, the radius for 90% cell edge reliability is given by

$$R = 10^{-(-95+10.3-50+140)/35.2} = 0.707 km$$

Thus, exact knowledge of the propagation parameters A', B, and σ is equivalent to the exact knowledge of R. The remainder of this paper deals with the details of how to estimate the parameters A', B, and σ from drive test data that has been corrupted with measurement error and the precision that results from doing so.

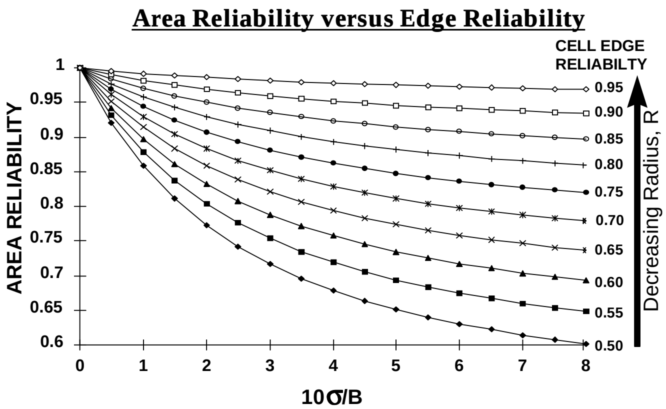

Figure 3. Area reliability (ordinate) and cell edge reliability (see parameter associated with each curve) versus$10\sigma/B$, where σ is the standard deviation of the RF signal and B is the propagation path loss exponent. For a given value of σ/B, knowledge of the cell edge reliability directly determines the area reliability. (The figure is redrawn from Chapter 2 of reference[2]). Note, increasing cell edge reliability is equivalent to decreasing the radius of coverage.

IV. AREA RELIABILITY ESTIMATION APPROACH

The relationship between the reliability of coverage over a circular area and the reliability of coverage on the perimeter of the circle was first established by D.O. Reudink (circa 1974) [2]. The main finding of this study was that cell area reliability and cell edge reliability obey the simple relationship illustrated in Figure 3. As long as the propagation path follows a power law this relationship is completely determined by the ratio of σ/B, where σ is the standard deviation of the lognormal fading within the cell and B is the path loss exponent (e.g., typical values are σ=8dB and B=35.2). As shown in Table 1, given exact knowledge of σ and B, the cell area reliability (and cell edge reliability) can be exactly computed (see also equation (a7) in the Appendix). Note that although 75% cell edge reliability approximately corresponds to 90% cell area reliability, and 90% cell edge reliability approximately corresponds to 97% cell area reliability, their exact values depend on the propagation parameters of each cell (i.e., σ and B)

σ	6	6	8	8	10	10
B	35	30	40	35	30	35
Edge Reliability	75%	90%	75%	90%	75%	90%
Area Reliability	91.65%	96.87%	90.72%	96.57%	87.4%	96.03%

Table 1. The relationship between area and edge reliability for various propagation parameters σ and B.

	Example 1	**Example 2**
Edge Reliability	75%	75%
σ	8	10
B	40	30
Area Reliability at 75% edge reliability	90.72%	87.4%
Radius at 75% edge reliability	1 km	0.9005 km
Radius at 90% area reliability	1.021 km	0.803 km
Edge reliability at 90% area reliability	73.5%	79.5%

Table 2. The relationship between cell area reliability, cell edge reliability and cell radius for different propagation parameters σ and B and the same transmit power.

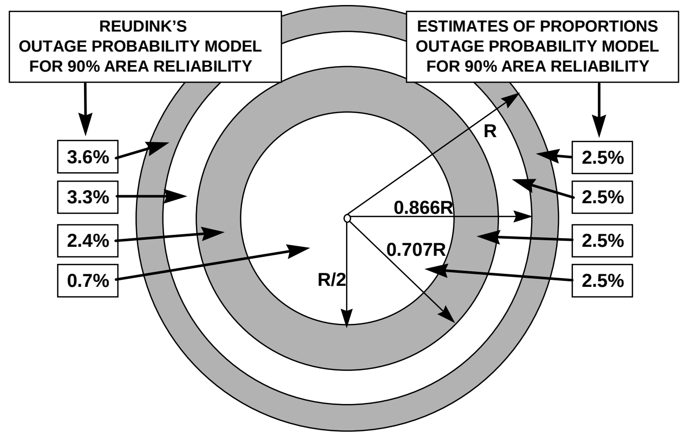

Figure 4. Four equal area portions of a cell and their corresponding outages for two probability models: Reudink's method assumes a linear path loss. The Estimate of Proportions method implicitly assumes no path loss. That is, outages directly under the base station and outages at the cell edge are equally likely. Reudink's path loss model is clearly a more valid assumption.

The relationship in Figure 3 is apparently independent of the absolute cell radius, as well as being independent of the transmit and receive power, which only serve to scale the radius. This seems to uncouple the problem of determining the coverage reliability from the problem estimating the size of a cell. However, the relationship in Figure 3 does not mean that the cell radius has no effect on coverage reliability. On the contrary, for the same two-way gain and transmit power, making the cell radius larger reduces the coverage reliability and decreasing the cell radius increases the coverage reliability. The dependency on cell radius is implicit through the desired edge reliability and equation (5). Table 2 demonstrates some of the relationships between cell radius, cell area reliability, and cell edge reliability for two cells designed with the same transmit power. In this table, the cell radius is computed from equation (5) and the area reliability is computed from equation (a7). For example 1, the cell radius at 75% edge reliability (90.72% area) is 1 km, the radius at 73.5% edge reliability (90% area) is 1.021 km. The results are similar for example 2. Observe that changing the cell radius can have a significant effect on the reliability of RF coverage.

In Reudink's original derivation of area reliability, the explicit dependence of coverage on cell radius was purposely eliminated. Since the cell radius is one of the estimated quantities of interest in this paper, it is reintroduced into Reudink's expression in the Appendix in equation (a7). This is the formula (i.e., $\hat{F}_u$) that is used throughout this paper to estimate the reliability of RF coverage over a circular area.

Thus, the approach for measuring area reliability in this study is as follows:

1) measure the propagation parameters $\hat{A}'$, $\hat{B}$, and $\hat{\sigma}$ for each cell via linear regression

2) use these parameters to estimate the cell radius $\hat{R}$ from equation (5)

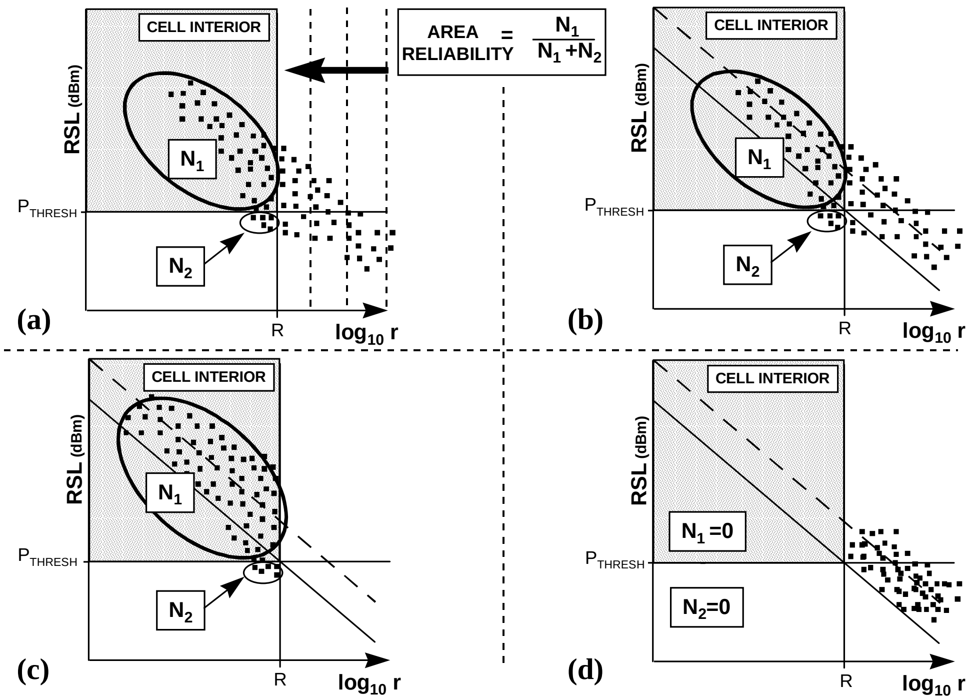

Figure 5. Comparison of area reliability estimators for three different drive test scenarios (a) Graphical approach to estimating the cell radius with the Estimate of Proportions method: the radius is reduced until the desired area reliability is reached **(b)** Most typical drive test; half of the signal strength measurements are within the cell; half of the signal strength measurements are outside the cell **(c)** Least likely(ideal) drive test; all of the signal strength measurements are within the cell **(d)** Worst case drive test; all of the signal strength measurements are outside the cell. The area reliability estimate based on Reudink's method can utilize the measurements outside the cell

3) use the radius and the propagation parameters to estimate the reliability of coverage, $\hat{F}_u$, over the cell area using Reudink's expression (see equation (a7)).
In the following section, this method of estimating coverage reliability is shown to be much more accurate than current approaches [3].

V. ESTIMATE OF PROPORTIONS COMPARED TO REUDINK'S APPROACH

In this section, the following two area reliability estimators are compared:

1) Reudink's approach with linear regression. We will refer to this technique as Reudink's method, even though his original study did not address the use of linear regression, or the significance of his results to verification [2].

2) Estimate of Proportions [3].

It is worthwhile to compare the path loss models that underlie both of these estimators. To facilitate this comparison, it is assumed that the measurements are independent and uniformly distributed throughout the area of the cell, both in range and in azimuth. Figure 4 shows the distribution of outages for Reudink's method, which assumes a linear path loss. The cell is divided into four equal area regions and outage probabilities for each region are generated, using equation (a7), for a typical 90% cell area reliability design. In Reudink's technique the median path loss is adaptively computed via linear regression within each cell. Observe that, as the cell edge is approached, the outage probability increases.

In contrast, the underlying assumption of the Estimate of Proportions technique is that outages are equally distributed throughout each cell, as also shown in this figure. The implicit assumption of the Estimate of Proportions approach is that there is no path loss. Specifically, outages at the cell edge and outages under the base station are equally likely events. Reudink's path loss model is clearly a more appropriate choice and this is the major reason that Reudink's area reliability estimates are always more precise.

The basic approach of the Estimate of Proportions method is illustrated in Figure 5(a). The cell radius is determined iteratively by reducing the radius until the desired area reliability (equation (6)) is reached. The precision of this method is compared with Reudink's technique in the analysis that follows.

Consider the following estimate of area reliability made by calculating the proportion of signal strength values that are above a quality threshold

$$\hat{F}_N = \frac{N_1}{N} = \frac{N_1}{N_1 + N_2} \tag{6}$$

where
N is the total number of signal strength measurements within the cell

N_1 is the number of signal strength values in the cell above P_{THRESH}

N_2 is the number of signal strength values in the cell below P_{THRESH}

Provided both NF_u>5 and $N(1-F_u)$>5, $\hat{F}_N$ is approximately a Normal random variable with a mean of F_u and a standard deviation as indicated in the following equation:

$$\hat{F}_N \sim N\left(F_u \ , \ \sqrt{\frac{F_u(1-F_u)}{N_1+N_2}} \right) \qquad (7)$$

where F_u is the true area reliability (this equation may be found in reference [3]). The error, ΔF_N, (one sided 95% confidence interval) of this coverage estimate is

$$\Delta F_N = 1.645 \sqrt{\frac{F_u(1-F_u)}{N_1+N_2}} \qquad (8)$$

The application of this expression to cellular verification is somewhat anomalous since this equation is completely independent of the amount of lognormal fading, σ, in the cell. This suggests that coverage estimation needs no more measurements in hilly terrain than for flat terrain, which is inconsistent with reality.

The precision of Reudink's method is empirically determined from the simulation results in section VII and expressed in equation (15). The relative precision of the area reliability estimates can be directly compared by dividing equation (8) by equation (15):

$$\frac{\Delta F_N}{\Delta F_u} \approx \left. \frac{1.645 \sqrt{\frac{F_u(1-F_u)}{N_1+N_2}}}{\frac{(0.1143\sigma + 0.2886)F_u^2(1-F_u)}{\sqrt{N}}} \right]_{N=N_1+N_2}$$

$$\boxed{\frac{\Delta F_N}{\Delta F_u} \approx \frac{1}{(0.0695\sigma + 0.1754)\sqrt{F_u^3(1-F_u)}}} \qquad (9)$$

where it is also assumed that Reudink's method is not using any signal strength samples outside of the cell (i.e., $N = N_1 + N_2$, Figure 5(c)). Given the above assumptions, Equation (9) can be shown to be a completely general expression, provided $6 \le \sigma \le 10$, $F_u \ge 90\%$ and $N \ge 100$. Equation (9) is independent of the number of signal strength samples, N, and dependent only on the area reliability and the lognormal fading within the cell, σ. The exact value of σ is unimportant to the point of this comparison; assume that σ=8 dB. Both area reliability estimators can be directly compared for the following two values of reliability F_u=90% and F_u=97%:

$$\left. \frac{\Delta F_N}{\Delta F_u} \right]_{\substack{\sigma=8 \\ F_u=0.90}} \approx 5.07 \quad \text{and} \quad \left. \frac{\Delta F_N}{\Delta F_u} \right]_{\substack{\sigma=8 \\ F_u=0.97}} \approx 8.27$$

By substituting different values of σ into equation (9), the reader can easily verify that Reudink's area reliability estimate is always more than four times the precision of the Estimate of Proportions approach.

Alternatively, Reudink's approach requires fewer measurements to achieve the same area reliability accuracy as the Estimate of Proportions technique. For the same assumptions as above, and using equation (9), the Estimate of Proportions method requires :

1) 26 times (=5.07^2) as many points as Reudink's method for a 90% area reliability design.

2) 68 times (=8.27^2) as many points as Reudink's method for a 97% area reliability design.

Hence, Reudink's method makes much better use of a finite set of signal strength measurements.

Figures 5(b), 5(c), and 5(d) show typical distributions of signal strength measurements within single cells. All three drive test scenarios in this figure are possible. The best scenario (and least likely) is in Figure 5(c) where all of the measurements are within the cell.

Reudink's method can also be used to predict coverage of sparsely driven areas. Often, during the drive data collection phase, the intended coverage area is not exactly known. Figures 5(b) and 5(d) have a significant number of measurements outside the cell. Reudink's method easily exploits this data, and thus can be very useful in optimizing cellular handoff performance.

The best solution to the scenario in Figure 5(d) is to retest the cell. However, this is not always possible and for these cases a tuned prediction of the coverage may be desired. Reudink's approach is ideal for this application.

In summary, it was shown that Reudink's area reliability estimator is more than four times the precision of approaches that are based on the Estimate of Proportion method. Equivalently, Reudink's technique requires an order of magnitude fewer measurements to achieve the same accuracy as the Estimate of Proportions method.

VI. RF PROPAGATION SIMULATION RESULTS

To test the validity of the radius estimation and area reliability estimation approaches, an RF propagation simulation was written, as shown in Figure 6. To reduce the computation, we use a single radial component that is uniformly sampled along its length. It is assumed that the simulated measurements result from a uniform azimuthal sampling of the cell. Hence, the samples along this single radial represent the composite path

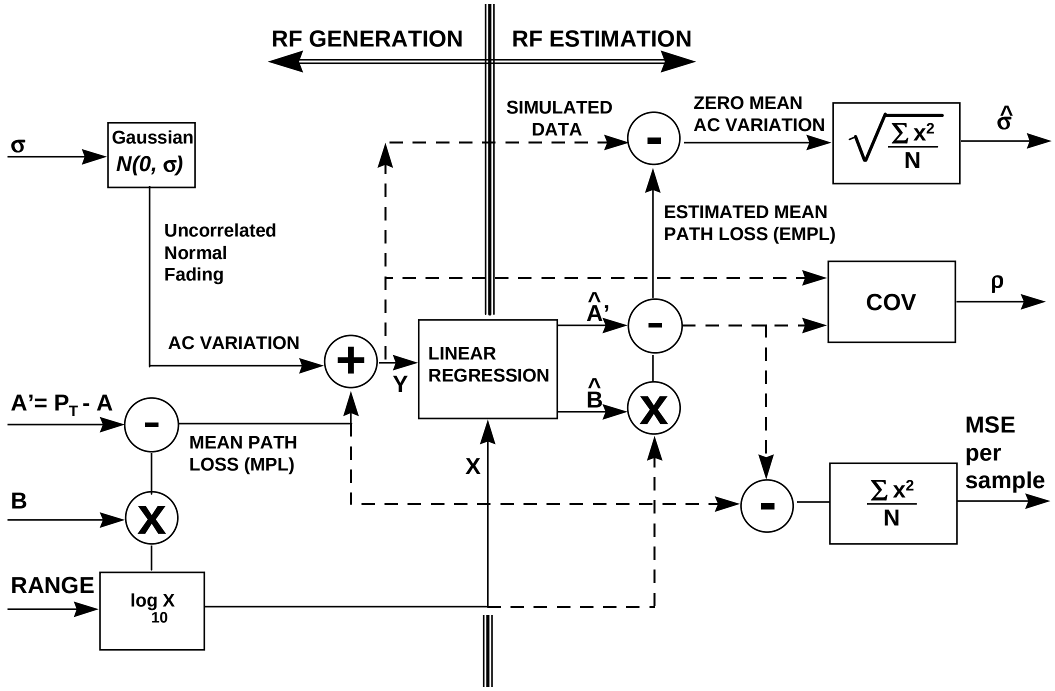

Figure 6. Block diagram of RF propagation simulation. The inputs to the simulation are *A'*, *B*, and σ. The corresponding outputs are estimated as shown above. The mean-square-error per sample between the best fitting line and the true mean path loss is used to measure the performance of the estimation process. The correlation coefficient, ρ, of the best fitting line with the data is also computed.

loss fluctuations of radials in all azimuth directions. The single radial of this model is thus considered to be a linear superposition of multiple radials that are uniformly spaced in azimuth (i.e., two-dimensional uniform area sampling). The simulation could explicitly calculate this superposition, but this is computationally inefficient and would have no effect on the results.

For a fixed azimuth angle, the fading is correlated along the radial from the base station. However, we assume that the distance between the measurements on a radial is large enough to neglect correlation effects. In addition, the fading between radial components at equally spaced azimuth angles is nearly uncorrelated. Since the regression is actually evaluated across all azimuth angles simultaneously, an uncorrelated Gaussian fading model is chosen. The standard deviation, σ, (typically 5-10 dB) is input into a Gaussian random number generator which produces uncorrelated normal random variables with zero mean and variance σ^2.

The mean path loss is computed for each range value as the product of the logarithm of range and the path loss coefficient, *B*, to which is added the intercept value *A'*. The variation due to fading is then added to the mean path loss (MPL) also shown in Figure 6. This concludes the RF signal generation portion of the simulation. The remainder of the simulation is concerned with estimating *A'*, *B*, and σ. Both $\hat{A}'$ and $\hat{B}$ are computed via linear regression. The estimated mean path loss is then subtracted from the simulated signal strength values and an estimate of the standard deviation, $\hat{\sigma}$, is made from the resulting zero-mean process. Two major criteria are used to evaluate the performance of the estimation procedure in the simulation:

1) the correlation coefficient, ρ, between the simulated data and the best fit line (EMPL). The closer ρ is to unity, the more linear the data. This measure is also used to characterize the reliability of the field data.

2) the mean square error (MSE) per sample between the best fit line (EMPL) and the true path loss in the simulation (MPL). This measure cannot be used in the field, since the true path loss is unknown.

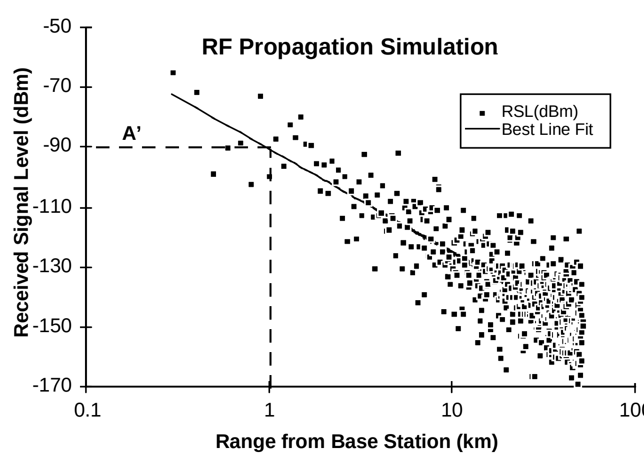

Figure 7. Simulated received signal strength versus distance from the base station and the best fitting linear approximation.

Typical results from the simulation are shown in Figure 7. The following parameters were the inputs used to generate the 515 data points in this figure:

A= 140

P_t= 50 dBm (EIRP)

hence

$A' = P_t - A$ =-90dBm

B= 35.2

σ = 10 dB

The corresponding outputs were

$\hat{A}'$ = -88.35

$\hat{B}$ = 36.35

$\hat{\sigma}$ =10.63 dB

ρ = 0.826 (correlation coefficient, where ρ =1 for a line)

MSE per sample = 0.239 dB

The value of ρ=0.826 is typical of that found in actual drive test data. The simulation estimated the input parameters very well since

A'- $\hat{A}'$=-1.65

B- $\hat{B}$=-1.15

σ- $\hat{\sigma}$=-0.63

MSE =0.239

The above four values can be made as close to zero as desired by increasing the number of simulated data points, *N*. Since these errors depend on the number of data samples used to compute the regression, a natural question is "How many data samples are necessary to achieve a given precision?" The accuracy of the measurement approach is examined in more detail in the next section.

VII. MEASUREMENT ACCURACY VERSUS THE NUMBER OF SAMPLES

This section deals with determining the measurement error of the overall estimation process. The simulation is used to determine the probability densities of the following two random variables:

$$e_{Fu} = \frac{F_u - \hat{F}_u}{F_u} \quad \text{and} \quad e_R = \frac{R - \hat{R}}{R} \tag{10}$$

where

e_{Fu} is the relative error of the area availability estimate as computed from equation (a7)

e_R is the relative error of the cell radius estimate

F_u is the true area reliability computed from equation (a7 $\hat{F}_u$ is the estimated area reliability computed from equation (a7)

R is the true cell radius computed from equation (5)

$\hat{R}$ is the cell radius estimate computed from equation (5)

The transformations specified by equation (10) allow a direct comparison of the cell radius estimate with the area reliability estimate, which would otherwise be difficult due to the differences in the dimensions of these two estimators (i.e., *R* is in kilometers and F_u is a percentage). Typical probability densities for e_{Fu} and e_R are shown in Figure 8. Observe that the error of the cell radius estimate, e_R, is comparable in Figures 8(a) and 8(b) (Note the scale change between the ordinates of these two figures). However, e_{Fu} is almost a factor of two smaller for the 90% cell edge reliability design. The evidence in Figure 8 and all of the histograms processed in this study demonstrate that both e_R and e_{Fu} are well modeled as zero-mean Normal random variables, and thus, only their respective variances are needed to characterize the precision of the estimates $\hat{R}$ and $\hat{F}_u$. These are determined empirically via Monte Carlo simulation.

We are interested in determining the inaccuracy, ΔR, of the estimate of the cell radius at a 95% confidence level. The inaccuracy is measured from empirical histograms by simulating e_R and determining ΔR such that

$$P\left(R - \Delta R \le \hat{R} \le R + \Delta R\right) = 95\%$$

The inaccuracy of the radius estimate, ΔR, is determined by the following two-sided test

$$c = F(z_c) = \frac{1}{\sqrt{2\pi}} \int_{-z_c}^{z_c} e^{-\frac{t^2}{2}} dt \tag{11}$$

where the z_c variable in equation (11) is chosen to yield the desired confidence level, *c*. For example, if *c*=95%, then z_c=1.96. Since e_R has a mean of zero, the corresponding two-sided normalized radius inaccuracy, δ_R , is

$$\pm \delta_R = \pm \frac{\Delta R}{R} = \pm 1.96\sqrt{VAR(e_R)} \tag{12}$$

where δ_R is a dimensionless percentage of the cell radius, *R*.

Likewise, the inaccuracy of the area reliability estimate, $\hat{F}_u$, is estimated from histograms of e_{Fu} and determining ΔF_u such that

$$P\left(\hat{F}_u \le F_u + \Delta F_u\right) = 95\%$$

Since e_{Fu} also has a mean of zero, the inaccuracy, ΔF_u, (one sided 95% confidence interval) of the coverage estimate is

$$\delta_F = \frac{\Delta F_u}{F_u} = 1.645\sqrt{VAR(e_{Fu})} \tag{13}$$

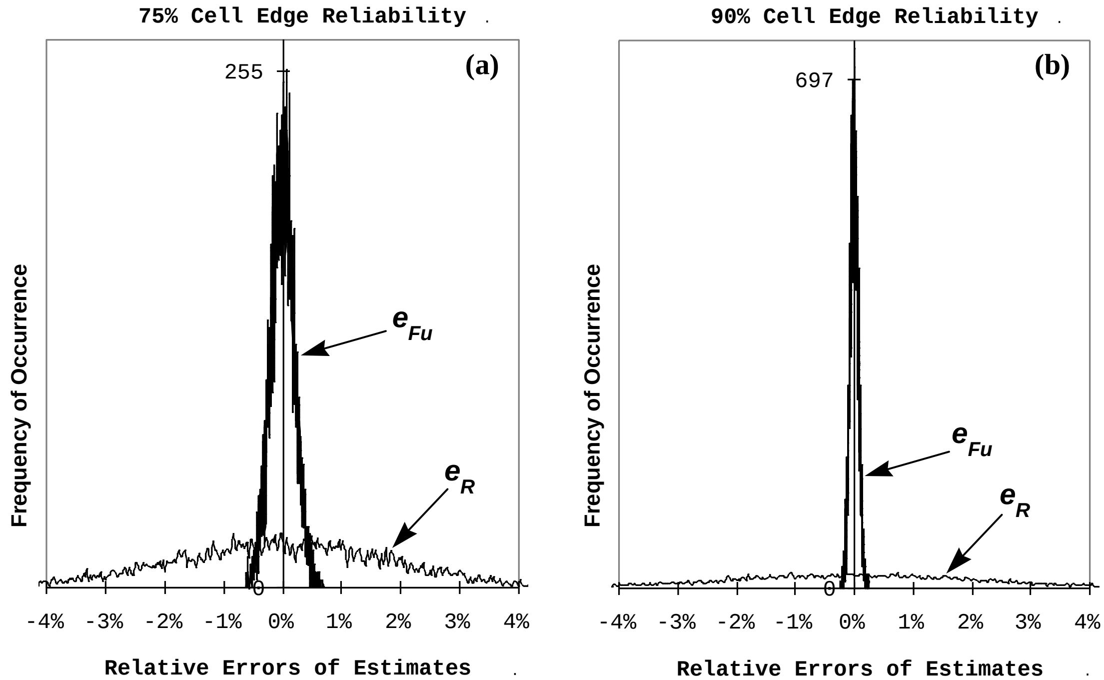

Figure 8. Histograms showing the simulated probability densities of the relative error e_{Fu} of the area availability estimate $\hat{F}_u$ and the relative error e_R of the cell radius estimate $\hat{R}$ for **(a)** a 75% cell edge reliability design **(b)** a 90% cell edge reliability design. The number of samples in the regression is 1000 and the standard deviation of the lognormal fading is $\sigma = 8$ dB.

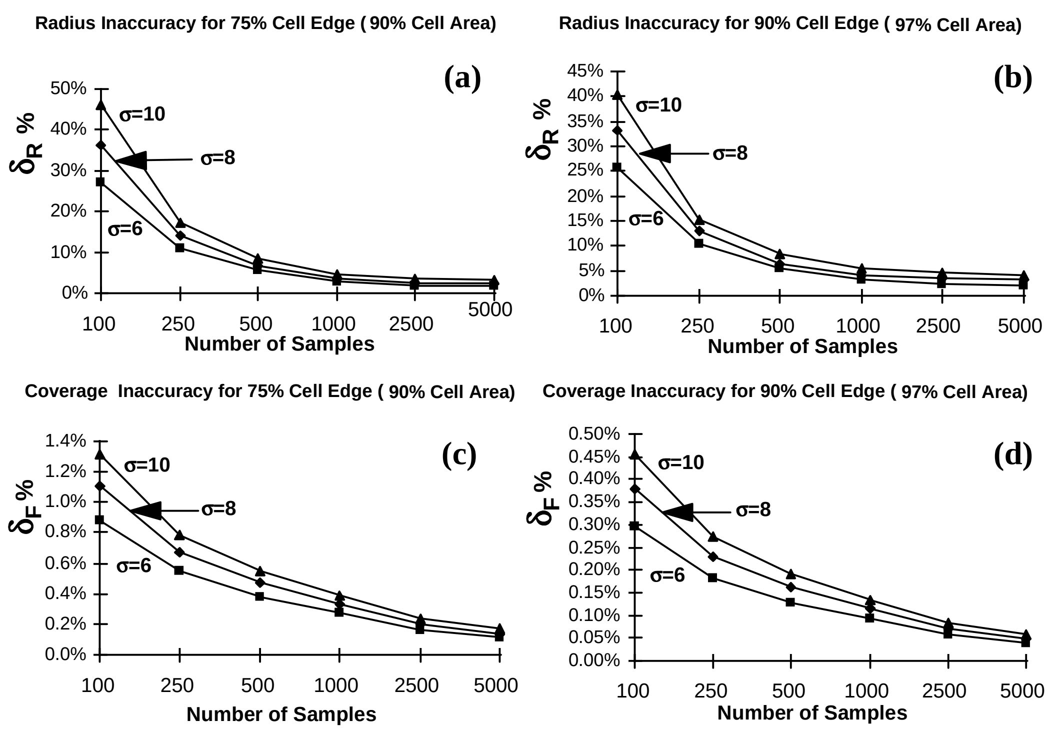

Figure 9. Simulated inaccuracy (95% confidence) of measurement techniques versus the number of samples in the regression: (a) cell radius estimate $\hat{R}$ of a 75% cell edge reliability design, **(b)** cell radius estimate $\hat{R}$ of a 90% cell edge reliability design, **(c)** area reliability estimate $\hat{F}_u$ of a 75% cell edge reliability design **(d)** area reliability estimate $\hat{F}_u$ of a 90% cell edge reliability design.

where δ_F is a dimensionless percentage of the area reliability, F_u.

Each point in the plots in Figure 9 represents the precision (at 95% confidence) that is obtained after simulating and processing five million signal strength values.

Close inspection of Figure 9 reveals that for a given number of signal strength samples, *N*, the area reliability is much more precise (by one to two orders of magnitude) than the estimate of the cell radius (compare Figure 9(a) with Figure 9(c)). For example, 1000 samples in the regression are needed for about a $\pm 3\%$ inaccuracy in the cell radius estimate. However, even with 500 samples in the regression, the area availability estimate is very precise. The inaccuracy of the area availability estimate is less than 0.5% for cells designed with 75% cell edge reliability and within 0.2% for cells designed with 90% cell edge reliability.

The inaccuracies of both of the estimates $\hat{R}$ and $\hat{F}_u$ can be approximated by the following expressions which were determined empirically (via least-squares) from the data in Figure 9

$$\delta_R = \frac{\Delta R}{R} \approx \frac{3.821\hat{\sigma} + 4.619}{N} \qquad (14)$$

$$\delta_F = \frac{\Delta F_u}{F_u} \approx \frac{(0.1143\hat{\sigma} + 0.2886)\hat{F}_u(1-\hat{F}_u)}{\sqrt{N}} \qquad (15)$$

where

N is the number of independent samples in the regression

$\hat{\sigma}$ is the estimated standard deviation of the lognormal fading in the cell

$\hat{F}_u$ is the estimated area reliability computed from equation (a7)

From equation (10), it is easy to show that equations (14) and (15) are completely general expressions, provided $6 \le \sigma \le 10$, $F_u \ge 90\%$ and $N \ge 100$. Observe that the area reliability inaccuracy in equation (15), ΔF_u, is inversely proportional to $\sqrt{N}$. It is interesting to compare the magnitudes of the inaccuracies of the above area reliability measurements with those computed by estimating a proportion of signal strength measurements that are above a desired threshold. From equation (8), for 500 samples and 90% cell edge reliability, the inaccuracy is about 2.2%. Thus, the area availability estimate, $\hat{F}_u$, shown in Figure 9(d) is about ten times the precision of estimates that are based on proportions of signal strengths (i.e., equation (6)).

The radius inaccuracy in equation (14), δ_R, is inversely proportional to the number of samples in the regression, *N*, and directly proportional to the amount of lognormal fading, σ, in the cell. Interestingly, radio survey engineers have long recognized the negative effects that widely varying terrain and clutter environments have on RF coverage tests. They usually compensate for these effects by taking many more measurements in these areas. Equation (14) is simply the mathematical expression of this practice, specifying the relationship between the desired coverage inaccuracy, δ_R, the number of independent signal strength measurements, *N*, and the terrain fading factor within the cell, σ. It should be noted that for real data, the radius inaccuracy will actually be less than specified by equation (14), since signal strength samples of adjacent bins are not completely independent. The fact that adjacent samples are correlated actually reduces the error of the cell radius estimate making equation (14) an upper bound.

The most important finding of this analysis is that it is the precision of the estimate of the cell radius (i.e., equation (14)) that is the limiting factor in determining the quality of RF coverage, not the precision of the area reliability estimate.

VIII. DISCUSSION

Given the abundance of other quality metrics such as adjacent channel interference, cochannel interference, dropped calls, hand-off failures, call access failures, bit error rate, frame error rate, etc., the experienced cellular engineer might question our focus on coverage. We justify our approach by arguing that, at least from a design perspective, degradations in these other performance metrics are simply a result of either not enough carrier power or too much interference power. Furthermore, we submit that all extraneous same-system interference is simply uncontrolled other-cell coverage which would not exist if the geographic extent of reliable coverage in each cell was properly designed in the first place. Hence, coverage estimation fundamentally remains the most critical step of the design of any network.

An accurate method of determining the radius ($\hat{R}$) of individual cells was presented. This led to an even more precise technique for estimating the reliability of coverage over the area of the cell ($\hat{F}_u$). For the same precision in area reliability, the Estimate of Proportions technique requires more than twenty times as many measurements than Reudink's method ($\hat{F}_u$). Thus, Reudink's area reliability estimator has some computational advantages in verification post-processing.

It was shown that it is possible to obtain an excellent estimate of the area reliability even if the number of samples is insufficient for estimating the cell radius. This raises an interesting question concerning the determination of service reliability: "What is the best metric to use in classifying the quality of RF coverage?" This study indicates that area reliability alone is insufficient. A major finding of this study is that the area reliability (F_u, equation (a7)) and the cell radius (*R*, equation (5)) are an equation pair. In RF verification, it is not possible to determine an area reliability without simultaneously computing a cell radius, and vice versa.

The results of this paper indicate that estimating the effective radius of a cell is the limiting factor in determining the RF coverage reliability. Specifically, it takes about fifty times as many signal strength samples to estimate the cell radius, *R*, than to estimate the area reliability, F_u. For a given propagation environment, computing the distance to the cell edge is deterministic (i.e., apply equation (5)). For real drive test data, the true cell radius is unknown and must be statistically estimated. It was shown that as long as the radius estimate is sufficiently precise, so is the area reliability estimate ($\hat{F}_u$).

If cell edge reliability is the desired coverage criterion, an accurate estimate of the cell radius is all that is needed since the cell edge reliability is ensured by the fade margin used to measure the radius.

However, if area reliability is the desired coverage criterion, then a minor adjustment to the cell edge reliability (and cell radius) must be made in each cell to compensate for the variation in the specific values of the propagation constants *A'*, *B*, and σ. This is easily done by first computing the propagation constants via linear regression and then computing the area reliability from equation (a7) at fine range increments (e.g., steps of $\Delta R/2$). The desired radius can then be found by inverse interpolation.

The proposed technique is best suited for macrocells. However, it can be modified to work equally well in microcells by eliminating measurements that have line-of-sight to the base station. In many cells, the path loss is better described by a composite of two line segments that intersect at some breakpoint distance near the base station. For these cases, this distance is approximated and measurements before this point are eliminated since virtually no outages occur over this region. The best linear fit to the path loss in the outer regions of the cell is extrapolated all the way to the base station. For most propagation scenarios, the error in the area reliability estimate due to this approximation is less than 0.5% (see Figure 4) and less than 1.5% for the radius estimate. Thus, the proposed method needs no additional parameters or modifications to accommodate dual-power law propagation environments. However, if these errors are not tolerable, they can easily be eliminated by incorporating breakpoint distance into Reudink's expression (equation (a7)).

Although the precision in this study was determined via simulation, we have processed signal strength measurements from hundreds of cell sites and have found that the results are completely consistent with those of our simulation.

We have also found cell radius inaccuracy to be very useful in determining the sampling requirements of cellular drive tests [5][6][7].

This verification approach is particularly useful to anyone involved with cell planning since this equates the problem of determining the reliability of RF coverage with that of determining the effective size of the cell. The latter concept is clearly more useful to the cellular network planner.

IX. CONCLUSIONS

Cell radius inaccuracy has been proposed as a new method for measuring RF cellular coverage. The technique measures the average distance from the base station to the cell edge (equation (5)) and quantifies the precision by also specifying the uncertainty of the radius estimate. In addition, the approach provides an estimate of the area reliability (equation(a7)) which was shown to be much more accurate than current methods that estimate the coverage from a proportion of signal strength measurements [3]. Empirical formulas are given that approximate the precision of both of these estimates (equations (14) and (15)).

The results of this paper show that area reliability is more useful in specifying a desired quality of RF coverage than in verifying that this quality is actually achieved.

The recommended verification technique, cell radius inaccuracy, uses linear regression to estimate the minimum mean square path loss within each cell, and is thus very tolerant to estimation errors due to terrain fluctuations (e.g., lognormal fading). The approach provides the best circular approximation to any equal power contour, at any desired reliability. Thus, this method is ideal for cell site planning with any wireless technology. For example, using standard CW drive test measurements, this technique can help verify that the RF design meets the proper amount of overlap in coverage needed to support the soft handoff regions of CDMA.

The conclusion of this study is that cell radius estimation and area reliability estimation should not be treated separately, and that cell radius inaccuracy is the more critical verification measure.

ACKNOWLEDGMENTS

This paper is dedicated to the memory Dr. Charles W. Bernardin and Elizabeth J. Bernardin.

The authors would like to thank the management of NORTEL Wireless Engineering Services for providing the funding and the environment necessary for this research. This study originated after several useful discussions about coverage estimation with Dr. Ahmad Jalali and Martin Kendal. We would like to acknowledge Dr. Richard Tang for originally suggesting the regression approach. Also, we would like to thank Professor Venugopal Veeravalli for his invaluable suggestions concerning the error performance of the area reliability estimation approach included here. We are especially grateful to Professor Veeravalli for performing the rigorous analysis presented in the Appendix that follows. We would also like to thank Dr. Sudheer Grandhi, Pulin Patel, Dr. Reid Chang, Muheiddin Najib, Mazin Shalash, and Mark Prasse for taking the time to critically evaluate this manuscript.

APPENDIX

AREA RELIABILITY AS A FUNCTION OF CELL RADIUS

This derivation is similar to D.O. Reudink's original analysis, which showed that the relationship between cell edge reliability and cell area reliability was theoretically independent of the absolute cell radius [2]. However, since cell radius is one of the estimated quantities of interest in this study, this dependency is purposely reintroduced.

Let the received power, P_r, at the edge of a cell, R, be given by

$$P_r(R) = A' - B\log_{10} R + X \qquad \text{(a1)}$$

where X is a normal zero mean random variable with variance σ^2.

Similarly, the received power at a distance, r, is

$$P_r(r) = A' - B\log_{10} r + X \qquad \text{(a2)}$$

where it will be assumed that $r<R$. The outage probability, $P_{out}(r)$, at a particular range, r, from the base station is given by

$$P_{out}(r) = P(A' - B\log_{10} r + X \le P_{THRESH})$$

$$= F\left(\frac{P_{THRESH} - A' + B\log_{10} r}{\sigma}\right)$$

$$= 1 - Q\left(\frac{P_{THRESH} - A' + B\log_{10} r}{\sigma}\right) \qquad \text{(a3)}$$

and the corresponding service reliability on a circular contour of radius, r, is

$$1 - P_{out}(r) = P(A' - B\log_{10} r + X > P_{THRESH})$$

$$= 1 - F\left(\frac{P_{THRESH} - A' + B\log_{10} r}{\sigma}\right)$$

$$= Q\left(\frac{P_{THRESH} - A' + B\log_{10} r}{\sigma}\right) \qquad \text{(a4)}$$

where P_{THRESH} is the desired threshold and

$$Q(x) = 1 - Q(-x)$$
$$Q(x) = 1 - F(x)$$
$$F(x) = P(\xi \le x)$$
$$\xi \approx N(0,1)$$

$$F(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt$$

$$\frac{d}{dx}Q(x) = -\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

Define $a = \dfrac{P_{THRESH} - A'}{\sigma}$ and $b = \dfrac{B\log_{10} e}{\sigma}$

Then from equation (a4), the service reliability on a circular contour of radius, r, is

$$1 - P_{out}(r) = Q(a + b\ln r) \qquad \text{(a5)}$$

The fraction of usable area, F_u, (i.e., area reliability) within the cell can be found by integrating the contour reliability across range

$$F_u = \frac{1}{\pi R^2}\int_0^R [1 - P_{out}(r)]\; 2\pi r dr$$

$$= \frac{2}{R^2}\int_0^R Q(a + b\ln r)\; r dr \qquad \text{(a6)}$$

Now consider the integral

$$\int_0^R Q(a + b\ln r)\; r dr$$

set

$$t = a + b\ln r$$
$$r = e^{\frac{t-a}{b}}$$
$$dr = \frac{e^{\frac{t-a}{b}}}{b}dt$$

Thus

$$\int_0^R Q(a + b\ln r)\; r dr$$

$$= \int_{t=-\infty}^{a+b\ln R} Q(t)\; e^{\frac{t-a}{b}}\;\frac{e^{\frac{t-a}{b}}}{b}dt$$

$$= \frac{1}{b}\int_{-\infty}^{a+b\ln R} Q(t)\; e^{\frac{2(t-a)}{b}}\; dt$$

$$= \frac{e^{\frac{-2a}{b}}}{b}\int_{-\infty}^{a+b\ln R} Q(t)\; e^{\frac{2t}{b}}\; dt$$

Now

$$\int_{-\infty}^{a+b\ln R} Q(t)\; e^{\frac{2t}{b}}\; dt \;=$$

$$Q(t)\frac{be^{2t/b}}{2}\bigg|_{-\infty}^{a+b\ln R} + \int_{-\infty}^{a+b\ln R} \frac{b}{2\sqrt{2\pi}} e^{-\frac{t^2}{2}}\; e^{\frac{2t}{b}}\; dt$$

$$= Q(a+b\ln R)\frac{b}{2}e^{\frac{2(a+b\ln R)}{b}}$$

$$+\frac{be^{\frac{2}{b^2}}}{2\sqrt{2\pi}}\int_{-\infty}^{a+b\ln R} e^{-\frac{1}{2}\left(t^2-\frac{4t}{b}+\frac{4}{b^2}\right)}dt$$

$$= Q(a+b\ln R)\frac{b}{2}e^{\frac{2a}{b}}R^2$$

$$+\frac{b}{2}e^{\frac{2}{b^2}}\int_{-\infty}^{a+b\ln R} \frac{e^{-\frac{1}{2}\left(t-\frac{2}{b}\right)^2}}{\sqrt{2\pi}}\; dt$$

$$= Q(a+b\ln R)\frac{b}{2}e^{\frac{2a}{b}}R^2$$

$$+\frac{b}{2}e^{\frac{2}{b^2}}\left[1-Q\left(a+b\ln R-\frac{2}{b}\right)\right]$$

Thus, the area reliability is

$$F_u = \frac{2}{R^2}\int_0^R Q(a+b\ln r)\;\; rdr$$

$$= \frac{2}{R^2}\frac{e^{\frac{-2a}{b}}}{b}\left[\frac{R^2 b}{2}e^{\frac{2a}{b}}Q(a+b\ln R)\right]$$

$$+\frac{2}{R^2}\frac{e^{\frac{-2a}{b}}}{b}\left[\frac{b}{2}e^{\frac{2}{b^2}}\left(1-Q\left(a+b\ln R-\frac{2}{b}\right)\right)\right]$$

And finally,

$$F_u = Q(a+b\ln R) + \frac{e^{\frac{-2a}{b}+\frac{2}{b^2}}}{R^2}\left[1-Q\left(a+b\ln R-\frac{2}{b}\right)\right] \quad \text{(a7)}$$

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Pete Bernardin (M' 90) received a B.S. in mathematics from Villanova University in 1973 and a Ph. D. in biomedical engineering from The Johns Hopkins University in 1979. His graduate work included research in the auditory system, applying mathematics and digital signal processing to the neural processes of hearing. He has had over 20 years experience in solving engineering and mathematical problems with digital methods. He has worked in the Radar and Digital Systems division of Texas Instruments, and the Advanced Technology Lab of RCA. He has worked as an image processing consultant for NASA, as a digital signal processing staff specialist at the Tandy Corporation and as a telecommunications engineer at Bell Northern Research. Currently, he is a manager in the Wireless RF Engineering Services group of Northern Telecom, Richardson, Texas.

Dr. Bernardin is the author of two books concerning numerical computation for engineering and scientific applications. He is also the author of several research papers, covering a broad range of technical areas which include image processing, radar, spectral estimation, and communications.

Meng F. Yee (M' 89) received a B. Sc. (Hons) in Electronic Engineering from University of Birmingham, England in 1975, a Master of Business Administration (MBA) from University of Warwick, England in 1992 and a Master of Engineering from the University of Toronto, Canada in 1994.

He has held numerous engineering and operations positions for Malaysian Telecom, providing both wireline and wireless network infrastructure throughout Malaysia. He has also held positions in Strategic Technology and Planning with Rogers Cantel, the largest cellular operator in Canada. In this position he participated in both the TIA and CTIA microcellular committees. Mr. Yee was also a member of the CTIA WBSS (Wideband Spread Spectrum) committee in 1992, which was formed to evaluate the use of spread spectrum technology for cellular applications.

Mr. Yee is currently a Senior Manager in CDMA RF Design and Technology Applications with NORTEL (Northern Telecom), Richardson, Texas and works on CDMA network optimization and RF performance.

Thomas Ellis has thirteen years experience, designing and implementing long haul and wireless systems. Additionally, he has several years experience developing software solutions for numerous communications and engineering problems. From 1985 to 1992 he worked for MCI Telecommunications, designing and implementing long haul microwave and fiber optic systems, as well as designing new two-way UHF systems for field engineering support. He joined BellSouth Enterprises in 1992 as a Senior Systems Engineer in their Worldwide Wireless International Engineering group. While at BellSouth, he was one of the initial system engineers that planned, designed and developed the technology that is currently known as BellSouth DCS. From 1993 to 1995 he was a consulting engineer with Mobile Systems International (MSI). In 1995 he left MSI and formed Wireless Software Design & Consulting. Since then he has been an engineering consultant to Nortel's CDMA Engineering group, and was the lead software developer on their CDMA and TDMA RF Optimizer tools.