1. State whether the following statements are true or false giving reasons or counterexamples in each case:

(a) An LP: \( \min cx \mid x \geq 0; Ax = b \) can be solved using graphical methods with a small amount of preprocessing if \( A \) is \( m \times n \) with \( n = m + 2 \) and has rank \( m \). In general, problems with two variables (having inequalities or equalities) can be solved graphically (i.e. pictorially)

(b) If in the above LP \( n = m + 1 \), then the number of feasible bases is at most 2.

(c) If a variable leaves the basis then it can not reenter at the next iteration.

(d) The value of the objective function changes iff the step is non-degenerate.

(e) If the original data are integral and the pivot elements are always 1 then all the canonical forms remain integral.

2. Consider the canonical form as discussed in class except that for now the \( b_i \) need not all be positive. Suppose \( \bar{c}_j < 0 \) for some nonbasic variable \( j \) and \( \bar{a}_{i,j} < 0 \) for all \( i \). Can we assert the LP has a feasible solution but no optimal solution?

3. Suppose we have a feasible canonical form. Let \( \bar{c}_s < 0 \) be the only nonbasic variable with this property. Can we assert: (i) we have only one step until termination; (ii) this variable will be positive in any optimal solution to the LP (assuming one exists).

4. How does one identify a redundant equation in the simplex method?

5. Do problem 2 of assignment #1 using the revised simplex method with the explicit inverse as well as the product form.

6. Challenge Problem: [Not to be turned in; Refer to Chvatal’s book] Klee—Minty Example: This shows that the simplex algorithm can be exponential. This example is found in Chvatal’s book. Consider the problem:

\[
\max \sum_{j=1}^{n} 10^{n-j} x_j \\
(2 \sum_{j=1}^{i-1} 10^{i-j} x_j) + x_i \leq 100^{i-1}, 1 \leq i \leq n \\
x_j \geq 0 \forall j.
\]

Solve the above problem for \( n = 3 \); Show the following by induction on \( n \):
(i) after $2^{n-1} - 1$ steps the objective row reads:

$$z = 10(100^{n-2} - \sum_{j=1}^{n-2} 10^{n-1-j}x_j - s_{n-1}) + x_n$$

(ii) after $2^{n-1}$ steps the same row reads:

$$z = 90.100^{n-2} + 10(\sum_{j=1}^{n-2} 10^{n-1-j}x_j + s_{n-1}) - s_n$$

(iii) after $2^n - 1$ steps this row reads:

$$z = 100^{n-1} - \sum_{j=1}^{n-1} 10^{n-j}x_j - s_n$$

(iv) after each step all the coefficients are integral for the objective row.