Assignment #1:
Due January 28

1. Which pairs of statements of the type \( f(n) = A(g(n)) \) are compatible/incompatible? Here \( A \in \{ \Theta, \Omega, O, o \} \)

2. Does the statement \([ f(n) = O(g(n)) ]\) imply the statement \([ 2^{f(n)} = O(2^{g(n)}) ]\)? Is the converse true? Give proofs.

3. Let \( f(n) = n^{\log_b a}(\log n)^k; b > 1; a \geq 1 \). Which of the following statements are true:
   
   (a) \( f(n) = O(n^{\log_b a-\epsilon}) \) for some \( \epsilon > 0 \)
   
   (b) \( f(n) = \Theta(n^{\log_b a}) \)
   
   (c) \( f(n) = \Omega(n^{\log_b a+\epsilon}) \) for some \( \epsilon > 0 \)

4. Exercises 3.2-4; 3.2-5

5. Problems: 3-1; 3-2. Show all work justifying your answers.

6. Challenge Problems: Do not turn it in. No answers will be provided.
   
   (a) Show that \( \sum_{i=1}^{n} i^k = \Theta(n^{k+1}) \) for any positive integer \( k \).
   
   (b) Show that \( \sum_{i=1}^{n} \frac{1}{i} = \Theta(\log n) \)