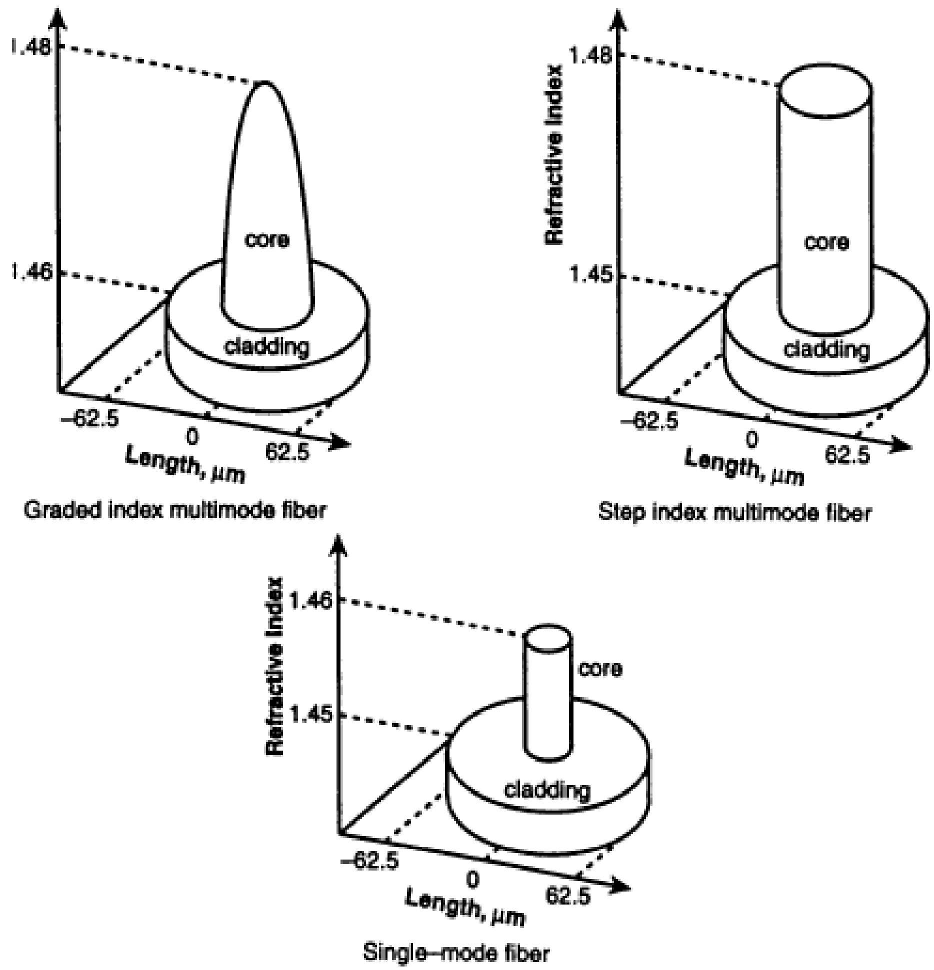
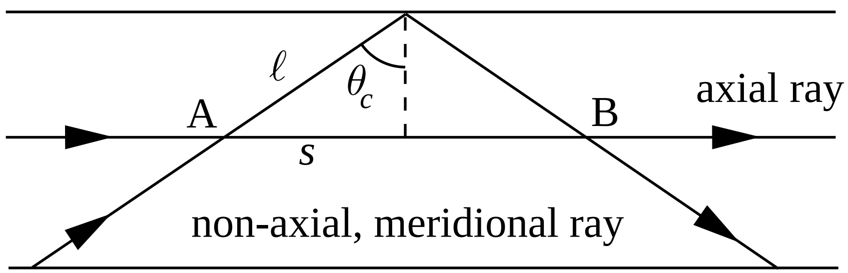
TYPES OF OPTICAL FIBER



INTERMODAL DISPERSION IN MULTIMODE FIBER (1)



- $\ell = \dfrac{s}{\sin\theta_c}$ where θ_c = critical angle and $\sin\theta_c = \dfrac{n_{\text{clad}}}{n_{\text{core}}}$
- For a fiber of length L, the difference in arrival times is

$$\Delta t = \frac{n_{\text{core}}L}{c}\left(1 - \frac{1}{\sin\theta_c}\right)$$

$$= \left(\frac{n_{\text{core}}L}{c}\right)\left(\frac{n_{\text{core}}}{n_{\text{clad}}}\right)\Delta$$

where $\Delta = \dfrac{n_{\text{core}} - n_{\text{clad}}}{n_{\text{core}}}$

INTERMODAL DISPERSION IN MULTIMODE FIBER (2)

- For a fiber of length L, the difference in arrival times of a pulse that travels along an axial ray and one that travels along a meridional ray at the critical angle is

$$\begin{aligned}\Delta t &= \frac{n_{\text{core}} L}{c}\left(1 - \frac{1}{\sin\theta_c}\right)\\ &= \left(\frac{n_{\text{core}} L}{c}\right)\left(\frac{n_{\text{core}}}{n_{\text{clad}}}\right)\Delta\end{aligned}$$

$$\text{where } \Delta = \frac{n_{\text{core}} - n_{\text{clad}}}{n_{\text{core}}}$$

▷ Numerical value for $n_{\text{core}} \approx n_{\text{clad}} \approx 1.5$ and $\Delta \approx 3 \times 10^{-3}$:

$$\frac{\Delta t}{L} \approx 15 \text{ ns/km}$$

INTERMODAL DISPERSION IN MULTIMODE FIBER (3)

- Requirement for minimal intersymbol interference:

$$B\,\Delta t < 1$$

where

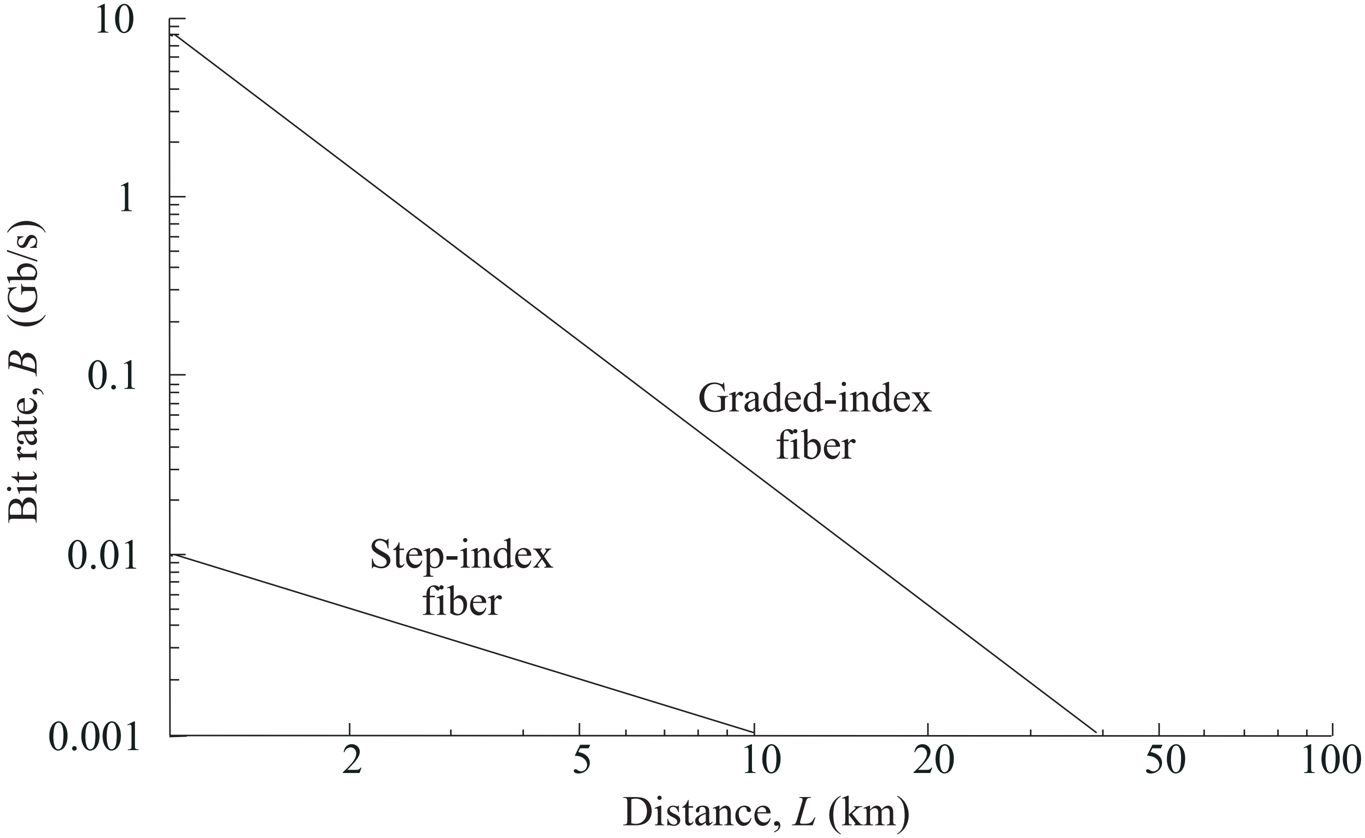
$$B = \text{bit rate}$$

▷ Maximum length bit-rate product:

$$BL < \frac{n_{\text{clad}}}{n_{\text{core}}^2}\frac{c}{\Delta}$$

▷ Numerical values for $n_{\text{core}} \approx n_{\text{clad}} \approx 1.5$:

- Clad multimode fiber ($\Delta \approx 3 \times 10^{-3}$): $BL < 67$ Mb-km/s
- Unclad multimode fiber ($\Delta \approx .33$): $BL < .4$ Mb-km/s


10
1
0.1
0.01
0.001
Bit rate, B (Gb/s)
Graded-index fiber
Step-index fiber
2
5
10
20
50
100
Distance, L (km)

GROUP VELOCITY DISPERSION (1)

- Difference in arrival times of waves at wavelengths λ and $\lambda + \Delta\lambda$:

$$\begin{aligned}\Delta t &= L \frac{d}{d\lambda}\left(\frac{1}{v_g}\right)\Delta\lambda \\ &= D L \,\Delta\lambda \\ D &= D_M + D_W\end{aligned}$$

  - ▷ Material dispersion: $D_M = \frac{1}{c}\frac{dn_{2g}}{d\lambda} = -\frac{2\pi}{\lambda^2}\frac{dn_{2g}}{d\omega}$
  - ▷ Waveguide dispersion: $D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega} - V\frac{d^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{dVb}{dV}\right]$

- Requirement for minimal intersymbol interference:

$$B\,\Delta t < 1 \Rightarrow BL|D|\Delta\lambda < 1$$

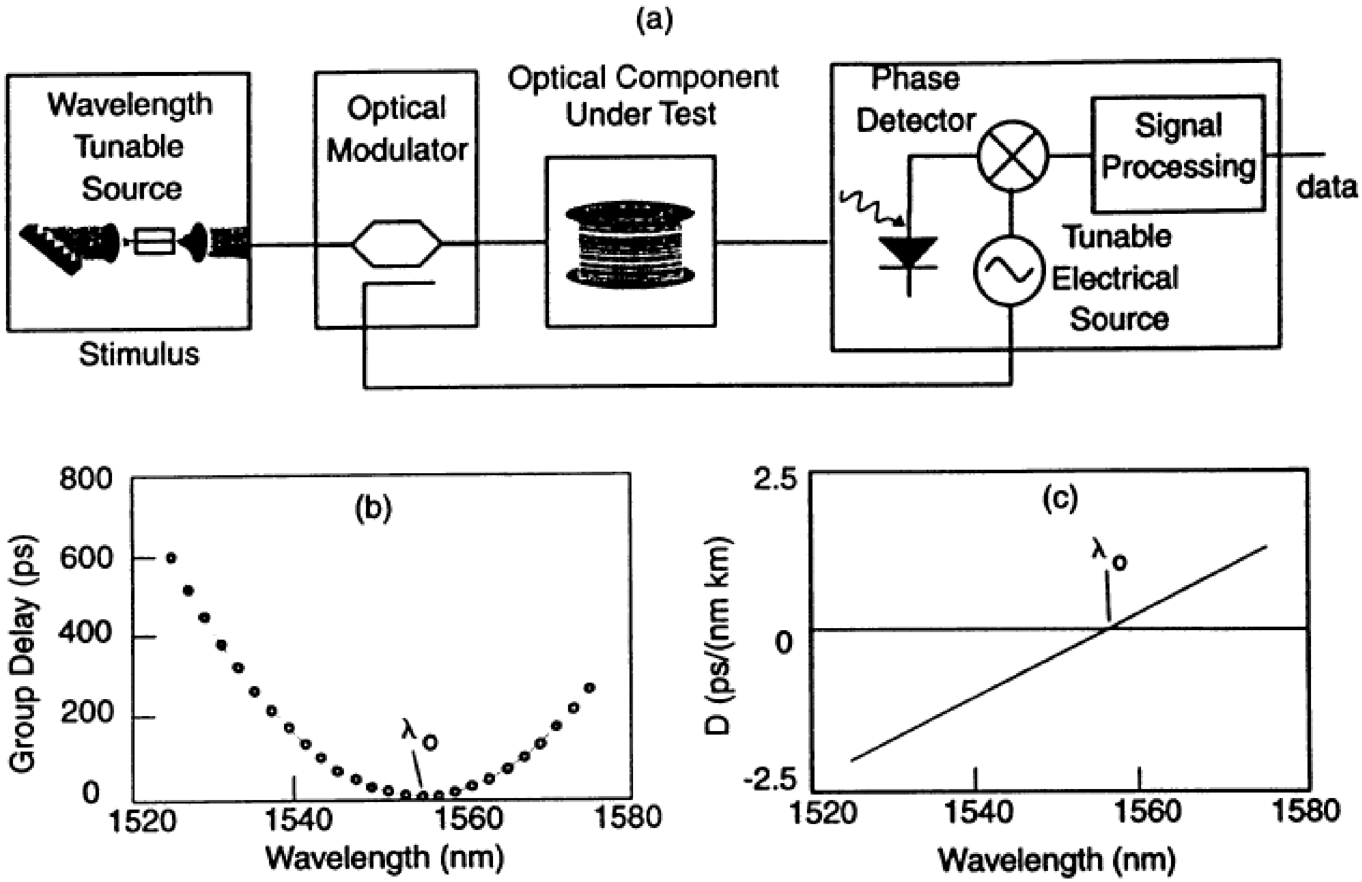
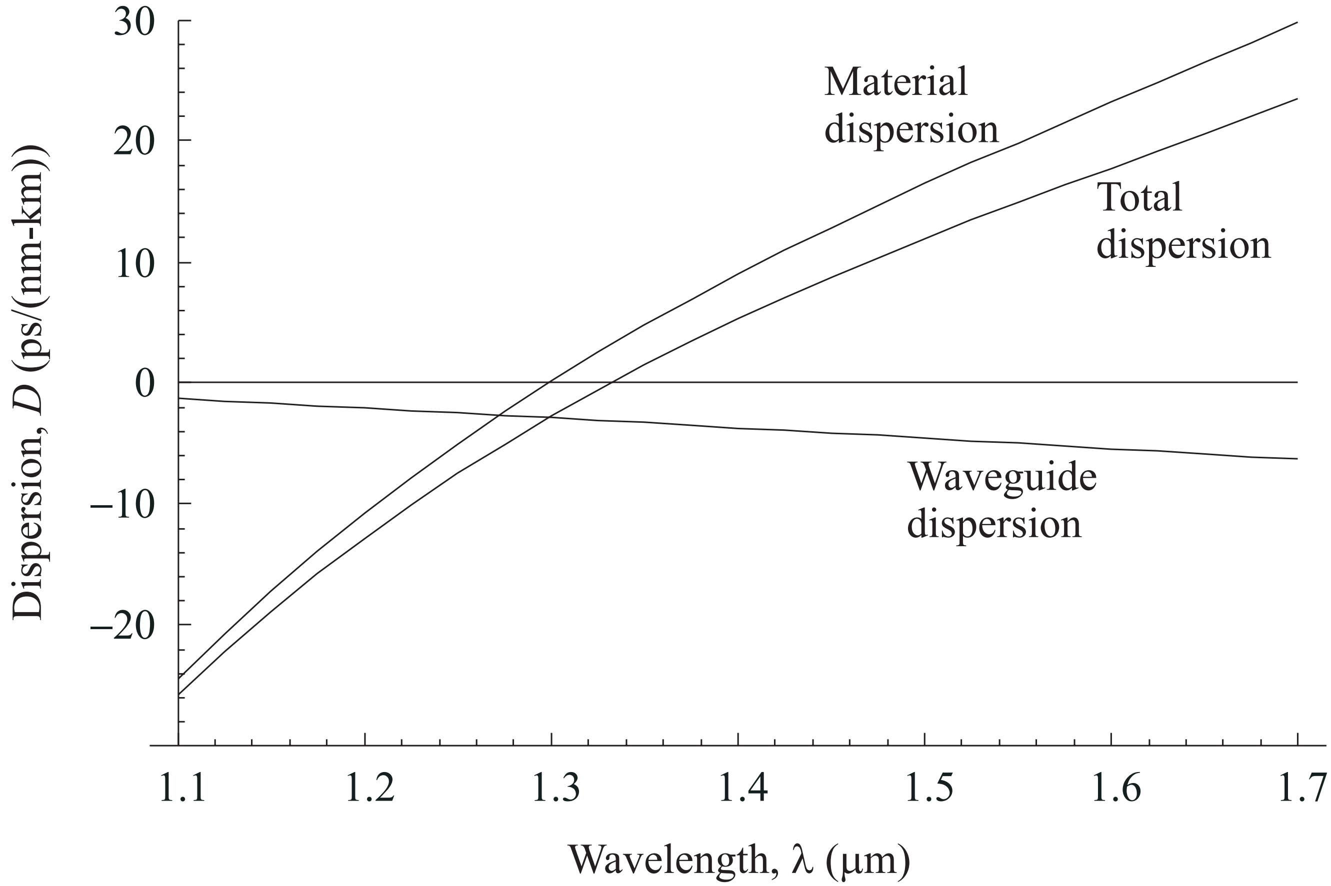


Figure 1.14 (a) Chromatic dispersion measurement of two-port optical devices. (b) Relative group delay versus wavelength. (c) Dispersion parameter versus wavelength.



30
20
10
0
−10
−20
Dispersion, D (ps/(nm-km))
1.1
1.2
1.3
1.4
1.5
1.6
1.7
Wavelength, λ (μm)
Material dispersion
Total dispersion
Waveguide dispersion

GROUP VELOCITY DISPERSION (2)

- Fiber paraxial wave equation without nonlinear effects:

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathcal{E}}(z',t') + \frac{\alpha}{2}\overline{\mathcal{E}} = 0$$

▷ Chromatic dispersion:

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda_{L,\text{vac}}^2}\beta_2 \quad \Rightarrow \quad \beta_2 = -\frac{\lambda_{L,\text{vac}}^2}{2\pi c}D$$

▷ Dispersion slope:

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda_{L,\text{vac}}^2}\right)^2\beta_3 + \frac{4\pi c}{\lambda_{L,\text{vac}}^3}\beta_2$$

$$\Rightarrow \quad \beta_3 = \left(\frac{\lambda_{L,\text{vac}}^2}{2\pi c}\right)^2\left(S + \frac{2}{\lambda_{L,\text{vac}}}D\right)$$

GROUP VELOCITY DISPERSION (3)

- Analytic studies of dispersion use the fiber paraxial wave equation without nonlinear effects or attenuation:

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathcal{E}}(z', t') = 0$$

  ▷ Introduce the Fourier transform of the field envelope, $\tilde{\mathcal{E}}$:

$$\overline{\mathcal{E}}(z', t') = \int_{-\infty}^{\infty} \tilde{\mathcal{E}}(z', \omega)\, e^{-i(\omega-\omega_0)t'}\, d\omega$$

  ▷ Differential equation for $\tilde{\mathcal{E}}$ and solution for $\overline{\mathcal{E}}$:

$$\left[\frac{\partial}{\partial z'} - i\left(\frac{\beta_2}{2}(\omega-\omega_0)^2 + \frac{\beta_3}{6}(\omega-\omega_0)^3\right)\right]\tilde{\mathcal{E}}(z', \omega) = 0$$

$$\overline{\mathcal{E}}(z', t') = \int_{-\infty}^{\infty} \tilde{\mathcal{E}}(0, \omega_0 + \omega')\, \exp\left[i\left(\frac{\beta_2}{2}\omega'^2 + \frac{\beta_3}{6}\omega'^3\right)z' - i\omega' t\right] d\omega'$$

GROUP VELOCITY DISPERSION (4)

- Analytic expression for $\mathcal{E}(z,t)$ in terms of the initial field $\mathcal{E}(0,t)$ when $\beta_3 = 0$:

$$\mathcal{E}(z,t) = \frac{e^{i\pi/4}}{\sqrt{2\pi\beta_2}} \int_{-\infty}^{\infty} \exp\left[i\left(\omega_0 t' - \frac{2}{\beta_2}[\beta_1 z - (t-t')]^2\right)\right] \mathcal{E}(0,t')\, dt'$$

▷ Assume a chirped Gaussian initial pulse:

$$\mathcal{E}(0,t) = \mathcal{E}_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^2\right]$$

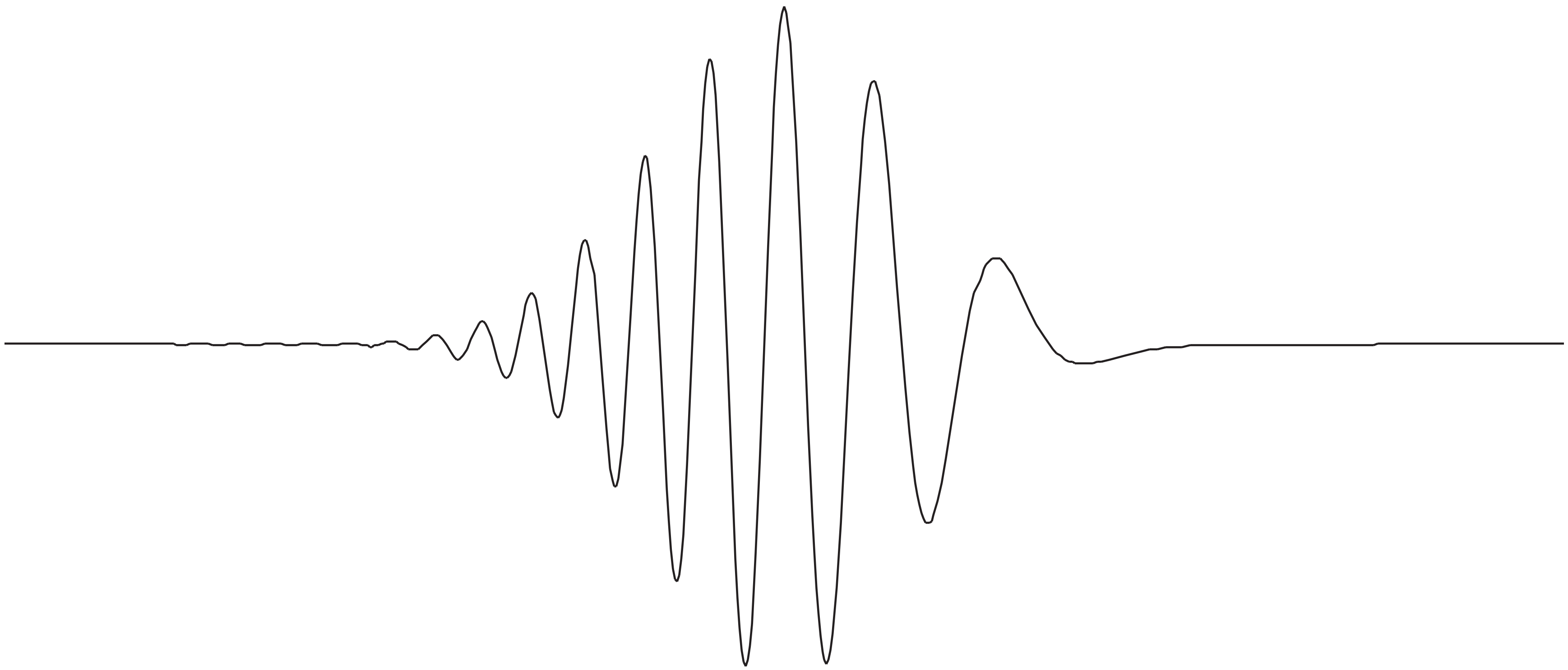
$$T_0 = \frac{1}{1.665} T_{\mathrm{FWHM}}$$

▷ C is the chirp rate:

$$\text{frequency at time } t = \omega_0 + \frac{C}{T_0^2} t$$

▷ Reference: D. Marcuse, *Applied Optics* **19**, 1653 (1980) and **20**, 3573 (1981)

A CHIRPED PULSE



GROUP VELOCITY DISPERSION (5)

- Spectrum of a chirped pulse:

$$\tilde{\mathcal{E}}(0,\omega) = \mathcal{E}_0 \left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2} \exp\left[-(1-iC)\frac{\omega^2 T_0^2}{2(1+C^2)}\right]$$

  ▷ Half-width of a chirped pulse at the $1/e$ intensities:

$$\Delta\,\omega_0 = \frac{\sqrt{1+C^2}}{T_0}$$

$$T_0 = \frac{1}{1.665} T_{\mathrm{FWHM}}$$

  ▷ Sources of chirp:

    ◦ Time-dependent gain saturation in SCDLs

    ◦ Self-phase modulation in fiber

GROUP VELOCITY DISPERSION (6)

- Analytic expression for the field envelope at z', t':

$$\mathcal{E}(z',t') = \frac{\mathcal{E}_0 T_0}{[T_0^2 - i\beta_2 z'(1+iC)]^{1/2}} \exp\left[-\frac{(1+iC)t'^2}{2[T_0^2 - i\beta_2 z'(1+iC)]}\right]$$

▷ Half-width at the $1/e$ intensities:

$$T_1 = T_0 \left[\left(1 + \frac{Cz}{L_D}\right)^2 + \left(\frac{z}{L_D}\right)^2\right]^{1/2}$$

▷ **Dispersion length**:

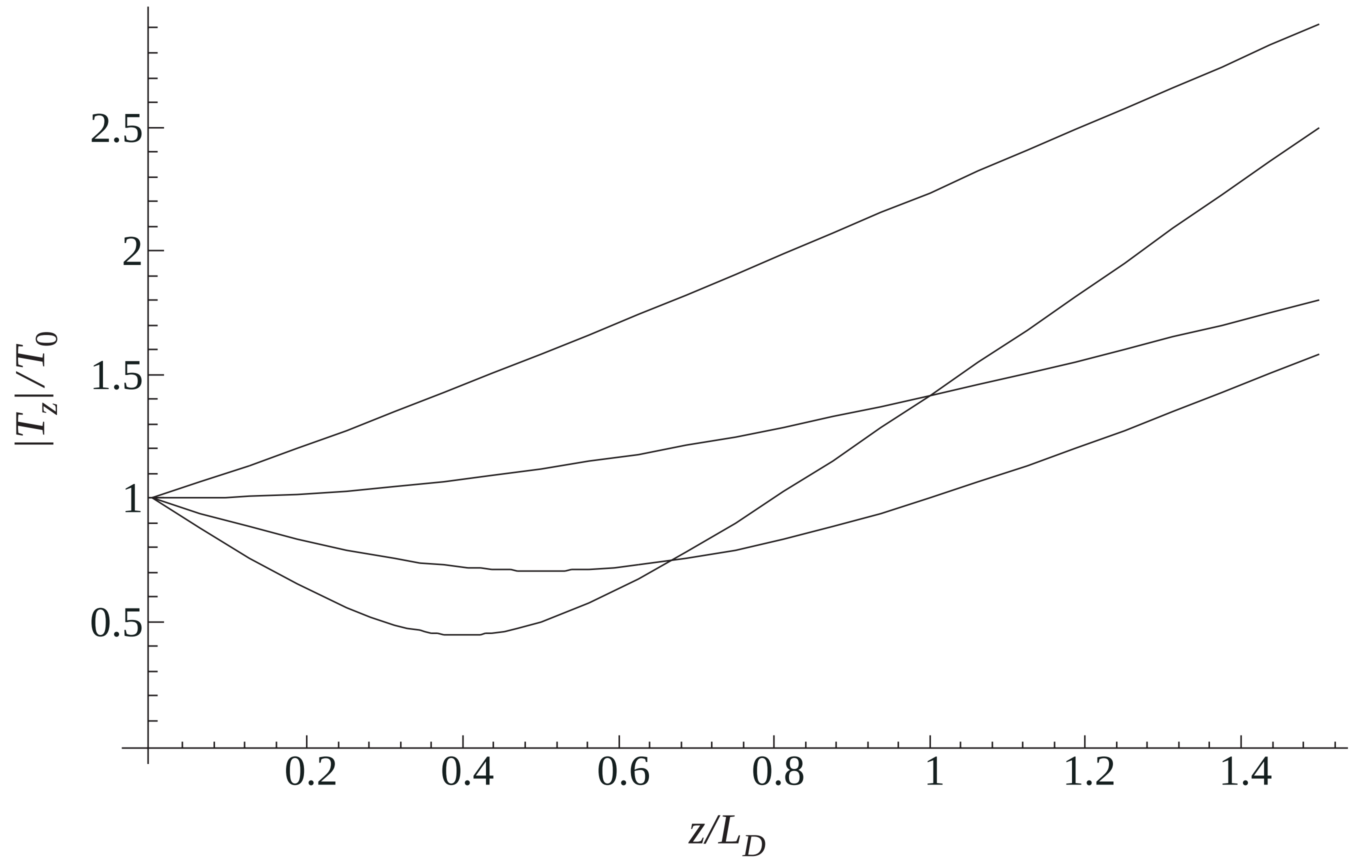
$$L_D := \frac{T_0^2}{|\beta_2|}$$

GROUP VELOCITY DISPERSION (7)

- Effects of chirp on pulsewidth when $\beta_3 = 0$:
  - ▷ Zero chirp ($C = 0$): Pulsewidth $\propto [1 + (z/L_D)^2]^{1/2}$
  - ▷ $C\beta_2 > 0$: Pulsewidth broadens faster than $[1 + (z/L_D)^2]^{1/2}$
  - ▷ $C\beta_2 < 0$: Pulsewidth decreases until $dT_1/dz = 0$, which occurs at

$$z_{\min} = \frac{|C|}{1 + C^2} L_D$$

$$T_1(z_{\min}) = \frac{T_0}{\sqrt{1 + C^2}}$$


2.5
2
1.5
1
0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
$|T_z|/T_0$
z/L_D

GROUP VELOCITY DISPERSION (8)

- Effects of finite laser bandwidth:
  - ▷ To allow for arbitrary pulse shapes, use

$$\sigma = \text{rms pulse width} = \left[\langle t^2 \rangle - \langle t \rangle^2\right]^{1/2}$$

$$\langle t^m \rangle = \frac{\int_{-\infty}^{\infty} t^m \, I(t) \, dt}{\int_{-\infty}^{\infty} I(t) \, dt}$$

$$\sigma_0 = \text{rms pulse width at input}$$

$$I_{\text{in}}(t) = I_0 e^{-t^2/T_0^2}$$

$$T_0^2 = 2\sigma_0^2$$

GROUP VELOCITY DISPERSION (9)

- Pulse broadening in the presence of a finite laser bandwidth:

$$\frac{\sigma}{\sigma_0} = \left[\left(1 + \frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + (1 + V_\omega^2)\left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + (1 + C^2 + V_\omega^2)^2 \frac{1}{2}\left(\frac{\beta_3 L}{4\sigma_0^3}\right)^2\right]^{1/2}$$

where

$$V_\omega = 2\sigma_\omega \sigma_0$$

$$\sigma_\omega = \text{laser rms spectral width}$$

$$D\sigma_\lambda = -\beta_2 \sigma_\omega$$

GROUP VELOCITY DISPERSION (10)

- Effects of dispersion on maximum bit rate achievable with an acceptable BER:
  - ▷ For a Gaussian pulse, if the bit-slot width is at least 2σ on each side of the maximum of a Gaussian pulse, then at least 95% of the energy is in the bit slot. Then

$$\sigma \leq \frac{1}{4B}$$

  - ▷ To estimate the maximum effects of dispersion, assume that

$$\sigma_0 = \frac{1}{4KB} \quad \text{where } K > 1$$

GROUP VELOCITY DISPERSION (11)

- Broadband source ($V_\omega \gg 1$), no chirp ($C = 0$):
  - ▷ Non-zero dispersion ($|D| \neq 0$):

$$\frac{\sigma}{\sigma_0} \approx \left[1 + \left(\frac{DL\sigma_\lambda}{\sigma_0}\right)^2\right]^{1/2}$$

In large-broadening limit:

$$\frac{\sigma}{\sigma_0} \approx 4KBL|D|\sigma_\lambda$$

GROUP VELOCITY DISPERSION (12)

- Broadband source ($V_\omega \gg 1$), no chirp ($C = 0$):
  - ▷ At the zero-dispersion wavelength:

$$\frac{\sigma}{\sigma_0} \approx \left[1 + \frac{1}{2}\left(\frac{SL\sigma_\lambda^2}{\sigma_0}\right)^2\right]^{1/2}$$

In large-broadening limit:

$$\frac{\sigma}{\sigma_0} \approx \frac{4K}{\sqrt{2}} BL|S|\sigma_\lambda^2$$

GROUP VELOCITY DISPERSION (13)

- Narrowband source ($V_\omega \ll 1$), no chirp ($C = 0$):
  - ▷ Non-zero dispersion ($|D| \neq 0$):

$$\frac{\sigma}{\sigma_0} \approx \left[1 + \left(\frac{DL\lambda_{L,\mathrm{vac}}^2}{4\pi c\sigma_0^2}\right)^2\right]^{1/2}$$

In large-broadening limit:

$$\frac{\sigma}{\sigma_0} \approx \frac{4K^2\lambda_{L,\mathrm{vac}}^2|D|B^2L}{\pi c}$$

  - ▷ σ/σ_0 depends on B^2L, not BL

GROUP VELOCITY DISPERSION (14)

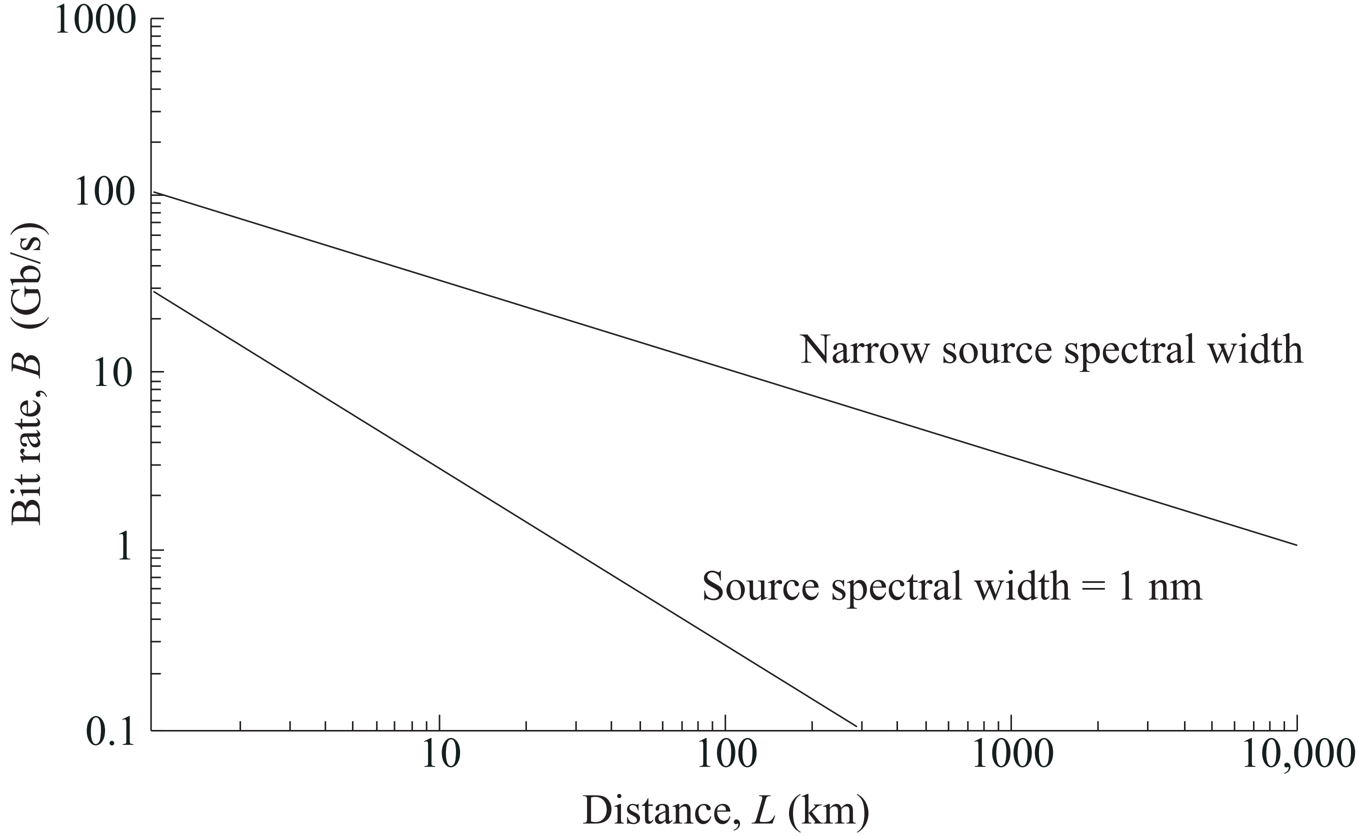
- Narrowband source ($V_\omega \ll 1$), no chirp ($C = 0$):
  - ▷ Zero dispersion:

$$\frac{\sigma}{\sigma_0} \approx \left[1 + \frac{1}{2}\left(\frac{\lambda_{L,\mathrm{vac}}^2}{2\pi c}\right)^4 \left(\frac{SL}{4\sigma_0^3}\right)^2\right]^{1/2}$$

In large-broadening limit:

$$\frac{\sigma}{\sigma_0} \approx 8\sqrt{2}\, K^3 \left(\frac{\lambda_{L,\mathrm{vac}}^2}{2\pi c}\right)^4 |S| B^3 L$$

  - ▷ σ/σ_0 depends on B^3L, not BL


1000
100
10
1
0.1
Bit rate, B (Gb/s)
Narrow source spectral width
Source spectral width = 1 nm
10
100
1000
10,000
Distance, L (km)

GROUP VELOCITY DISPERSION (15)

- Generalized nonlinear Schrödinger equation:

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathfrak{F}}(z', t') = -\frac{\alpha}{2}\overline{\mathfrak{F}} + i\gamma\left(1 + i\frac{2}{\omega_0}\frac{\partial}{\partial t}\right)|\overline{\mathfrak{F}}|^2\overline{\mathfrak{F}}$$

▷ Define scaled variables:

$$\tau = \frac{t'}{T_0},$$

$$\zeta := \frac{z'}{L_{NL}} = \gamma P_0 z',$$

$$e^{-\alpha z'/2}\sqrt{P_0}\,U(\zeta, \tau) := \overline{\mathfrak{F}}(z', t'),$$

$$s := \frac{2}{\omega_0 T_0}$$

GROUP VELOCITY DISPERSION (16)

- Scaled generalized nonlinear Schrödinger equation:

$$\left[\frac{\partial}{\partial\zeta} + \left(i\,\mathrm{sgn}(\beta_2)\frac{L_{NL}}{2L_D}\frac{\partial^2}{\partial\tau^2} - \mathrm{sgn}(\beta_3)\frac{L_{NL}}{6L'_D}\frac{\partial^3}{\partial\tau^3}\right)\right]U(\zeta,\tau) = ie^{-\alpha L_{NL}\zeta}\left(1 + is\frac{\partial}{\partial\tau}\right)|U|^2U$$

  ▷ Dispersion lengths:

$$L_D := \frac{T_0^2}{|\beta_2|},$$

$$L'_D := \frac{T_0^3}{|\beta_3|}$$