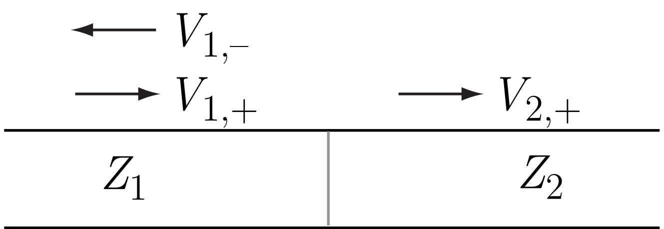
REFLECTION AT NORMAL INCIDENCE (1)

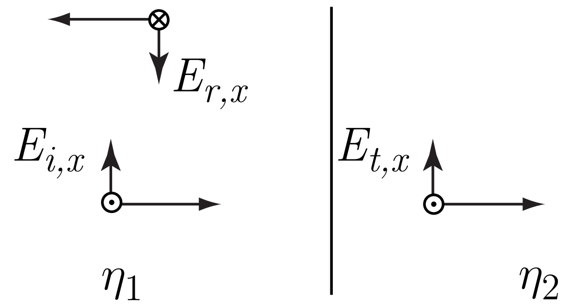
- Goal: Understand how to calculate the reflection coefficient at the interface between two dielectrics in terms of the indexes of refraction
- For normal incidence ($\theta_i = 0°$), the reflection coefficient of a plane wave is independent of the wave's polarization
  - ▷ A reflected wave propagates back into the medium in which the incident wave propagates
  - ▷ The **E** and **H** fields of the reflected and transmitted waves can be found by either of two methods:
    - ◦ Brute-force analysis of the boundary conditions (physics method)
    - ◦ Impedance analysis borrowed from transmission-line theory (EE method)

REFLECTION AT NORMAL INCIDENCE (2)

- Boundary conditions at a dielectric interface:
  - ▷ Continuity of the normal components of **D** and **B**
  - ▷ Continuity of the tangential components of **E** and **H**
- Fields:
  - ▷ In medium 1, $\mathbf{E} = \mathbf{E}_i + \mathbf{E}_r$
  - ▷ In medium 2, $\mathbf{E} = \mathbf{E}_t$
- For a plane wave at normal incidence ($\theta_i = 0°$), all of the fields are parallel (tangential) to the interface
  - ▷ The transmission-line approach is easy to justify and apply
  - ▷ The transmission-line equations are really just integral forms of Maxwell's equations
  - ▷ $E_{i,x}$ is analogous to $V_{1,+}$, $H_{i,y} = E_{i,x}/\eta$ is analogous to $I_{1,+} = V_{1,+}/Z_0$, η is analogous to Z_0, $E_{r,x}$ is analogous to $V_{1,-}$, etc.

REFLECTION AT NORMAL INCIDENCE (3)





- Boundary conditions:
  - ▷ V and I are continuous at the transmission-line interface
  - ▷ E_x and H_y are continuous at the dielectric interface
- Voltage reflection coefficient:

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

- Electric-field reflection coefficient:

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{n_2} - \frac{1}{n_1}}{\frac{1}{n_2} + \frac{1}{n_1}} = -\frac{n_2 - n_1}{n_2 + n_1}$$

REFLECTION AT NORMAL INCIDENCE (4)

- Power (and intensity) reflection coefficient:

$$\rho^2 = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

- Power (and intensity) transmission coefficient:

$$1 - \rho^2 = 1 - \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

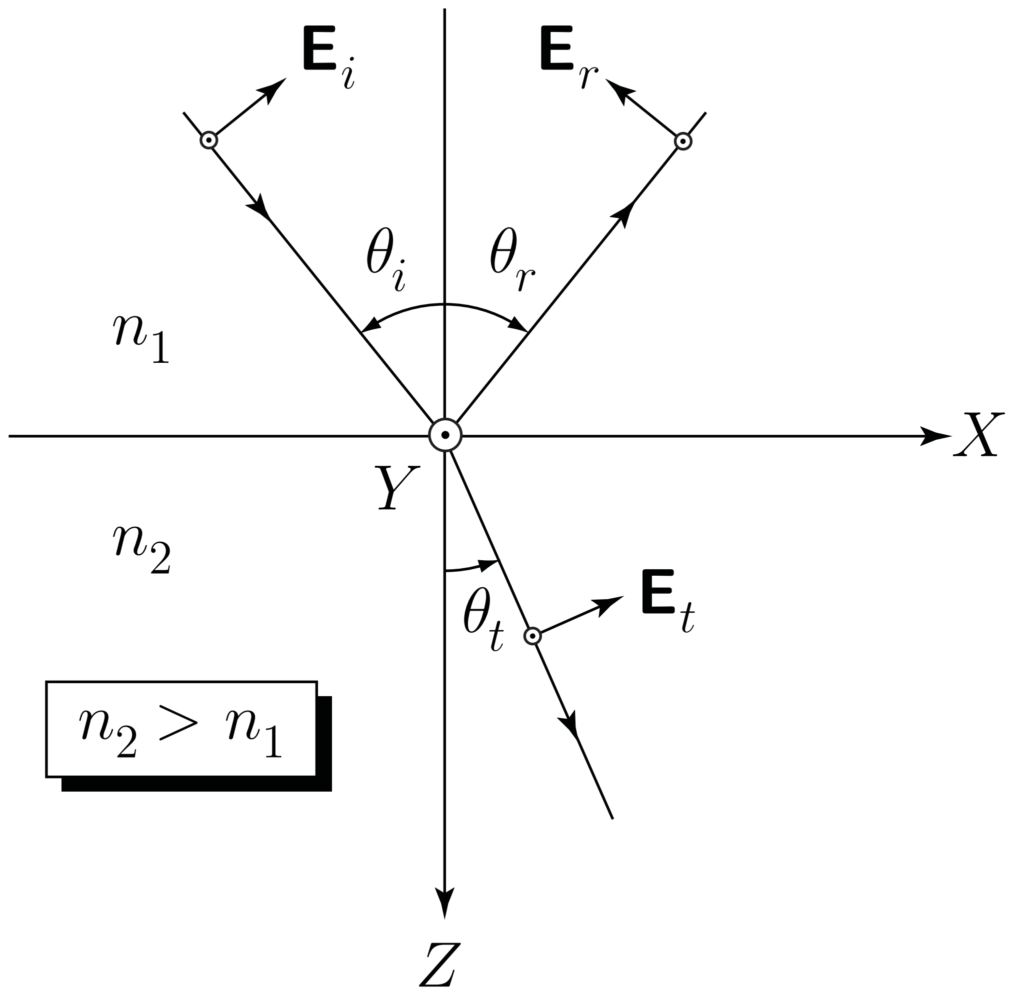
▷ Example: Glass-air interface ($n_1 = 1.0$, $n_2 = 1.5$)

$$\rho^2 = 0.04$$

REFLECTION AT OBLIQUE INCIDENCE (1)

- For all angles of incidence except 0° (normal incidence), the reflection coefficient of a plane wave depends on the wave's polarization
  - ▷ The useful orthogonal states of polarization for this situation are called TE and TM
    - ◦ In TE (or $\perp$) polarization, the **E** field is perpendicular to the plane of incidence
      - ◇ The **plane of incidence** is the plane defined by the direction of the incident field and the normal to the dielectric surface
    - ◦ In TM (or $\|$) polarization, the **E** field is in the plane of incidence

TM REFLECTION AT OBLIQUE INCIDENCE



REFLECTION AT OBLIQUE INCIDENCE (2)

- Impedance analysis
  - ▷ Apply to the tangential components of **E** and **H**
    - ∘ The tangential components are continuous at a dielectric interface, like V and I in a transmission line
  - ▷ In the medium from which the wave is incident, for the TM (∥) case,

$$Z_{1,\|} = \frac{E_{i,x}}{H_{i,y}} = -\frac{E_{r,x}}{H_{r,y}} = \eta_1 \cos\theta_i$$

$$Z_{2,\|} = \frac{E_{t,x}}{H_{t,y}} = \eta_2 \cos\theta_t$$

(the plane of incidence is the $x - z$ plane)

  - ▷ Reflection coefficient for TM wave:

$$\rho_\| = \frac{Z_{2,\|} - Z_{1,\|}}{Z_{2,\|} + Z_{1,\|}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

REFLECTION AT OBLIQUE INCIDENCE (3)

- TE ($\perp$) case
  - ▷ In the medium from which the wave is incident,

$$Z_{1,\perp} = -\frac{E_{i,y}}{H_{i,x}} = \frac{E_{r,x}}{H_{r,y}} = \eta_1 \sec\theta_i$$

$$Z_{2,\perp} = -\frac{E_{t,y}}{H_{t,x}} = \eta_2 \sec\theta_t$$

(the plane of incidence is the $x - z$ plane)

  - ▷ Reflection coefficient for TE wave:

$$\rho_\perp = \frac{Z_{2,\perp} - Z_{1,\perp}}{Z_{2,\perp} + Z_{1,\perp}} = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_i}$$

FRESNEL FORMULAS

- The normal to a dielectric interface and the incident ray define the **plane of incidence**
  - ▷ **E** components that are in (parallel to) the plane of incidence do not interfere with field components that are perpendicular to the plane of incidence
  - ▷ Transmission and reflection coefficients:

$$\tau_{\parallel} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$\tau_{\perp} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$\rho_{\parallel} = -\frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\rho_{\perp} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

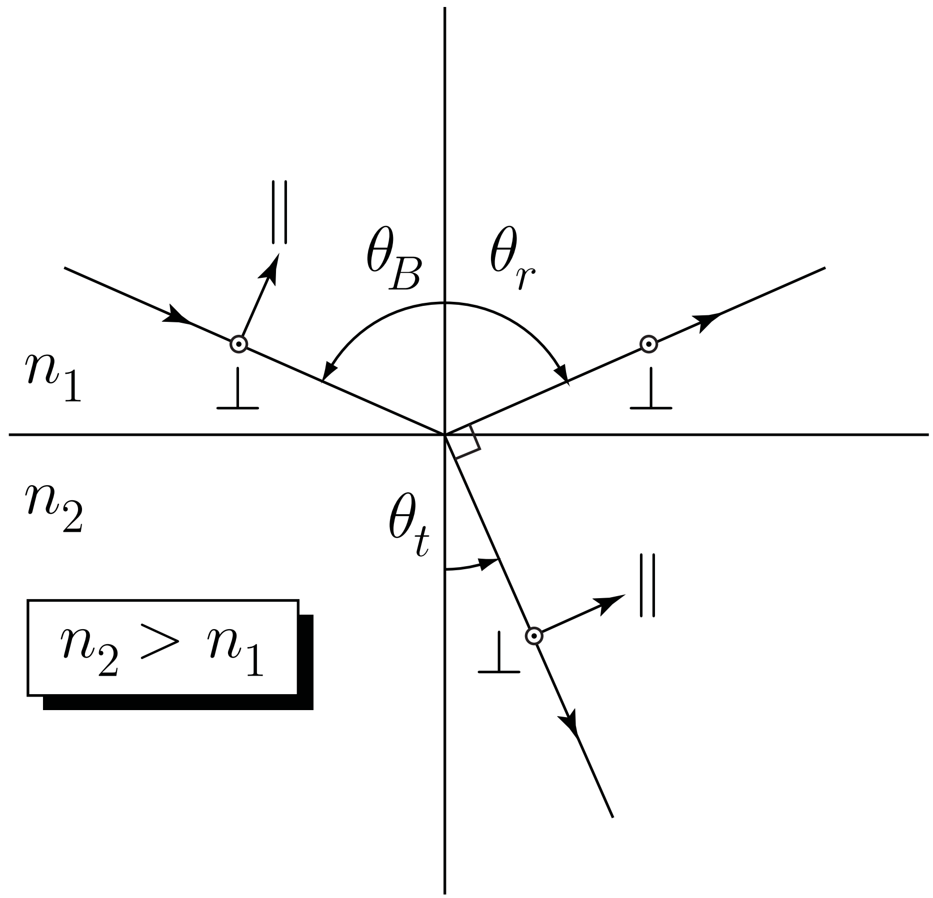
BREWSTER'S ANGLE (1)

- When $\theta_i + \theta_t = \pi/2$, the **E** component in the plane of incidence is not reflected ($\rho_\| = 0$)
  - ▷ A more usable formula for Brewster's angle:

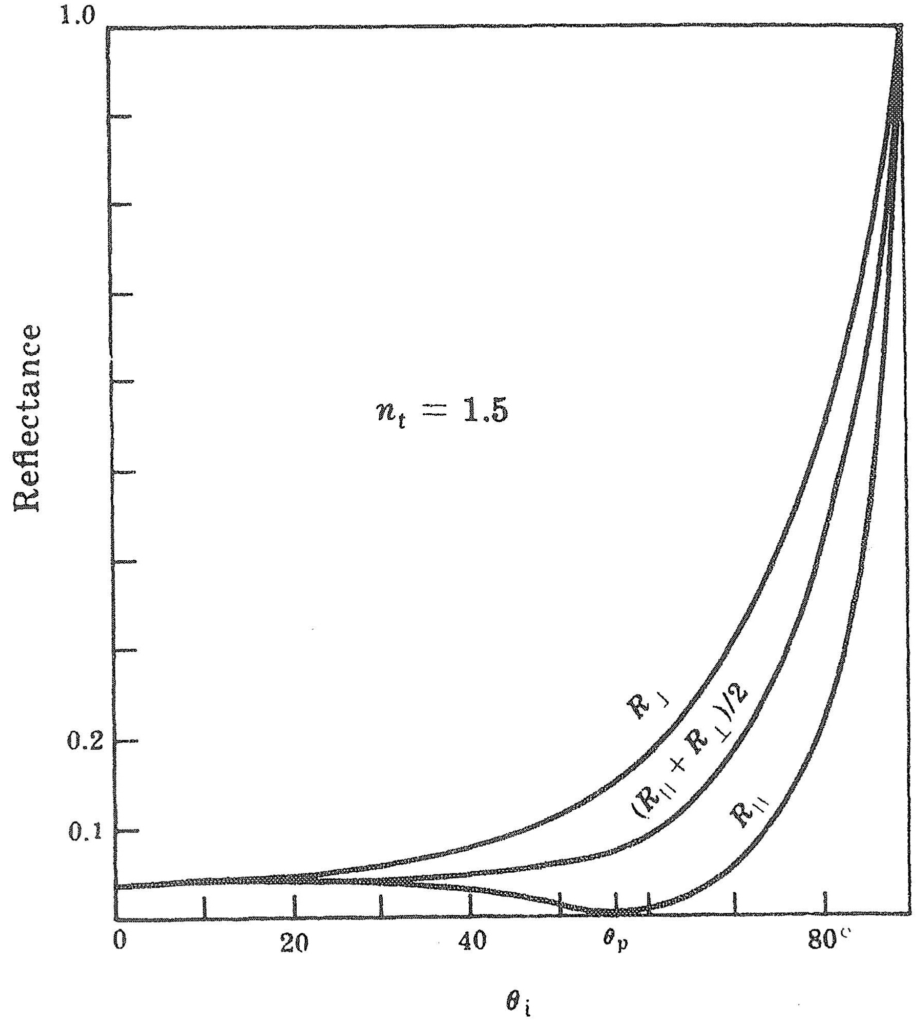
$$\tan\theta_B = \frac{n_t}{n_i}$$

  - ▷ At Brewster's angle, the reflected light is fully polarized perpendicular to the plane of incidence
    - ∘ Polaroid sunglasses and camera filters
    - ∘ End faces on lasers

BREWSTER'S ANGLE (2)



BREWSTER'S ANGLE (3)



Hecht, *Schaum's Outline of Optics*