DIFFRACTION (1)

- **Diffraction** is the name given to the deviation of light from rectilinear propagation
  - ▷ A property of all waves
  - ▷ Usually most evident at the edges of shadows
  - ▷ Vital for a quantitative understanding of free-space optical beam propagation
    - ◦ Light exiting into air from a waveguide diffracts strongly
- Diffractive effects are already included in guided modes, so don't need to treat diffraction separately for light inside a waveguide
- Important special cases
  - ▷ Diffraction by a circular aperture — Airy disk
  - ▷ Diffraction by a rectangular aperture
  - ▷ Optics of Gaussian beams

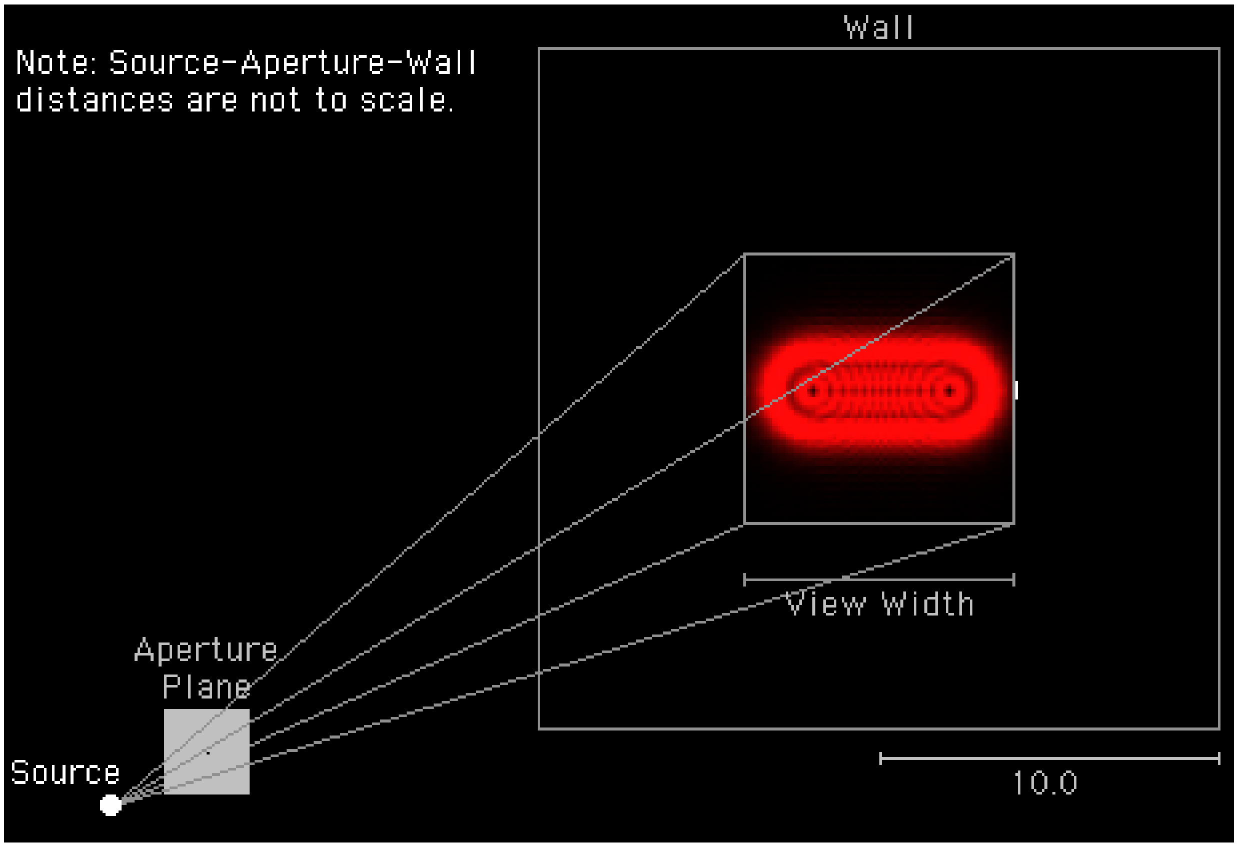
DIFFRACTION (2)

- The details of diffraction patterns depend on the **Fresnel number**

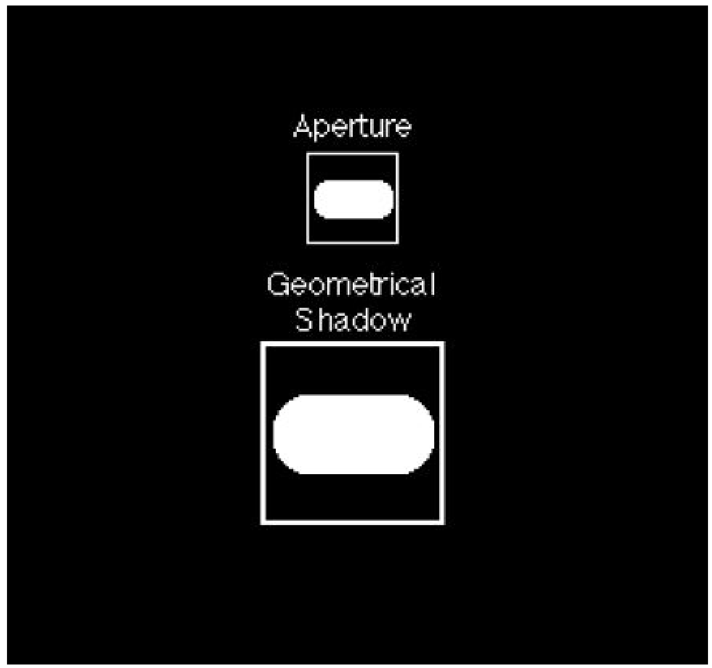
$$N = \frac{a^2}{\lambda d}$$

  - ▷ a is the radius of the aperture (we're assuming a circular aperture here...)
  - ▷ d is the distance to the observation point, or the focal length of a lens, if one is used; λ is the wavelength
  - ▷ Large Fresnel numbers ($N \gg 1$): **Fresnel regime**
    - ∘ Wavefront curvature is an essential part of the physics
    - ∘ Crudely, Fresnel diffraction by an aperture or obstacle results in a shadow with lots of bright and dark fringes
  - ▷ Small Fresnel numbers ($N \ll 1$): **Fraunhofer regime**
    - ∘ Plane-wave limit
    - ∘ The diffraction pattern is the Fourier transform of the aperture
    - ∘ The focal plane of an ideal lens is in the Frauhofer regime — a lens is a Fourier transformer (see Jack Gaskill's book)

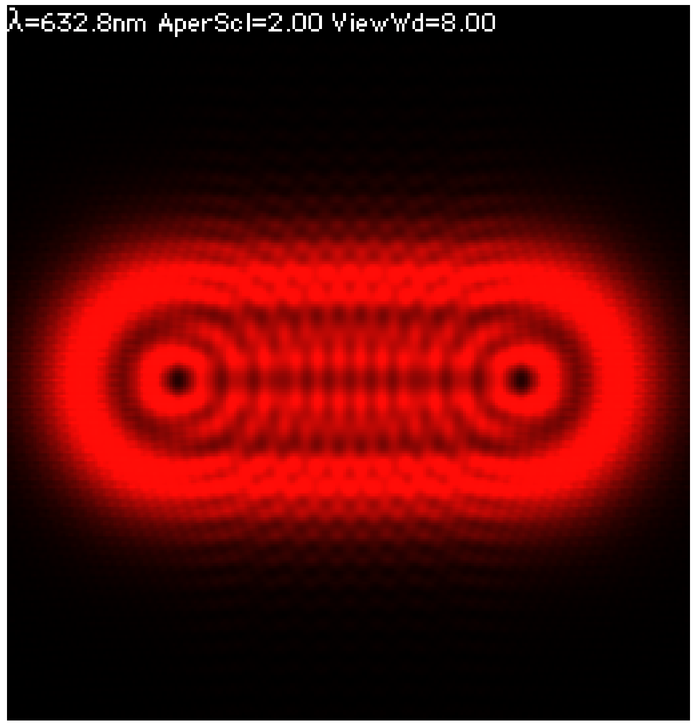
FRESNEL DIFFRACTION EXPERIMENT



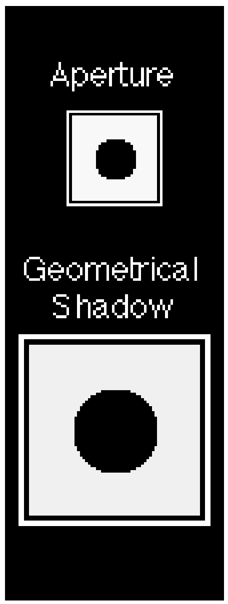
OBLONG APERTURE AND GEOMETRICAL SHADOW



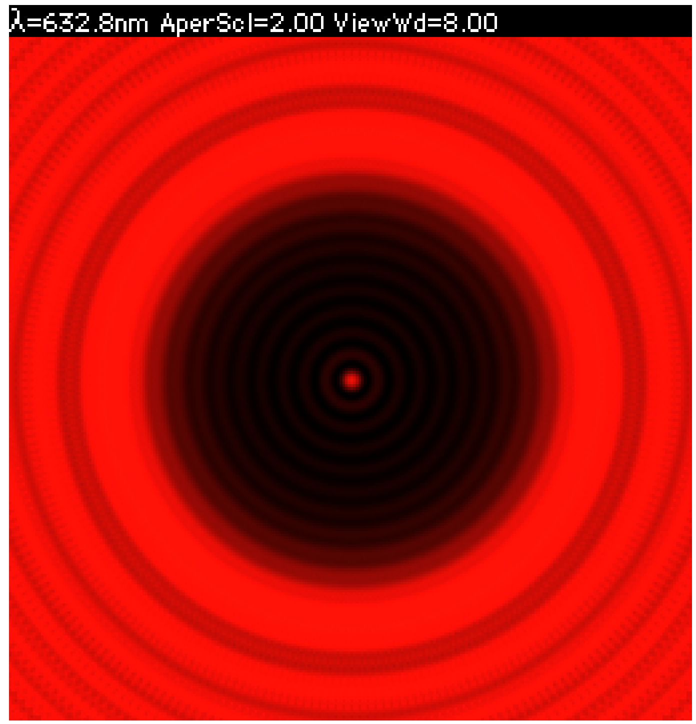
FRESNEL DIFFRACTION BY AN OBLONG APERTURE



FRESNEL DIFFRACTION BY A CIRCULAR OBSTACLE

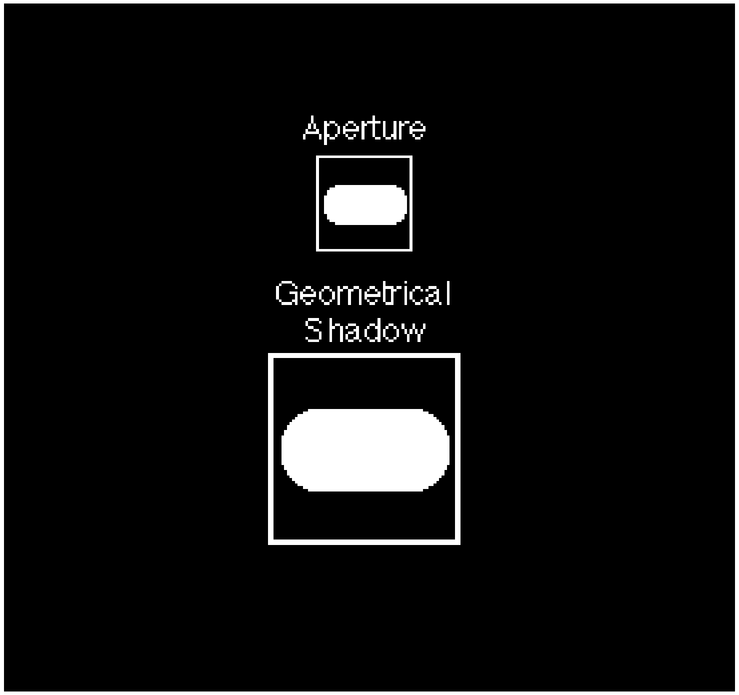


Fresnel diffraction pattern

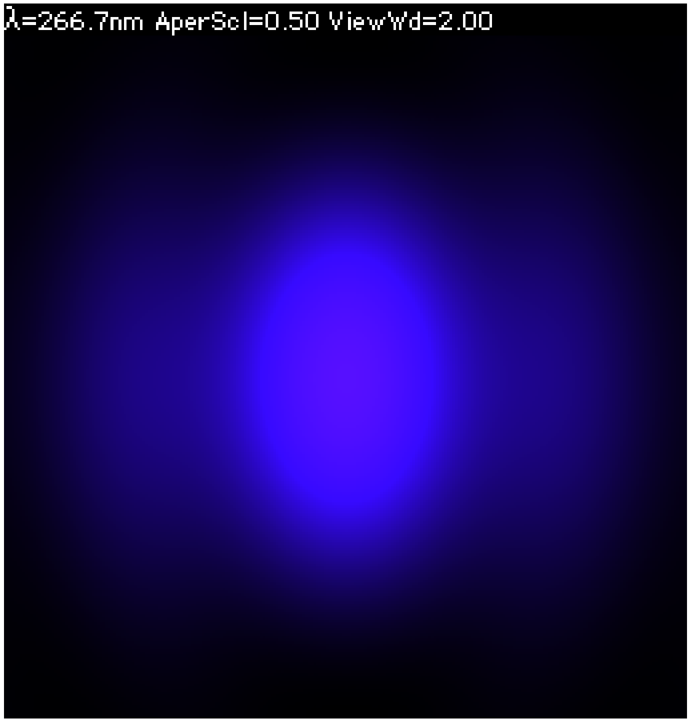


Notice the "Poisson bright spot" in the center of the geometrical shadow!

FRAUNHOFER DIFFRACTION BY AN OBLONG APERTURE



Fraunhofer diffraction pattern



Note that the diffraction pattern is widest in the direction in which the aperture is narrowest

FRAUNHOFER DIFFRACTION

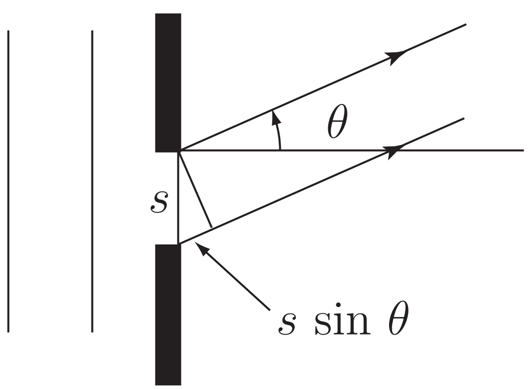
- Plane-wave limit
- Criterion for destructive interference (minimum of the diffraction pattern):

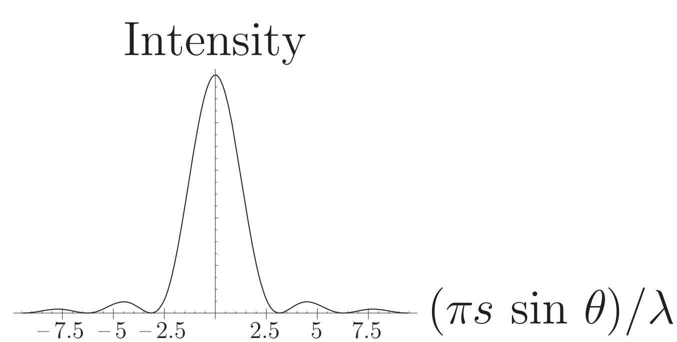
$$s \sin\theta \approx m\lambda$$

For small angles ($\theta \ll 1$), and for the first minimum,

$$\theta \approx \frac{\lambda}{s}$$

- Fraunhofer diffraction by a slit of width s:



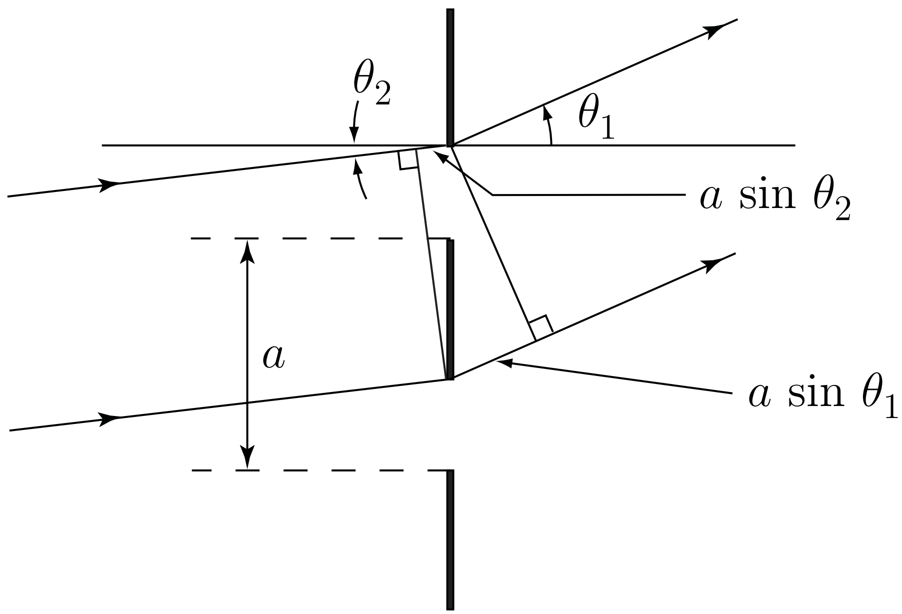


DIFFRACTION GRATINGS (PHYSICS 2 APPROACH)

- The grating shown below consists of a repeating pattern of opaque screen + aperture, with period a
- The path difference for parallel rays through adjacent apertures is

$$\Delta L = a \sin\theta_1 - a \sin\theta_2$$

- Constructive interference occurs when $\boxed{\Delta L = m\lambda}$ (where m = integer)



ORDERS OF DIFFRACTION

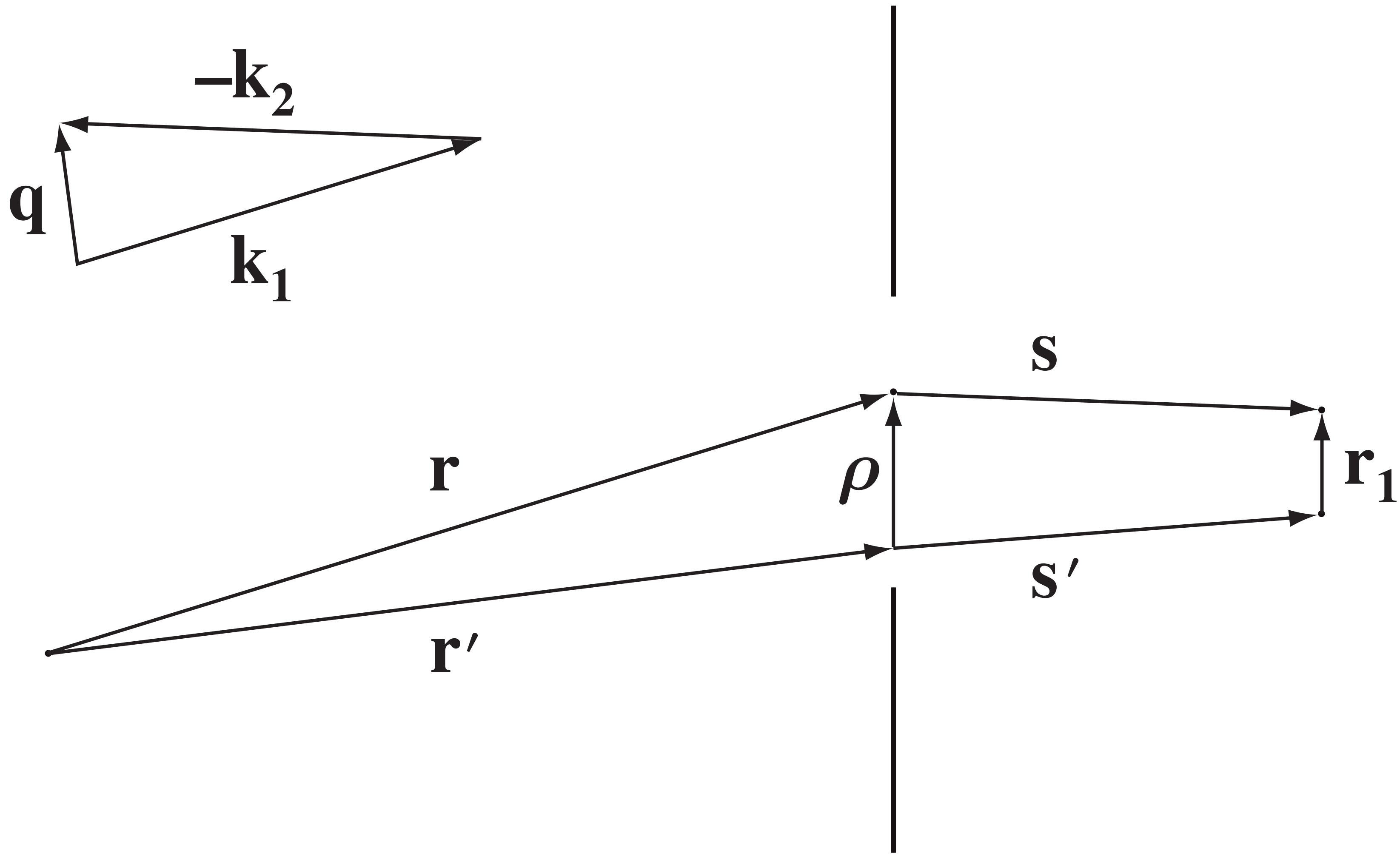
- Constructive interference occurs when $\boxed{\Delta L = m\lambda}$
- The integer $|m|$ is called the **order of diffraction**
  - ▷ The zero order ($m = 0$) corresponds to transmission straight through the grating
  - ▷ The wavelength interval $\Delta\lambda_{\mathrm{FSR}}$ between successive orders of diffraction that exactly overlap (have the same θ_1 and θ_2), such that

$$\lambda_m = a \sin\theta_1 - a \sin\theta_2 = \lambda_{m+1} + \Delta\lambda_{\mathrm{FSR}},$$

is called the **free spectral range** of the grating:

$$\Delta\lambda_{\mathrm{FSR}} = \frac{\lambda_{m+1}}{m} \approx \frac{\lambda}{m}$$

COORDINATES FOR THE DIFFRACTION INTEGRAL



DIFFRACTION GRATINGS (1)

- Fraunhofer diffraction by a screen:

$$E(\mathbf{r}_1) = C \int_S \tau(\boldsymbol{\rho})\, e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS$$

where $\mathbf{q} := \mathbf{k}_1 - \mathbf{k}_2$, $\mathbf{k}_1 := k\hat{\mathbf{r}}'$, $\mathbf{k}_2 := k\hat{\mathbf{s}}'$ (see preceding slide), τ is the transmission function of the screen, $\boldsymbol{\rho}$ is the position vector in the screen, and C is a constant

▷ For our simple diffraction-grating model, $\tau = 0$ in the opaque regions, and $\tau = 1$ in the transparent regions

▷ Fraunhofer diffraction by a screen with N identical apertures (a grating!):

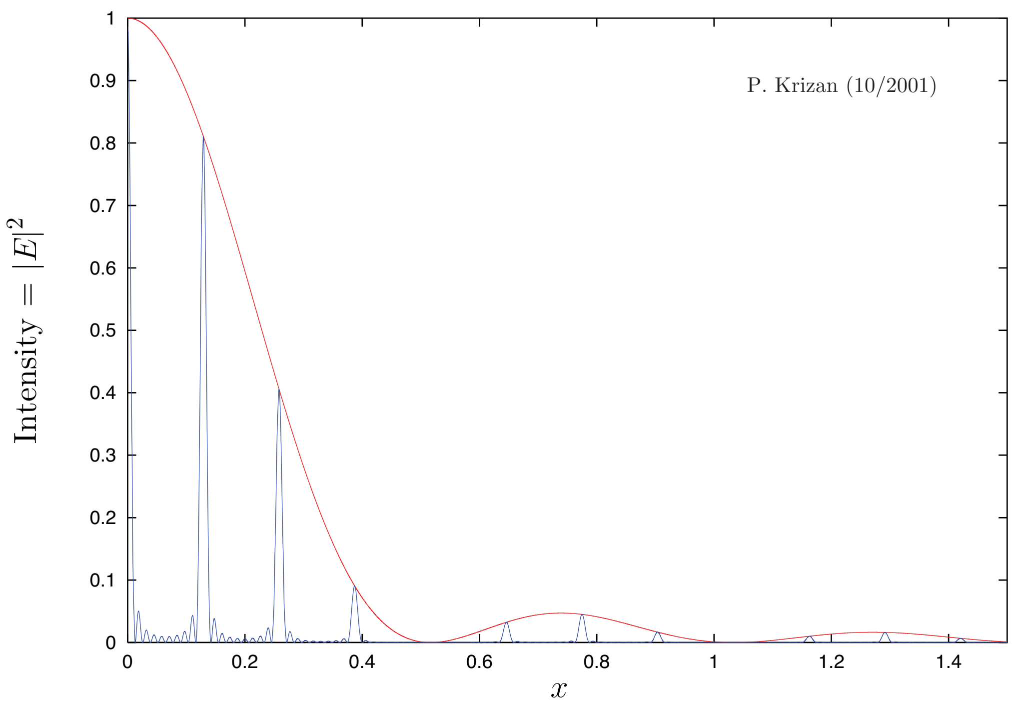
- Aperture A_n is displaced from aperture A_1 by a vector $\mathbf{a}_n = (n-1)\mathbf{a}$
- Amplitude of diffracted field is

$$E(\mathbf{r}_1) = C \int_S \tau(\boldsymbol{\rho})\, e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS = \sum_{n=1}^{N} e^{i\mathbf{q}\cdot\mathbf{a}_n} \int_{A_1} e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS$$

DIFFRACTION GRATINGS (2)

- Amplitude of field diffracted by a screen with N identical, periodically arranged apertures:

$$E(\mathbf{r}_1) = \underbrace{\sum_{n=1}^{N} e^{i\mathbf{q}\cdot\mathbf{a}_n}}_{\text{multiple-beam interference pattern}} \times \underbrace{\int_{A_1} e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS}_{\text{diffraction pattern of one aperture}}$$



BRAGG REFLECTION

- Plane-wave limit
- **Bragg's law** for constructive interference of plane waves reflected from a refractive-index grating:

$$2\Lambda \sin\theta = m\lambda/\bar{n}$$

  - ▷ Λ is the grating period, m is an integer, and $\bar{n}$ is the mean refractive index of the medium