# EE 6340: Introduction to Telecommunications Networks

## Project 2

A. Probability of queuing in a $M/M/m$ queue:

1. **Plot #1:** plot the probability of queueing ($P_Q$) for an $M/M/m$ system having $m$ servers, versus $\lambda/\mu$. Perform the experiment for $m = 2, 4$, $\lambda \in (0, 1)$, and $\mu = 1$.
2. From plot #1, it is possible to compare the probability of queueing for two systems with the same customer arrival rate and service rate, but with a different number of servers. Which system provides the minimum probability of queueing and why?
3. Assume that the customer arrival rate is increased proportionally to the number of servers, i.e., $\lambda' = m\lambda$, while the service rate of each one of the $m$ servers remains the same (i.e., $\mu$). **Plot #2:** plot the probability of queueing ($P_Q$) for this $M/M/m$ system versus the utilization factor $\rho' = \lambda'/m\mu$. Perform the experiment for $m = 2, 4$, $\lambda \in (0, 1)$, and $\mu = 1$.
4. From plot #2, it is possible to compare the probability of queueing for two systems having the same customer arrival rate *per server* and the same service rate, but with a different number of servers. Which system provides the minimum probability of queueing and why?

B. Comparison of one versus multiple channels in statistical multiplexing:

1. Consider a communication link serving $m$ independent Poisson traffic streams with overall rate $\lambda$. Suppose that the link is divided into $m$ separate channels with one channel assigned to each traffic stream. However, if a traffic stream has no packet awaiting transmission, its corresponding channel is used to transmit a packet of another traffic stream. The transmission times of packets on each of the channels are exponentially distributed with mean $1/\mu$. The system can be modeled as a $M/M/m$ queue. **Plot #3:** plot the average delays per packet of this system versus $\rho = \lambda/m\mu$ with arrival rate $\lambda$ and service rate $\mu$ for each server. In the same figure, add the plot of the average delays per packet versus $\rho$ for a statistical multiplexing system with one channel having $m$ times larger capacity, i.e., a $M/M/1$ system with arrival rate $\lambda$ and service rate $m\mu$. Perform the experiments for $m = 4$, $\rho \in (0, 1)$, and $\mu = 1$.
2. What is the behavior of the two systems under different loads? Which system has the smallest delay per packet under light load and why?

C. Recursive Erlang-B formula:

1. The probability ($p_m$) that an arrival in a $M/M/m/m$ system will find $m$ servers busy and will therefore be lost is given by the well-known Erlang-B formula. Derive the recursive equation for Erlang-B formula (i.e., $p_m = f(p_{m-1})$). [Hints: 1. derive the final formula using the recursion of the inverse, i.e., $p_m^{-1} = f(p_{m-1}^{-1})$; 2. check the correctness by verifying that the conditions $p_0 = 1$ and $p_1 = \frac{\rho}{1+\rho}$ hold in the final recursive formula, i.e., $p_1 = f(p_o)$.]
2. A telephone company receives calls with rate $\lambda = 10$ calls/minute. The durations of the calls are independent and exponentially distributed with mean of $1/\mu = 15$ minute/call. Use the Erlang-B recursive formula to determine the minimum number ($m$) of circuits to ensure that an attempted call is blocked with probability $P < 10^{-4}$. It is assumed that calls are blocked because all the circuits are busy and blocked class are lost, i.e., not attempted again.