

BASIC PROBLEMS

4.21. Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t]u(t)$, $\alpha > 0$

(b) $e^{-3|t|} \sin 2t$

(c) $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT)$, $|\alpha| < 1$

(e) $[te^{-2t} \sin 4t]u(t)$

(f) $\left[\frac{\sin \pi t}{\pi t} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$

(g) $x(t)$ as shown in Figure P4.21(a)

(h) $x(t)$ as shown in Figure P4.21(b)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

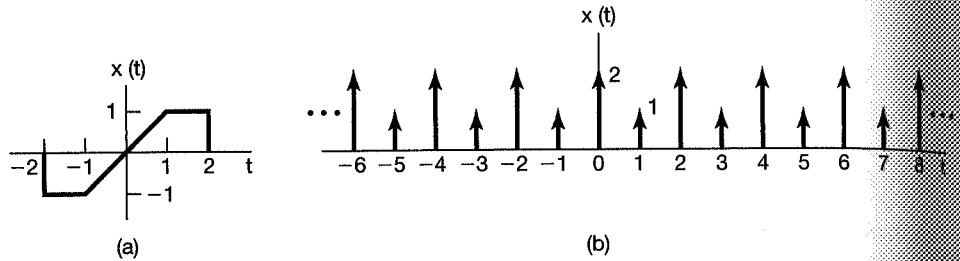


Figure P4.21

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

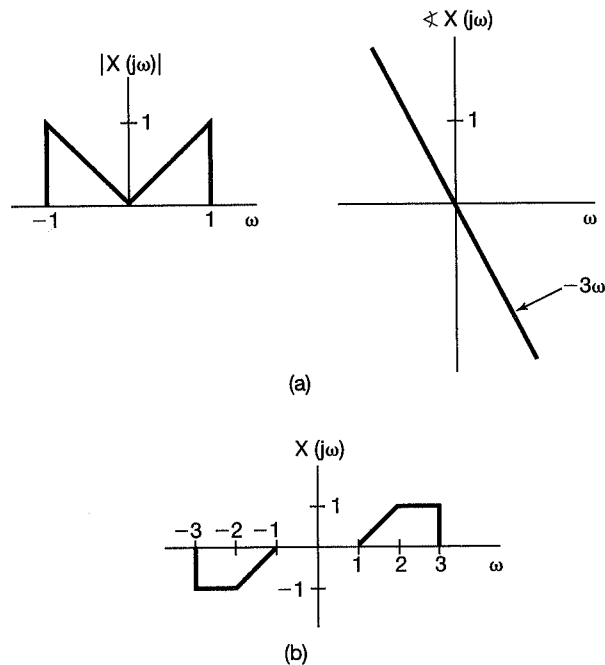


Figure P4.22

- (a) $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$
- (b) $X(j\omega) = \cos(4\omega + \pi/3)$
- (c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
- (d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
- (e) $X(j\omega)$ as in Figure P4.22(b)

4.23. Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure P4.23. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.

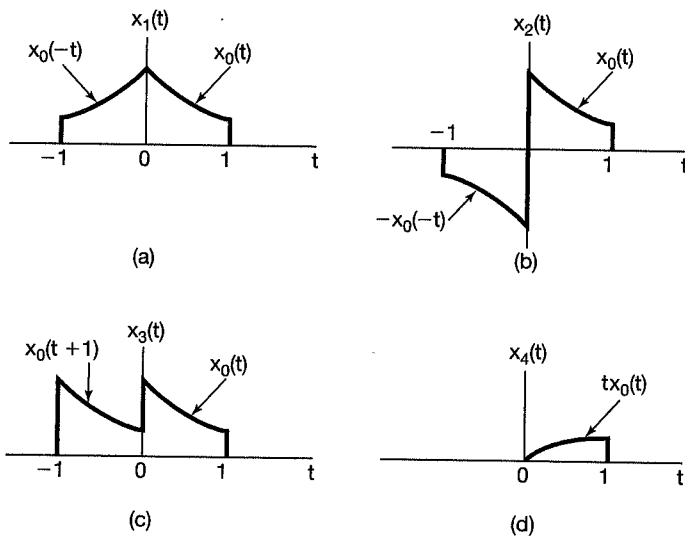


Figure P4.23

- 4.24. (a) Determine which, if any, of the real signals depicted in Figure P4.24 have Fourier transforms that satisfy each of the following conditions:
- (1) $\Re\{X(j\omega)\} = 0$
 - (2) $\Im\{X(j\omega)\} = 0$
 - (3) There exists a real α such that $e^{j\alpha\omega} X(j\omega)$ is real
 - (4) $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$
 - (5) $\int_{-\infty}^{\infty} \omega X(j\omega) d\omega = 0$
 - (6) $X(j\omega)$ is periodic
- (b) Construct a signal that has properties (1), (4), and (5) and does *not* have the others.

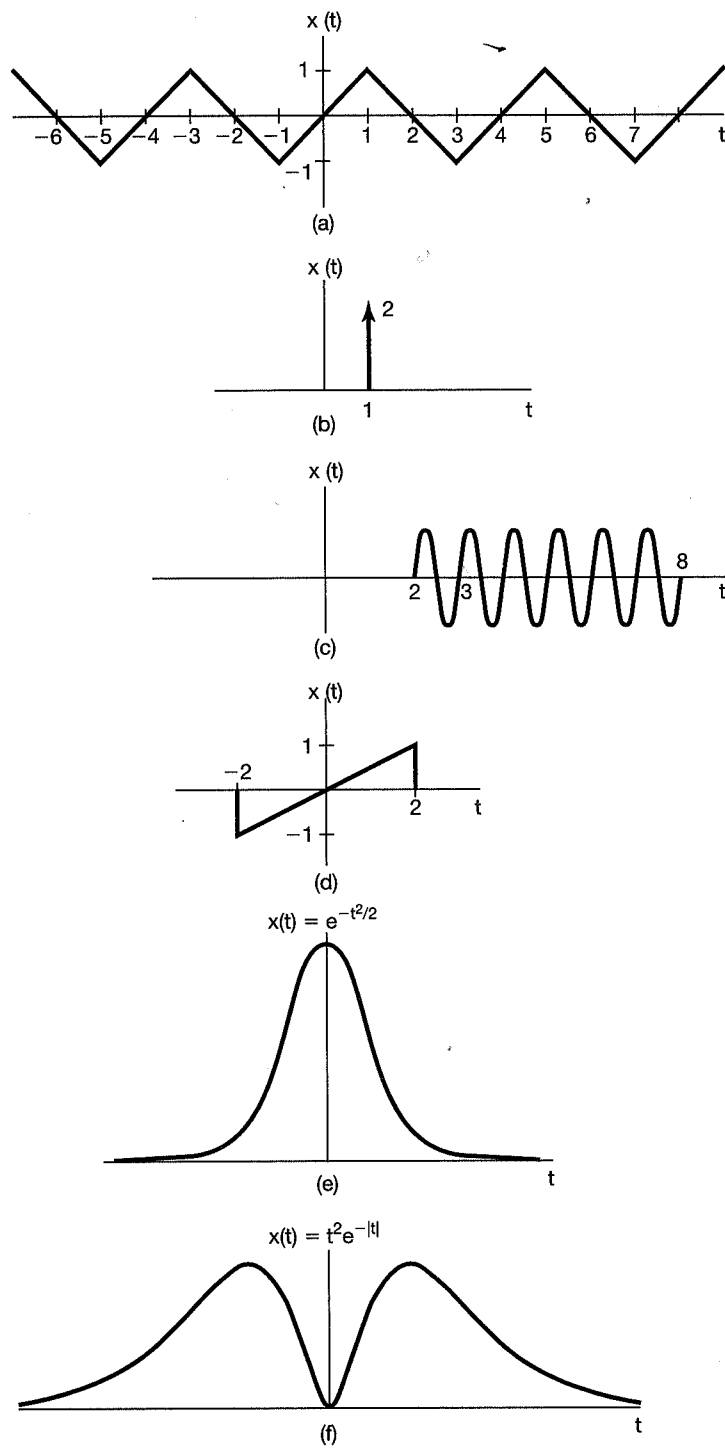


Figure P4.24