

this chapter we have derived and examined many of these properties. Among them are two that have particular significance for our study of signals and systems. The first is the convolution property, which is a direct consequence of the eigenfunction property of complex exponential signals and which leads to the description of an LTI system in terms of its frequency response. This description plays a fundamental role in the frequency-domain approach to the analysis of LTI systems, which we will continue to explore in subsequent chapters. The second property of the Fourier transform that has extremely important implications is the multiplication property, which provides the basis for the frequency-domain analysis of sampling and modulation systems. We examine these systems further in Chapters 7 and 8.

We have also seen that the tools of Fourier analysis are particularly well suited to the examination of LTI systems characterized by linear constant-coefficient differential equations. Specifically, we have found that the frequency response for such a system can be determined by inspection and that the technique of partial-fraction expansion can then be used to facilitate the calculation of the impulse response of the system. In subsequent chapters, we will find that the convenient algebraic structure of the frequency responses of these systems allows us to gain considerable insight into their characteristics in both the time and frequency domains.

Chapter 4 Problems

The first section of problems belongs to the basic category and the answers are provided in the back of the book. The remaining three sections contain problems belonging to the basic, advanced, and extension categories, respectively.

BASIC PROBLEMS WITH ANSWERS

- 4.1. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:
 (a) $e^{-2(t-1)}u(t-1)$ (b) $e^{-2|t-1|}$
 Sketch and label the magnitude of each Fourier transform.
- 4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:
 (a) $\delta(t+1) + \delta(t-1)$ (b) $\frac{d}{dt}\{u(-2-t) + u(t-2)\}$
 Sketch and label the magnitude of each Fourier transform.
- 4.3. Determine the Fourier transform of each of the following periodic signals:
 (a) $\sin(2\pi t + \frac{\pi}{4})$ (b) $1 + \cos(6\pi t + \frac{\pi}{8})$
- 4.4. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transforms of:
 (a) $X_1(j\omega) = 2\pi \delta(\omega) + \pi \delta(\omega - 4\pi) + \pi \delta(\omega + 4\pi)$

$$(b) X_2(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2 \\ -2, & -2 \leq \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$$

4.5. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transform of $X(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)}$, where

$$|X(j\omega)| = 2\{u(\omega + 3) - u(\omega - 3)\}, \\ \angle X(j\omega) = -\frac{3}{2}\omega + \pi.$$

Use your answer to determine the values of t for which $x(t) = 0$.

4.6. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in Table 4.1.

(a) $x_1(t) = x(1 - t) + x(-1 - t)$

(b) $x_2(t) = x(3t - 6)$

(c) $x_3(t) = \frac{d^2}{dt^2} x(t - 1)$

4.7. For each of the following Fourier transforms, use Fourier transform properties (Table 4.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) $X_1(j\omega) = u(\omega) - u(\omega - 2)$

(b) $X_2(j\omega) = \cos(2\omega) \sin(\frac{\omega}{2})$

(c) $X_3(j\omega) = A(\omega)e^{jB(\omega)}$, where $A(\omega) = (\sin 2\omega)/\omega$ and $B(\omega) = 2\omega + \frac{\pi}{2}$

(d) $X(j\omega) = \sum_{k=-\infty}^{\infty} (\frac{1}{2})^{|k|} \delta(\omega - \frac{k\pi}{4})$

4.8. Consider the signal

$$x(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ t + \frac{1}{2}, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 1, & t > \frac{1}{2} \end{cases}$$

(a) Use the differentiation and integration properties in Table 4.1 and the Fourier transform pair for the rectangular pulse in Table 4.2 to find a closed-form expression for $X(j\omega)$.

(b) What is the Fourier transform of $g(t) = x(t) - \frac{1}{2}$?

4.9. Consider the signal

$$x(t) = \begin{cases} 0, & |t| > 1 \\ (t + 1)/2, & -1 \leq t \leq 1 \end{cases}$$

(a) With the help of Tables 4.1 and 4.2, determine the closed-form expression for $X(j\omega)$.

(b) Take the real part of your answer to part (a), and verify that it is the Fourier transform of the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$?

BASIC PROBLEMS

4.21. Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t]u(t)$, $\alpha > 0$

(b) $e^{-3|t|} \sin 2t$

(c) $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT)$, $|\alpha| < 1$

(e) $[te^{-2t} \sin 4t]u(t)$

(f) $\left[\frac{\sin \pi t}{\pi t} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$

(g) $x(t)$ as shown in Figure P4.21(a)

(h) $x(t)$ as shown in Figure P4.21(b)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

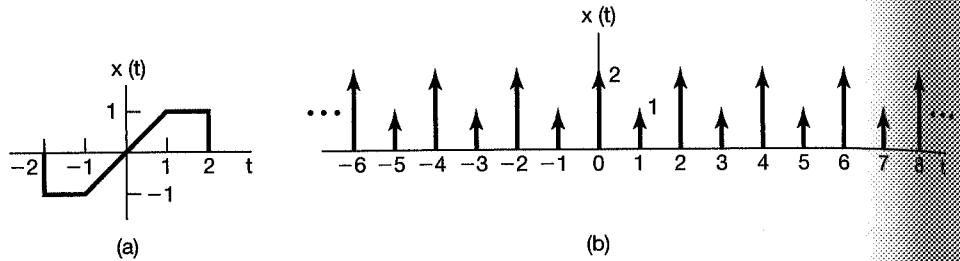


Figure P4.21

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

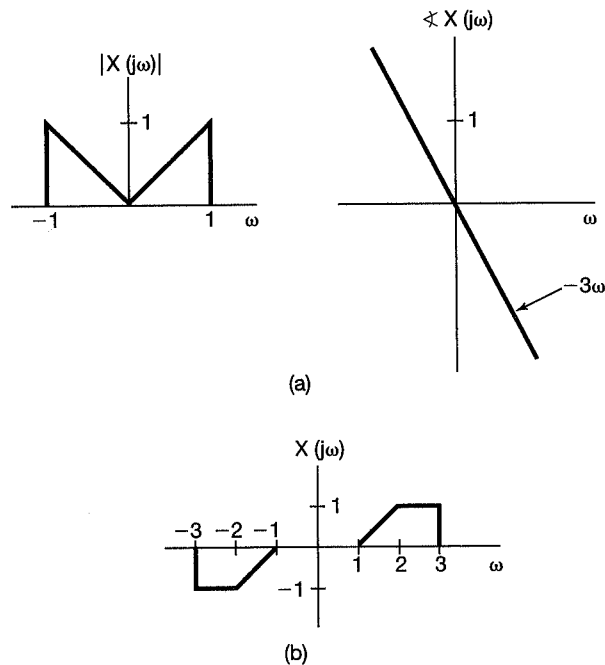


Figure P4.22

- (a) $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$
- (b) $X(j\omega) = \cos(4\omega + \pi/3)$
- (c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
- (d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
- (e) $X(j\omega)$ as in Figure P4.22(b)

4.23. Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure P4.23. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.

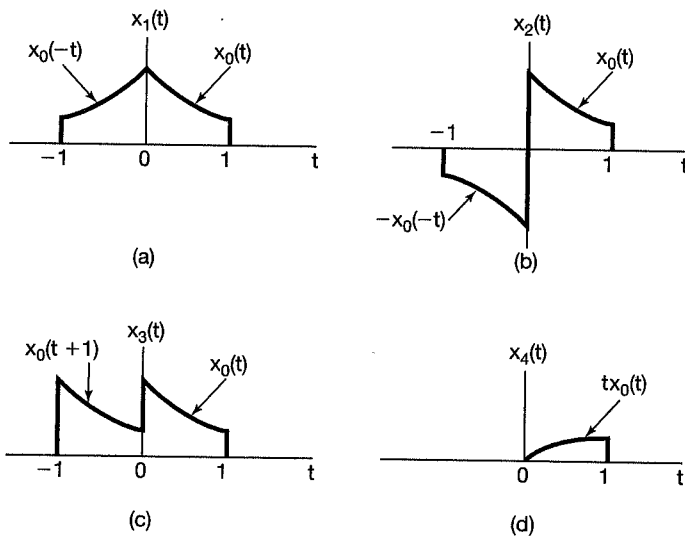


Figure P4.23

- 4.24. (a) Determine which, if any, of the real signals depicted in Figure P4.24 have Fourier transforms that satisfy each of the following conditions:
- (1) $\Re\{X(j\omega)\} = 0$
 - (2) $\Im\{X(j\omega)\} = 0$
 - (3) There exists a real α such that $e^{j\alpha\omega} X(j\omega)$ is real
 - (4) $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$
 - (5) $\int_{-\infty}^{\infty} \omega X(j\omega) d\omega = 0$
 - (6) $X(j\omega)$ is periodic
- (b) Construct a signal that has properties (1), (4), and (5) and does *not* have the others.