**(a)** Find the differential equation relating $x(t)$ and $y(t)$.
**(b)** Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.
**(c)** Determine the output $y(t)$ if $x(t) = \sin(t)$.

## BASIC PROBLEMS

**3.21.** A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1}^* = j,\ a_5 = a_{-5} = 2.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(w_k t + \phi_k).$$

**3.22.** Determine the Fourier series representations for the following signals:
**(a)** Each $x(t)$ illustrated in Figure P3.22(a)–(f).
**(b)** $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1$$

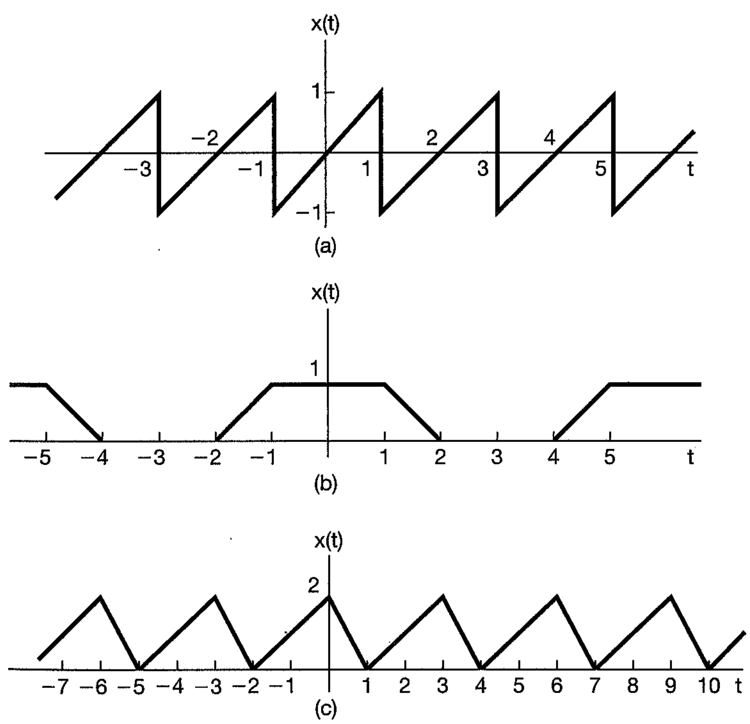


Figure P3.22

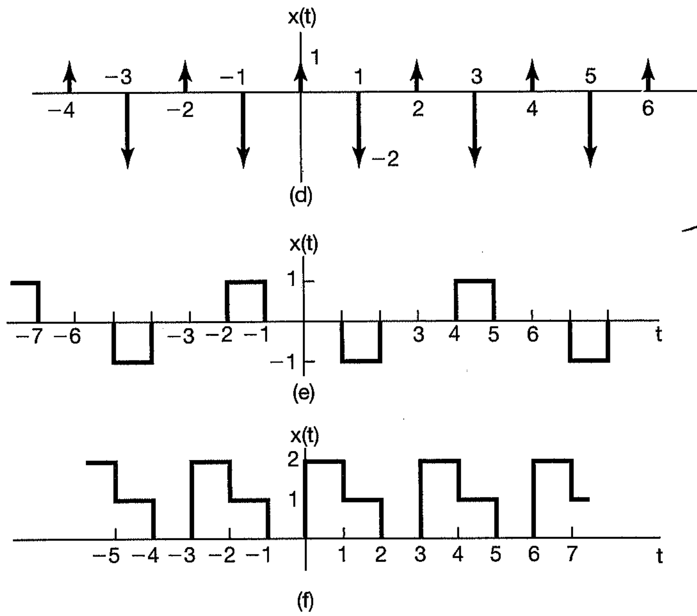


**Figure P3.22** *Continued*

**(c)** $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \leq t \leq 2 \\ 0, & 2 < t \leq 4 \end{cases}$$

**3.23.** In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

**(a)** $a_k = \begin{cases} 0, & k = 0 \\ (j)^k \frac{\sin k\pi/4}{k\pi}, & \text{otherwise} \end{cases}$

**(b)** $a_k = (-1)^k \frac{\sin k\pi/8}{2k\pi}, \quad a_0 = \frac{1}{16}$

**(c)** $a_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$

**(d)** $a_k = \begin{cases} 1, & k \text{ even} \\ 2, & k \text{ odd} \end{cases}$

**3.24.** Let

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2 - t, & 1 \leq t \leq 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients $a_k$.

**(a)** Determine the value of $a_0$.

**(b)** Determine the Fourier series representation of $dx(t)/dt$.

**(c)** Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

**3.25.** Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = \cos(4\pi t),$$
$$y(t) = \sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

**(a)** Determine the Fourier series coefficients of $x(t)$.
**(b)** Determine the Fourier series coefficients of $y(t)$.
**(c)** Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.
**(d)** Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

**3.26.** Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}.$$

Use Fourier series properties to answer the following questions:
**(a)** Is $x(t)$ real?
**(b)** Is $x(t)$ even?
**(c)** Is $dx(t)/dt$ even?

**3.27.** A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$$a_0 = 2, a_2 = a_{-2}^* = 2e^{j\pi/6}, \quad a_4 = a_{-4}^* = e^{j\frac{\pi}{3}}.$$

Express $x[n]$ in the form

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k).$$

**3.28.** Determine the Fourier series coefficients for each of the following discrete-time periodic signals. Plot the magnitude and phase of each set of coefficients $a_k$.
**(a)** Each $x[n]$ depicted in Figure P3.28(a)–(c)
**(b)** $x[n] = \sin(2\pi n/3)\cos(\pi n/2)$
**(c)** $x[n]$ periodic with period 4 and

$$x[n] = 1 - \sin\frac{\pi n}{4} \quad \text{for } 0 \le n \le 3$$

**(d)** $x[n]$ periodic with period 12 and

$$x[n] = 1 - \sin\frac{\pi n}{4} \quad \text{for } 0 \le n \le 11$$