(a) $x_1[n] = (\frac{1}{4})^n u[n+5]$
(b) $x_2[n] = \delta[n+3] + \delta[n] + 2^n u[-n]$
(c) $x_3[n] = (\frac{1}{2})^{|n|}$

10.20. Consider a system whose input $x[n]$ and output $y[n]$ are related by

$$y[n-1] + 2y[n] = x[n].$$

(a) Determine the zero-input response of this system if $y[-1] = 2$.
(b) Determine the zero-state response of the system to the input $x[n] = (1/4)^n u[n]$.
(c) Determine the output of the system for $n \geq 0$ when $x[n] = (1/4)^n u[n]$ and $y[-1] = 2$.

BASIC PROBLEMS

10.21. Determine the z-transform for each of the following sequences. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether or not the Fourier transform of the sequence exists.

(a) $\delta[n+5]$ **(b)** $\delta[n-5]$
(c) $(-1)^n u[n]$ **(d)** $(\frac{1}{2})^{n+1} u[n+3]$
(e) $(-\frac{1}{3})^n u[-n-2]$ **(f)** $(\frac{1}{4})^n u[3-n]$
(g) $2^n u[-n] + (\frac{1}{4})^n u[n-1]$ **(h)** $(\frac{1}{3})^{n-2} u[n-2]$

10.22. Determine the z-transform for the following sequences. Express all sums in closed form. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether the Fourier transform of the sequence exists.

(a) $(\frac{1}{2})^n \{u[n+4] - u[n-5]\}$ **(b)** $n(\frac{1}{2})^{|n|}$
(c) $|n|(\frac{1}{2})^{|n|}$ **(d)** $4^n \cos[\frac{2\pi}{6}n + \frac{\pi}{4}]u[-n-1]$

10.23. Following are several z-transforms. For each one, determine the inverse z-transform using both the method based on the partial-fraction expansion and the Taylor's series method based on the use of long division.

$$X(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}}, \quad |z| < \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{(1 - \frac{1}{2}z^{-1})^2}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{(1 - \frac{1}{2}z^{-1})^2}, \quad |z| < \frac{1}{2}.$$

10.30. Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[n+3] * x_2[-n+1]$$

where

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \quad \text{and} \quad x_2[n] = \left(\frac{1}{3}\right)^n u[n].$$

Given that

$$a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \quad |z| > |a|,$$

use properties of the z-transform to determine the z-transform $Y(z)$ of $y[n]$.

10.31. We are given the following five facts about a discrete-time signal $x[n]$ with z-transform $X(z)$:

1. $x[n]$ is real and right-sided.
2. $X(z)$ has exactly two poles.
3. $X(z)$ has two zeros at the origin.
4. $X(z)$ has a pole at $z = \frac{1}{2}e^{j\pi/3}$.
5. $X(1) = \frac{8}{3}$.

Determine $X(z)$ and specify its region of convergence.

10.32. Consider an LTI system with impulse response

$$h[n] = \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Determine the output $y[n]$ by explicitly evaluating the discrete convolution of $x[n]$ and $h[n]$.

(b) Determine the output $y[n]$ by computing the inverse z-transform of the product of the z-transforms of the input and the unit sample response.

10.33. **(a)** Determine the system function for the causal LTI system with difference equation

$$y[n] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] = x[n].$$

(b) Using z-transforms, determine $y[n]$ if

$$x[n] = \left(\frac{1}{2}\right)^n u[n].$$