Quiz 2

Name:

(you may leave your answers in the form of integrals)

In an experiment, we have two equally-likely scenarios (A and B). In scenario A, the pdf of an observed signal obeys the following

\[ f_y(y|A) = \frac{1}{\sqrt{2\pi}\sigma_A^2} \exp\left(-\frac{(y-m_A)^2}{2\sigma_A^2}\right) \]

If scenario B happens, then the observed signal obeys:

\[ f_y(y|B) = a e^{-ay}, y \geq 0 \]

1. What is the average signal level for scenario A?
2. What is the variance of the observed signal for scenario A?
3. What is the average signal level for scenario B?
4. What is the variance of the observed signal for scenario B?
5. What is the average signal level?
6. What is the pdf of y?

122) \[ E(y|A) = m_A; \ Var(y|A) = \sigma_A^2 = E(y^2|A) - (E(y|A))^2 \]

3) \[ E(y|B) = \int_0^\infty y e^{-\frac{ay}{\alpha}} dy = \frac{1}{a} = m_B \]

4) \[ E(y^2|B) = \int_0^\infty y^2 e^{-\frac{ay}{\alpha}} dy = \frac{2}{\alpha^2}; \ Var(y|B) = \frac{2}{\alpha^2} - \frac{1}{\alpha^2} = \frac{1}{\alpha^2} \]

5) \[ E[y] = \frac{1}{2} E(y|A) + \frac{1}{2} E(y|B) = \frac{m_A}{2} + \frac{1}{2\alpha} \]

6) \[ f_y(y) = P(A)f(y|A) + P(B)f(y|B) \]
   \[ = \frac{1}{\sqrt{2\pi\sigma_A^2}} e^{-\frac{(y-m_A)^2}{2\sigma_A^2}} + \frac{1}{2} a e^{-ay} \ u(y) \]
Quiz 1

Name:

Close-book and notes

(Show all work in the space provided and do not provide multiple answers. Cross out wrong answers)

A radio station switch board receives calls according to a Poisson process at the rate of 1 calls/sec. Then, each call with probabilities 0.8, 0.1, and 0.1 either gets answered, blocked, or sent to voicemail, respectively.

1. What is the probability of receiving 11 calls in 10 seconds?

\[ P(\lambda) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} = (10 \times 1)^{11} e^{-10} = \frac{10^{11}}{11!} e^{-10} \]

2. What is the probability that all 11 calls get answered?

\[ P(B|A) = \frac{11!}{11! \times 0 \times 0!} (0.8)^{11} (0.1)^0 (0.1)^0 \]

3. Given that 11 calls have been received in 10 seconds, what is the probability that 5 get answered, 4 are blocked, and 2 are sent to voicemail?

\[ P(C|A) = \frac{11!}{5! \times 4! \times 2!} (0.8)^5 (0.1)^4 (0.1)^2 \]

4. What is the joint probability of getting 11 calls in 10 seconds and having 5 of them answered, 4 blocked, and 2 sent to voicemail?

\[ P(C, A) = P(C|A)P(A) = \frac{11!}{5! \times 4! \times 2!} (0.8)^5 (0.1)^4 (0.1)^2 \]

\[ = \frac{10^{11}}{5! \times 4! \times 2!} \times \frac{10^{10}}{11!} e^{-10} \]