A linear system whose system function is $H[\omega]$ has $X[n] + V[n]$ as its input. Assume that $X[n]$ and $V[n]$ are two zero-mean, mutually uncorrelated Gaussian random sequences whose correlation functions are $R_x[n]$ and $R_v[n]$, respectively.

1. Find the output power spectrum of this system. Show all the work.
2. Give an expression for the pdf of the output sequence $Y[n]$ for all $n$. (you can leave your answer in the form of an integral).

\[ Z[n] = X[n] + V[n], \quad E[Z[n]] = 0 \]

$Z[n]$ is Gaussian

\[ R_Z[n] = R_X[n] + R_V[n] \]

\[ S_Z[\omega] = \sum_{n=-\infty}^{n=\infty} R_Z[n] e^{-j\omega n} \]

\[ S_Y[\omega] = S_Z[\omega] |H[\omega]|^2 \]

\[ \text{Power} = E[Y[n]^2] = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_Y[\omega] d\omega = P = R_X[0] \]

2) \[ f_Y(y; n) = \frac{1}{\sqrt{2\pi} \nu} \exp\left[ -\frac{y^2}{2\nu^2} \right] \]