Solution to Quiz 3
The Date

There are essentially 6 regions for $y$:

1. $y > a$, $P \{ Y \leq y \} = F_Y (y) = 1$
2. $y = a$, $F_Y (a) = P \{ X \geq a \} = 1 - F_X (a)$
3. $0 < y \leq a$; $F_Y (y) = F_X (a)$
4. $y = 0$; $F_Y (0) = F_X (a) - F_X (-b)$
5. $-b < y \leq 0$; $F_Y (y) = F_X (-b)$
6. $y < -b$; $F_Y (y) = F_X (y)$

Notice that the inverse function has two discontinuities; one at $y = 0$ and one at $y = a$. Hence, we have two delta functions at those points. That is,

- $y > a$, $f_Y (y) = 0$
- $y = a$, $f_Y (a) = P \{ X \geq a \} = (1 - F_X (a)) \delta (y - a)$
- $0 < y \leq a$; $f_Y (y) = 0$
- $y = 0$; $f_Y (y) = \{ F_X (a) - F_X (-b) \} \delta (y)$
- $-b < y \leq 0$; $f_Y (y) = 0$
- $y < -b$; $f_Y (y) = f_X (y)$